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Coordination Mechanisms in Supply Chain under Asymmetric Quality Information

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ABSTRACT
This paper studies three supply chain contracts with asymmetric supplier quality information  
employing game theory: optimal contracts in centralized control setting, profit margin contract  
and profit sharing contract in decentralized control setting. The numerical analyses show that  
profit sharing contract is a coordination mechanism.

KEYWORDS: Supply Chain Contract, Game Theory, Asymmetric Information

INTRODUCTION
Research in quality management has traditionally often focused on internal enterprise process  
competition now is not only found at the firm level, but also exists as supply chains. This level of  
competition requires a much greater level of coordination among suppliers, distributors,  
producers, and customers.

LITERATURE REVIEW
Liu and Wang (2015) developed a quality control game model for logistics service supply chain  
and analyzed the impact of different risk attitude of a logistics service integrator and a functional  
logistics service provider. Yan et al. (2015) analyzed the first-mover right in quality contracting  
by considering two different strategies for the buyer: the quality requirement (QR) strategy and  
quality promise (QP) strategy. They found that QP always led to the first-best quality efforts  
from the supplier(s) while QR limits their efforts. Ma et al. (2013) investigated the issue of  
channel coordination for a two-stage supply chain with one retailer and one manufacturer. They  
identified the optimal level of retail sales effort, optimal level of quality-improvement effort and  
optimal supply chain profit.
Supply chain contracts are considered as a useful tool to structure the costs and rewards of all  
of its members so as to achieve coordination in a decentralized situation. Leng and Parlar (2009)  
developed a simple profit-sharing contract in a two-level supply chain involving a manufacturer  
and a retailer to achieve the maximum system-wide profit. Li et al. (2015) considered a supply  
chain with two heterogeneous suppliers and a common retailer whose type is either low-volume  
or high-volume, and characterized the equilibrium contract menus offered by the suppliers to the  
retailer.
In related works on supply chain coordination under asymmetry information research literature,  
Gümü et al. (2013) considered a supplier who offers a revenue sharing contract to two  
competing retailers, one of whom has private information about uncertain market potential and  
orders first. Li et al. (2012) analyzed two supply chain inventory models (the vendor has  
complete information about the buyer’s cost structure, and the buyer possesses private cost
information), they designed the coordination mechanism by using principal agent model to induce the buyer to report his true cost structure.

Coordination Mechanisms in Centralized Control Setting Under Asymmetric Supplier Quality Information

We consider the standard setting with a single supplier and single manufacturer who sells the supplier’s product to the final market. Manufacturer orders from supplier according to market demand $D$. The situation is described as follows. The market demand function is given by linear demand function in quality and price: $D(p, x) = \alpha - \beta p + \varepsilon x$, where $\alpha > 0$, $\beta > 0$ and $\varepsilon > 0$ are known parameters. The supplier’s variable costs are $s$, $s = \lambda + \delta x$. $\lambda$ denotes the variable production cost, $x$ denotes the supplier quality level, the unit variable cost increases (decreases) by $|\delta x|$. The manufacturer’s internal variable costs are $m$, the wholesale price is $w$, and the retail price selling to the customer is $p$. The manufacturer’s order quantity is $Q$, without loss of generality, we assume that the supplier follows a lot-for-lot policy, i.e., the supplier’s production lot size is equal to the lot size shipped to the manufacturer. Let $\pi$ and $\Pi$ denote the profit and the expected profit respectively.

In general, the manufacturer doesn’t know the supplier’s quality level $x$; we assume the manufacturer holds a prior distribution function $H(x)$ with continuous density function $h(x)$, and mean value of $\mu$. $H(x)$ is differentiable, strictly increasing and is defined on $[x, \bar{x}]$, where $x \in [0, \infty)$. Let $H(0) = 0$ and $\bar{H}(x) = 1 - H(x)$. All parameters except $x$ are common knowledge.

We first investigate the centralized situation where one central decision maker seeks to maximize total system profits. Then the supply chain system profit can be written as

$$\Pi(p) = \int_{x}^{\bar{x}} \left[ (\alpha - \beta p + \varepsilon x)(p - s - m) \right] h(x) dx.$$  

From the first optimality condition $\frac{\partial \Pi(p)}{\partial p} = 0$, we obtain Lemma1.

