ABSTRACT

Based on a linear deterministic demand model, the profit function of a bivariate quadratic form is formulated in this paper to determine the optimal pricing scheme for advance and spot selling for a service provider in a monopolistic environment. The impacts of price sensitivities on the optimal pricing scheme are analytically examined. A numerical study is conducted to demonstrate the superiority of the optimal pricing scheme in two experimental scenarios.

KEYWORDS: Pricing, Advance selling, Yield management, Optimization

INTRODUCTION

Advance selling occurs when sellers allow buyers to make purchases at a time preceding consumption (Xie & Shugan, 2001). The practice of advance selling is a well-known phenomenon in service industries. Thanks to technological advances such as electronic tickets and online prepayment, service providers sell their service capacity to consumers not only at the time of consumption but also in advance. For example, airlines may sell seats on a flight several months in advance and at the time of the flight’s departure; hotels may sell room nights a few weeks in advance and at the time of check-in as well.

The concept of yield management is closely related to the practice of advance and spot selling (see, Lee & Ng, 2001; Zhang & Mesak, 2010; Zeng, 2013). The primary goal of yield management is to maximize revenues by selling capacity at optimal prices. Several notable studies are relevant to our paper in the context of yield management. Xie and Shugan (2001) propose the pricing strategies for advance selling under certain conditions of service capacity. In another study based on a two-period model, Shugan and Xie (2005) explore the impact of competition on advance selling driven by consumer uncertainty about future consumption states and conclude that advance selling can be a very effective marketing tool in a competitive setting. Also using a two-period model, Png (1989) argues that a service reservation provides insurance for risk-averse buyers against the uncertainty in service valuation and unavailability of service capacity. Png (1991) further suggests that service providers should practice advance selling to maximize profits through premium advance prices and a discount price for the remaining capacity as the time of consumption approaches. Lee and Ng (2001), incorporating price sensitivity in their modeling framework, analytically determine the optimal allocation of service
capacity over a two-period planning horizon and corresponding pricing strategies for a monopolistic service provider. Based on a two-period model of advance selling in a market composed of experienced and inexperienced consumers, Zeng (2013) develops multiple pricing strategies for a typical retailer. In addition to the above papers that are of general orientation, some related studies focus on certain service industries. Ladany (1996), using a dynamic programming approach, presents a market segmentation strategy that optimizes the number of market segments, the corresponding prices, and the number of hotel rooms allocated to each segment. Ladany and Arbel (1991) determine for a cruise liner the optimal segmentation of total and unused capacities for certain cases. These notable studies shed interesting lights on the issue of service capacity allocation and pricing.

Our paper is in the spirit of the studies by Lee & Ng (2001) and Zeng (2013), but different in three significant ways. First, in our paper the market served by a monopolistic service provider is divided into two segments — the segment for advance selling and the other for spot selling. A linear bivariate demand function is employed to model the demand of each segment dictated by both the advance and spot selling prices. Second, the necessary and sufficient conditions are provided for the service provider to set the optimal advance and spot selling prices. Third, a numerical study is conducted to demonstrate the superiority of the optimal pricing scheme in two experimental scenarios.

In the next section, we model the aggregate demand of each segment in the market and then formulate the profit function. The optimal advance and spot selling prices are analytically determined in the third section. The fourth section presents the results of the numerical study conducted for nine pricing schemes. Finally, the paper concludes with a summary of its findings, managerial implications, limitations, and directions for future research in the fifth section.

MODEL FORMULATION

Consider a service provider in a monopolistic environment, who must determine the prices of the service capacity allocated for advance and spot selling, respectively. The market served by the service provider is divided into two segments: one composed of the consumers who purchase the service capacity in advance and the other of those who make their purchases at the time of consumption.

The problems that we intend to solve in this paper can be specifically stated as follows: (i) What is the best pricing strategy in a monopolistic environment to price the service capacities for advance and spot selling so that the service provider’s total profit will be maximized? (ii) What are the impacts of the price sensitivities of each segment on the optimal advance and spot selling prices?

We make the following basic assumptions while addressing the two strategic issues stated above:

(i) All the consumers in the two-segment market are well aware of the advance and spot selling prices charged by the service provider.

(ii) The aggregate demand of each segment is affected by the advance and spot selling prices.

(iii) The service provider can fully satisfy the aggregate demand of each segment.

