ABSTRACT

This research focuses on an inventory optimization problem with uncertain demand. The sensitivity analysis of the inventory parameters is conducted to provide managerial insights for inventory managers. The numerical example demonstrates the performance of inventory management under different scenarios. Higher variation of demand results in higher costs in most scenarios.

KEYWORDS: Sensitivity analysis, Inventory management, Demand uncertainty, Supply chain management (SCM)

INTRODUCTION

Supply chain management (SCM) is a complicated task involving coordination and decision making across organizational boundaries. In the past, management would concentrate on making each entity of a supply chain efficient. It has been realized now that efficiency at each entity does not result in the supply chain as a whole operating optimally. Different entities along a supply chain typically operate subject to different sets of constraints and objectives (Strader and Lin, 2001).

In recent years, it has become clear that many companies have reduced manufacturing costs as much as practically possible. Many of these companies are discovering the magnitude of savings that can be achieved by planning and managing their supply chain more efficient and effective. Nowadays, new technology provides access to comprehensive data from all components of the supply chain. Current trend of improving information sharing and decision making in supply chain is to implement supply chain integration. Unfortunately, supply chain integration is difficult for two main reasons. One is that different entities along in the supply chain may have different, conflicting, objectives. For instance, the manufacturers’ objective of making large production batches typically conflicts with the objective of warehouse and distribution centers to reduce inventory. The other reason is that supply chain is a dynamic system that evolves uncertainty over time. In fact, both customer demands and supplier capabilities keep changing over time (Simchi-Levi et al, 2000).

Classic inventory theory tells us that in order to increase customer service level, the firm must increase inventory which will lead to extra costs. Surprisingly, recent developments in information and communications technologies together with a better understanding of supply chain strategies, have led to various innovative approaches that allow the firm to improve both objectives, better service and lower inventory, simultaneously. The complexity of supply chain is the main obstacle to prevent us to go further from current limited information sharing stage to manage or optimize the supply chain system as a whole (Simchi-Levi et al, 2000).

In this research, we focus on demand uncertainty. More specifically we conduct a research on sensitivity analysis of inventory management parameters, such as inventory holding cost, ordering
cost, the feature of demand functions, and their effects on supply chain inventory decision making processes.

THE MODEL

We first present the models each supply chain entity used to management their own profits. A multi-period supply chain model developed for testing the effects of demand uncertainty and sensitivity analysis.

Basic Model

We assume the demand is affected by the selling (market) price determined by the buyer (manufacturer). The price-elasticity demand function determines the mean of the expected demand,

\[ m(p) = d - ap, \quad d, a > 0 \quad d/a \geq p \geq 0 \]

The supplier’s profit function at the period \( t \) is as follows.

\[ SUP(Q_t) = (w_t - c)Q_t - eQ_t^2 \]

\[ (w_t - c)Q_t \]

is the supplier’s profit from selling the products to the buyer and extracting the basic production cost. \( eQ_t^2 \) is the supplier’s cost associated with diseconomies such as scheduling problems, productivity loss due to congestion, overtime premium, etc. The order quantity \( Q_t \) can be described as the function of expected demand, \( Q_t = k_t(\theta)m_t(p_t) \). The buyer’s expected profit is described as follows.

\[ BUP(Q_t) = \int_0^{Q_t + I_t} p_t uf_t(u)du + \int_{Q_t + I_t}^{\infty} p_t (Q_t + I_t)f_t(u)du - w_t Q_t \]

The first term is the buyer’s expected profit when the demand is less than \( Q_t + I_t \). The second term is the buyer’s expected profit when the demand is greater than \( Q_t + I_t \). The third term is the purchasing costs. The holding cost for overstock and the penalty for the shortage are described as follows.

\[ L(Q_t) = h \int_0^{Q_t + I_t} (Q_t + I_t - u)f_t(u)du + \pi \int_{Q_t + I_t}^{\infty} (u - Q_t - I_t)f_t(u)du \]

Supply Chain Model

We propose a multiple-period model for a one supplier and one buyer supply chain with dynamic and price-sensitive demand. We define the profit for the whole supply chain at period \( t \) as \( TSP(Q_t) \) below. In the last period, the unit salvage price is \( r \) for the overstock product instead of a holding cost.
Moreover, for any missed demand, a shortage cost $\pi$ will occur. Thus, the total profit in $N$ periods for the supply chain is a function of order quantities, $Q_i$, for each period.

$$\max_{Q_i} \sum_{i=1}^{N} TSP(Q_i) = BUP(Q_i) + SUP(Q_i) - L(Q_i)$$

$$= \sum_{i=1}^{N} \left[ \int_{0}^{Q_i} p_i f_i(u) du + \int_{Q_i}^{\infty} p_i (Q_i + I_i)f_i(u) du - c Q_i - c Q_i^2 \right]$$

$$- h \int_{0}^{Q_i} (Q_i + I_i - u) f_i(u) du - \pi \int_{Q_i}^{\infty} (u - Q_i - I_i) f_i(u) du$$

$$= \sum_{i=1}^{N} \left[ \int_{0}^{Q_i} p_i f_i(u) du + \int_{Q_i}^{\infty} p_i (Q_i + I_i)f_i(u) du - c Q_i - c Q_i^2 \right]$$

$$- h \int_{0}^{Q_i} (Q_i + I_i - u) f_i(u) du - \pi \int_{Q_i}^{\infty} (u - Q_i - I_i) f_i(u) du$$

(4)