**Lemma1.** The optimal contract in centralized control setting under asymmetric supplier quality information is following.

The optimal retail price is

$$p^c = \frac{\alpha + \beta (\lambda + m) + (\varepsilon + \beta \delta) \mu}{2\beta}$$  \hspace{1cm} (1)

The optimal order quantity is

$$Q^c = \frac{\alpha - \beta (\lambda + m) + (\varepsilon - \beta \delta) \mu}{2}$$  \hspace{1cm} (2)

Therefore, the maximum supply chain expected profit is given by

$$\Pi^c = \frac{\left[ \alpha - \beta (\lambda + m) + (\varepsilon - \beta \delta) \mu \right]^2}{4\beta}$$  \hspace{1cm} (3)
Coordination Mechanisms in Decentralized Control Setting Under Asymmetric Supplier Quality Information

A. Profit Margin Contract (no coordination situation)

Just as a dominated supplier will declare a unit wholesale price $w$, the economics and marketing literature has long recognized that a dominated manufacturer can declare a required profit margin. The manufacturer’s profit function can be written as: $\pi_m = Q(p - w - m)$. After the dominant manufacturer declares his required profit margin $M$, i.e., $M = \pi_m / Q$, the supplier knows that, for whatever she quotes, the unit retailer price will be $p = w + m + M$. Hence, the supplier’s profit function is $\pi_s(w) = \left[\alpha - \beta(w + m + M) + \epsilon x \right](w - s)$. From the first optimality condition $\frac{\partial \pi_s(w)}{\partial w} = 0$, we obtain the optimal wholesale price

$$w(M) = \frac{\alpha + \beta(\lambda - m - M) + (\epsilon + \beta \delta) x}{2\beta}.$$  

Substituting $w(M)$ into the manufacturer’s expected profit function $\Pi_m(M) = \int \pi_m(M) h(x) dx$, solving $\frac{\partial \Pi_m(M)}{\partial M} = 0$, we obtain optimal profit margin $M^d$. By calculating the above function, we can derive Lemma 2.

**Lemma 2.** The profit margin contract in decentralized control setting under asymmetric supplier quality information is following.

The optimal profit margin is

$$M^d = \frac{\alpha - \beta(\lambda + m) + (\epsilon - \beta \delta) \mu}{2\beta} \quad (4)$$

The optimal wholesale price is

$$w^d = \frac{\alpha + \beta(3\lambda - m) + (\epsilon + 3\beta \delta) \mu}{4\beta} \quad (5)$$

The optimal manufacturer price is:

$$p^d = \frac{3\alpha + \beta(\lambda + m) + (3\epsilon + \beta \delta) \mu}{4\beta} \quad (6)$$

The optimal order quantity is:

$$Q^d = \frac{\alpha - \beta(\lambda + m) + (\epsilon - \beta \delta) \mu}{4} \quad (7)$$

The manufacturer’s expected profit is:

$$\Pi_m^d = \frac{\left[\alpha - \beta(\lambda + m) + (\epsilon - \beta \delta) \mu\right]^2}{8\beta} \quad (8)$$

The supplier’s expected profit is:

$$\Pi_s^d = \frac{\left[\alpha - \beta(\lambda + m) + (\epsilon - \beta \delta) \mu\right]^2}{16\beta} \quad (9)$$

The supply chain system’s expected profit is:
\[
\Pi^d = \frac{3(\alpha - \beta (\lambda + m) + (\varepsilon - \beta \delta) \mu)^2}{16 \beta} 
\]  

(10)

By calculating, we can obtain \( \Pi^d < \Pi^c \), so supply chain don’t achieve coordination in profit margin contract.