To improve exposition, the segment for advance selling is denoted as Segment 1 and that for spot selling as Segment 2, respectively. In general, a service provider operates with a high fixed
cost, $C$, which is much higher than the variable cost of capacity. As in the study of Lee and Ng (2001), we only consider the case in which $C$ is a constant exogenously determined and variable costs are sufficiently small to be ignored. Examples of such a case can be found in the airline and hotel industries (Desiraju & Shugan 1999). Several terms used to model the demand of each segment and formulate the profit function are defined below:

- $\pi$ the service provider’s total profit in the entire market;
- $P_i$ the price of a unit of capacity charged to Segment $i$ by the service provider;
- $d_i$ the aggregate demand of Segment $i$ $(i = 1, 2)$.

A linear form of the demand function is extensively employed in both theoretical and empirical studies (see Lee & Ng, 2001, for a review). Zufryden (1975) stresses that linear formulations are more appealing because of the relative ease of parameter estimation through classical statistical methods. As it is assumed that the aggregate demand of each segment is affected by both the advance and spot selling prices, we thus employ a linear demand model introduced by Huang et al. (2012) to model the aggregate demand $d_i$ $(i = 1, 2)$ in our study:

$$d_i = \alpha_i - \beta_i P_i + \beta_{12} P_2,$$  \hspace{1cm} (1)

$$d_2 = \alpha_2 - \beta_2 P_2 + \beta_{21} P_1,$$  \hspace{1cm} (2)

where, $\alpha_i$, $\alpha_2$, $\beta_1$, $\beta_2$, $\beta_{12}$, $\beta_{21} > 0$.

For $i = 1, 2$, the constant $\alpha_i$ captures the part of the aggregate demand of Segment $i$ that does not vary with the advance and spot selling prices (i.e., $P_1$ and $P_2$); $\beta_i$ measures the price sensitivity of demand of Segment $i$ to changes in the price charged to the same segment, $P_i$. $\beta_{12}$ measures the price sensitivity of demand of Segment 1 to changes in the price charged to Segment 2, $P_2$. Similarly, $\beta_{21}$ measures the price sensitivity of demand of Segment 2 to changes in $P_1$.

Expression (1) shows that the aggregate demand of Segment 1, $d_1$, is decreasing in $P_1$ but increasing in $P_2$. In contrast, the aggregate demand of Segment 2, $d_2$, is decreasing in $P_2$ but increasing in $P_1$, as noted in expression (2). These functional relationships could be found in some service industries. For example, given the advance selling price of a seat on a flight remaining unchanged, an increase in the spot selling price could lead more air travelers to purchase the flight tickets in advance. On the other hand, if the spot selling price is unchanged but the advance selling price is rising, more air travelers would likely purchase the tickets when the time for departure is near.

It was assumed earlier that the aggregate demand of each segment can be fully satisfied by the service provider. If she allocates her service capacity exactly matching the demand of each segment, her total profit earned in the two-segment market is given by expression (3):

$$\pi = P_1 d_1 + P_2 d_2 - C.$$  \hspace{1cm} (3)

Substituting expressions (1) and (2) in (3) yields:

$$\pi = \alpha_1 P_1 - \beta_1 P_1^2 + \beta_{12} P_1 P_2 + \alpha_2 P_2 - \beta_2 P_2^2 + \beta_{21} P_2 P_1 - C.$$  \hspace{1cm} (4)
Expression (4) shows that the service provider’s total profit, $\pi$, is a bivariate quadratic function of the advance and spot selling prices, $P_1$ and $P_2$. Based on (4), we employ an approach of differential calculus to find out the optimal advance and spot selling prices, $P_1^*$ and $P_2^*$, at which the service provider’s total profit is maximized.

OPTIMAL ADVANCE AND SPOT SELLING PRICES

In this section, we first determine the optimal advance and spot selling prices in closed forms, and subsequently examine the impacts of changes in the four types of price sensitivity on the service provider’s optimal selling prices. Five propositions are introduced below for which the proofs are found in the Appendix.