In order to decide the transfer price ($w^*$) between the supplier and the buyer, we first calculate the optimal order quantities from the supplier and buyer’s perspective separately. Both parties will uniquely reach an agreement only when the order quantity that the buyer desires to place is equal to the quantity the supplier wants to provide. Then we can get the optimal transfer price to enforce the supplier and the buyer to follow the supply chain optimal solution we derive aforementioned. For example, the supplier’s profit described in Eq. (1) and the optimal quantity form supplier’s perspective to maximize his profit can be solved below. We derivative the supplier’s profit

$$\frac{dSUP(Q_i)}{dQ_{i,s}} = w_i - c_i - 2c_i Q_i =0$$

$$Q_{i,s}^* = \frac{w_i - c_i}{2c_i}, \text{ hence, (5)}$$

is the supplier’s optimal order quantity. Similar to the above analysis, we can get the optimal order quantity from the buyer’s perspective.

$$\max_{Q_i} \quad$$

$$- h \int_{0}^{Q_i} (Q_i + I_i - u) f_i(u) du - \pi \int_{Q_i}^{\infty} (u - Q_i - I_i) f_i(u) du$$

(6)

**NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS**

Consider a numerical example for the discussion presented in previous sections. We assume $d = 1000$, $a = 20$, $e = 0.01$, $\theta = 95\%$, $h = 2$, $\pi = 10$, $c = 5$, and $\sigma = \mu/10$. We assume the demand follows normal distribution and $\mu$. We get,

$$\frac{dTSP(Q_i)}{dQ_i} = 0$$

$$Q_i^* = \frac{37.53 - 0.073534764I_i}{0.93534764}, \text{ hence, (7)}$$
In the price-elastic demand function, the expected demand \( m(p) \) is defined as a linear function with intersection parameter \( d \) and slop \( a \),

\[
m(p) = d - ap \quad (d, a > 0, d/a > p)
\]

The intersection and slop are obtained based on usually linear regression analysis of historical data. What we find out is that the accuracy of the linear fitting has major influence to the system inventory level and the total cost. Figure 1 presented three cases for different \( d \) values with same value for \( a \).

![Figure 1: Price-elastic demand vs. sales price](image1)

Figure 2 illustrates the relationship between optimal order quantity and the parameter \( d \). We can see the optimal order quantity is a linear increasing function of parameter \( d \).

![Figure 2: System profit vs. order quantity with different parameter \( d \)](image2)

Figure 3 shows three price-elastic demand curves with different values of \( a \).

\[
a = -\frac{dm(p)}{dp}
\]

Figure 3: Price-elastic demand vs. sales price with different parameter \( a \).
Parameter \( e \) denotes the supplier’s cost associated with diseconomies such as scheduling problems, productivity loss due to congestion, overtime premium, etc. We illustrate the relationship between expected system profit and order quantities with different \( e \) values. In Figure 4 below, we can see that both the optimal order quantity and the expected supply chain profit are higher with smaller \( e \). We derive the equation to express the relationship between supply chain profit and the value of \( e \).

\[
TSP(Q_t) = -0.36767 * Q_t^2 - e * Q_t^2 + 37.53 * Q_t
\]

(9)

\( \theta \) is the predefined minimal customer service level. We described optimal order quantity as a function of expected demand,

\[
Q_t = k(\theta)m(p_t) = m(p_t)
\]

Where \( m(p_t) \) is the price-elasticity demand function for period \( t \) and \( d_t, a_t > 0, \ d_t / a_t \geq p_t \geq 0 \). Since both service level \( \theta \) and demand deviation \( \sigma \) are related to a function of \( k(\theta) \), we consider the sensitivity analysis for \( \theta \) and \( \sigma \) together. Firstly, we derive the equation of the optimal sales price to service level under the assumption defined at the beginning of this Chapter. We also illustrate several optimal sales prices vs. service level curves under different standard deviation level of demand distribution in Figure 5.

\[
p^*_t = 50 - \frac{37.53}{1.71336 + 0.4k(\theta)}
\]

(10)

Figure 4: Expected supply chain profit vs. order quantity with different \( e \)
From the above figure, we can see the optimal sales price increases with the increasing of the service level under various standard deviation level. The reason is that the higher service level will require more inventory. Therefore, the total holding cost will increase. If the cost increases, the sales price will increase with it. Due to the assumption of price-elastic demand, the expected demand will decrease when the sales price increases. Figure 6 illustrates the relationship between optimal order quantity and the service level.

\[ Q^*_t = \frac{37.53 * k(\theta)}{0.85668 + 0.02 * k(\theta)} \]  

(11)
From Figure 6 and Eq. (11), we can see although the increasing of sales price causes the demand decreasing, the optimal order quantity still increases with the increasing of service level. In turn the expected supply chain profit will increases with the increasing of the service level. We derive the equation of maximum expected system profit and the service level in Eq. (12) and illustrate the relationship between them in Figure 7.

\[
TSP(Q_t) = -\frac{0.85668}{20* k(\theta)} * Q_t^2 - 0.01 * Q_t^2 + 37.53 * Q_t
\]

(12)

Figure 7: Supply chain profit vs. service level with different demand standard deviation

The larger standard deviation of demand causes a wider variant range for optimal order quantity, optimal unit sales prices and maximum expected system profit. We illustrate this idea discussed above in Figure 8.

CONCLUSION

This study examines the changes of supply chain performance with the changes of several cost and other business environment parameters. The findings offer managers information about which direction they need to move when the business environment changed. The sensitivity analysis conducted in this study extends the flexibility of classic supply chain models on inventory management and production planning. We find that higher variation of demand results in higher supply chain costs in most scenarios.

REFERENCES

References available upon request.