**B. Profit Sharing Contract (coordination situation)**

We now design a profit sharing contract to coordinate the supply chain. Under the profit sharing contract, let \( \psi \) be the fraction of channel expected profit the retailer keeps, \( \psi \in [0,1] \), so \( 1 - \psi \) is the fraction the supplier earns. The manufacturer’s expected profit is \( \Pi^p_m = \psi \Pi^p \), and the supplier’s expected profit is \( \Pi^p_s = (1 - \psi) \Pi^p \). With coordination, the system expected profit is \( \Pi^c = \Pi^c \), the optimal order quantity is \( Q^c = Q^c \), and the optimal retail price is \( p^c = p^c \).

The manufacturer’s expected profit function can be written as \( \Pi^m = E[Q \cdot M] \), it also can be written as \( \Pi^m = E[Q^c \cdot M^c] = \psi \Pi^c \). By calculating the above function, we can derive Lemma3.

**Lemma3.** The profit sharing contract in decentralized control setting under asymmetric supplier quality information is following.

The optimal profit margin is:

\[
M^p = \frac{\psi \left[ \alpha - \beta (\lambda + m) + (\varepsilon - \beta \delta) \mu \right]}{2 \beta} 
\]

(11)

The optimal wholesale price is:

\[
w^p = \frac{(2 - \psi) (\alpha - \beta m) + (2 + \psi) \beta \lambda + \left[ (2 - \psi) \varepsilon + (2 + \psi) \beta \delta \right] \mu}{4 \beta} 
\]

(12)

The optimal manufacturer price is:

\[
p^c = \frac{\alpha + \beta (\lambda + m) + (\varepsilon + \beta \delta) \mu}{2 \beta} 
\]

(13)

The optimal order quantity is:

\[
Q^c = \frac{\alpha - \beta (\lambda + m) + (\varepsilon - \beta \delta) \mu}{2} 
\]

(14)

The manufacturer’s expected profit is:

\[
\Pi^m = \frac{\psi \left[ \alpha - \beta (\lambda + m) + (\varepsilon - \beta \delta) \mu \right]^2}{4 \beta} 
\]

(15)

The supplier’s expected profit is:

\[
\Pi^s = \frac{(1 - \psi) \left[ \alpha - \beta (\lambda + m) + (\varepsilon - \beta \delta) \mu \right]^2}{4 \beta} 
\]

(16)

The supply chain system’s expected profit is:

\[
\Pi^p = \frac{\left[ \alpha - \beta (\lambda + m) + (\varepsilon - \beta \delta) \mu \right]^2}{4 \beta} 
\]

(17)
Results

In this section, we give several numerical examples to analyze the effects of the supplier quality level on the order quantity and supply chain system’s expected profit. Let $\alpha = 150$, $\beta = 5$, $\epsilon = 10$, $\lambda = 10$, $\delta = 1$. We assume the value of $\mu$ varies from 1 to 10.

1. Impact on optimal order quantity

Fig. 1 illustrates the impact of varying $\mu$ on the optimal order quantity. From this figure, we can see that optimal order quantity is a linearly increasing function of $\mu$.

Comparing the optimal order quantity in profit margin contract and profit sharing contract in decentralized control setting in Fig. 1, the latter is more than the former. This implies that latter is a more optimal and adaptable instrument for channel coordination.

2. Impact on system’s expected profit

Fig. 2 illustrates the impact of varying $\mu$ on the supply chain system’s expected profit. From this figure, we can see that the system’s expected profit is a non-linearly increasing function of $\mu$.

Comparing the system’s expected profit in profit margin contract and profit sharing contract in decentralized control setting in Fig. 2, the latter is more than the former. This implies that latter is a more optimal and adaptable instrument for channel coordination.

![Fig.1 $\mu$ versus optimal order quantity](image1.png)  
![Fig.2 $\mu$ versus system's expected profit](image2.png)

Fig. 1 $\mu$ versus optimal order quantity  
Fig. 2 $\mu$ versus system's expected profit

Conclusion

In this paper, we have investigated coordination mechanisms in a supplier-manufacturer supply chain in centralized and decentralized setting under asymmetric supplier quality information. From the analysis above, we can draw a conclusion that the profit sharing contract could achieve supply chain channel coordination in decentralized setting, besides, the order quantity and the expected profit are increasing functions of supplier quality level.

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