Proposition 1. Given $4\beta_1\beta_2 - (\beta_{12} + \beta_{21})^2 > 0$, the service provider’s total profit, $\pi$, reaches its maximal level at

$$P_1^* = \frac{\alpha_1(\beta_{12} + \beta_{21}) + 2\alpha_2\beta_1}{4\beta_1\beta_2 - (\beta_{12} + \beta_{21})^2},$$

and

$$P_2^* = \frac{\alpha_2(\beta_{12} + \beta_{21}) + 2\alpha_2\beta_2}{4\beta_1\beta_2 - (\beta_{12} + \beta_{21})^2}.$$

Proposition 2. The optimal price for advance selling, $P_1^*$, is monotonically decreasing in $\beta_1$ and $\beta_2$, respectively.

Proposition 3. The optimal price for advance selling, $P_1^*$, is monotonically increasing in $\beta_{12}$ and $\beta_{21}$, respectively.

Proposition 4. The optimal price for spot selling, $P_2^*$, is monotonically decreasing in $\beta_1$ and $\beta_2$, respectively.

Proposition 5. The optimal price for spot selling, $P_2^*$, is monotonically increasing in $\beta_{12}$ and $\beta_{21}$, respectively.

Proposition 1 provides the necessary and sufficient conditions for the service provider to optimize her total profit in the two-segment market. It is noted in expressions (5) and (6) that the optimal advance and spot selling prices, $P_1^*$ and $P_2^*$, are determined by the price sensitivities of each segment and the two constants, $\alpha_1$ and $\alpha_2$, as well. With optimal prices $P_1^*$ and $P_2^*$ being established, the service provider can determine the demand of each segment, $d_i^*$ ($i = 1, 2$) by substituting $P_1^*$ and $P_2^*$ in expressions (1) and (2), respectively. If she sells the service capacity exactly at the levels of $d_1^*$ and $d_2^*$, the total profit ($\pi$) will reach the optimal level, $\pi^* = P_1^*d_1^* + P_2^*d_2^* - C$.

Propositions 2 – 5 offer guidelines for the service provider to fine-tune the optimal pricing scheme for advance and spot selling as the price sensitivities change. For example, as the price sensitivity $\beta_1$ or $\beta_2$ increases, the service provider should lower both the advance and spot
selling prices. In contrast, she should raise advance and spot selling prices if the price sensitivity $\beta_{12}$ or $\beta_{21}$ goes up.

**NUMERICAL ILLUSTRATIONS**

A numerical study is presented in this section to (i) compare the service provider’s total profit yielded by nine pricing schemes for advance and spot selling, and (ii) show the impacts of the intra-segment price-sensitivities (i.e., $\beta_1$ and $\beta_2$) on the optimal pricing scheme. The values of the parameters in the demand and profit functions (1), (2) and (4) chosen for the numerical study are given in Table 1. In the table, the first set of the parameter values represents the scenario in which the intra-segment price sensitivity of Segment 1 is twice that of Segment 2. The second set of parameter values, on the other hand, describes another scenario in which the intra-segment price sensitivity of Segment 2 is twice that of Segment 1.

| Table 1. Parameter settings for the numerical study in two experimental scenarios |
|-------------------------------|-------------------------------|
|                               | Set 1                          | Set 2                          |
|                               | $\alpha_1 = 1,500$ units       | $\alpha_1 = 1,500$ units       |
|                               | $\alpha_2 = 1,800$ units       | $\alpha_2 = 1,800$ units       |
|                               | $\beta_1 = 0.8$                | $\beta_1 = 0.4$                |
|                               | $\beta_2 = 0.4$                | $\beta_2 = 0.8$                |
|                               | $\beta_{12} = 0.3$             | $\beta_{12} = 0.3$             |
|                               | $\beta_{21} = 0.3$             | $\beta_{21} = 0.3$             |
|                               | $C = $1,000,000                | $C = $1,000,000                |

The nine pricing schemes are related to the two experimental scenarios described above. The elements of each pricing scheme $(P_1, P_2)$ are selected such that $P_i \in \{P_i', P_i' +$ $500, P_i' -$ $500\}$ for $i = 1, 2$, where the $P_i'$ s are determined by expressions (5) and (6), respectively. The resultant demands in the two-segment market and the total profit are calculated for each pricing scheme based on expressions (1), (2) and (4). The computational results are reported in Tables 2 and 3.

As shown in Table 2, the optimal pricing scheme $(P_1^*, P_2^*) = ($2478.26, $4108.70)$ is calculated for the first set of the parameter values. It is noted that the optimal pricing scheme yields the highest total profit compared to all the other pricing schemes. Table 3 shows that given the second set of the parameter values, the optimal pricing scheme is $(P_1^*, P_2^*) = ($3782.61, $2543.48)$, which also yields the highest total profit. The experimental findings of the numerical study are consistent with Proposition1 as expected.

As Table 2 shows, the optimal spot selling price is higher than the optimal advance selling price (i.e., $P_2^* > P_1^*$), mainly because the intra-segment price sensitivity of the segment for spot selling, $\beta_2$, is lower than its counterpart of the segment for advance selling, $\beta_1$. In Table 3, $P_2^*$ is lower than $P_1^*$, as $\beta_2$ is greater than $\beta_1$ in the second experimental scenario. These findings offer directions for the service provider to fine-tune her advance and spot selling prices over time in an ever-changing market.
Table 2. The profits and demands generated by nine pricing schemes based on the first parameter set

<table>
<thead>
<tr>
<th>Pricing Schemes</th>
<th>Prices ($/unit)</th>
<th>Demands (units)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1$</td>
<td>$d_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>Scheme 1</td>
<td>2478.26*</td>
<td>750*</td>
<td>4108.70*</td>
</tr>
<tr>
<td>Scheme 2</td>
<td>2978.26</td>
<td>350</td>
<td>4108.70*</td>
</tr>
<tr>
<td>Scheme 3</td>
<td>1978.26</td>
<td>1150</td>
<td>4108.70*</td>
</tr>
<tr>
<td>Scheme 4</td>
<td>2478.26*</td>
<td>900</td>
<td>4608.70</td>
</tr>
<tr>
<td>Scheme 5</td>
<td>2478.26*</td>
<td>500</td>
<td>3608.70</td>
</tr>
<tr>
<td>Scheme 6</td>
<td>2978.26</td>
<td>700</td>
<td>4608.70</td>
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<tr>
<td>Scheme 7</td>
<td>2978.26</td>
<td>200</td>
<td>3608.70</td>
</tr>
<tr>
<td>Scheme 8</td>
<td>1978.26</td>
<td>1300</td>
<td>4608.70</td>
</tr>
<tr>
<td>Scheme 9</td>
<td>1978.26</td>
<td>1000</td>
<td>3608.70</td>
</tr>
</tbody>
</table>

Table 3. The profits and demands generated by nine pricing schemes based on the second parameter set

<table>
<thead>
<tr>
<th>Pricing Schemes</th>
<th>Prices ($/unit)</th>
<th>Demands (units)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1$</td>
<td>$d_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>Scheme 1</td>
<td>3782.61*</td>
<td>750*</td>
<td>2543.48*</td>
</tr>
<tr>
<td>Scheme 2</td>
<td>4282.61</td>
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<td>2543.48*</td>
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<tr>
<td>Scheme 3</td>
<td>3282.61</td>
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<td>2543.48*</td>
</tr>
<tr>
<td>Scheme 4</td>
<td>3782.61*</td>
<td>900</td>
<td>3043.48</td>
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<td>Scheme 5</td>
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<td>600</td>
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<td>Scheme 6</td>
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<td>1100</td>
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<tr>
<td>Scheme 9</td>
<td>3282.61</td>
<td>800</td>
<td>2043.48</td>
</tr>
</tbody>
</table>

CONCLUDING REMARKS

In this paper, a linear deterministic demand function is employed to describe the relationship between the demand of each segment in the market and advance and spot selling prices. We first address the problem of determining the optimal advance and spot selling prices for a monopolistic service provider to maximize her total profit and as a result, the necessary and sufficient conditions of optimizing prices are established. The impacts of four types of price sensitivity in the two-segment market on the optimal pricing scheme are then analytically examined.

The analytical findings presented in the third section imply that the service provider should take all the four types of price sensitivity into consideration when making her pricing decisions. As
any of the price sensitivities changes, both the advance and spot selling prices must be adjusted in the right direction.

There are several directions for future research. First, the demand function employed in our model has a deterministic structure. A probabilistic demand function may be chosen for model development. Second, in this exploratory study we focus on a service provider in a monopolistic environment. Incorporating competition into the model would be a possible extension. Third, the optimal pricing scheme is determined for a two-segment market for advance and spot selling. A rather challenging direction for future research would be to address optimal pricing over a general n-segment market.

In sum, this paper tackles two strategic issues of pricing decisions and opens avenues for managerial applications.

APPENDIX

Proof of Proposition 1

The first-order necessary condition for optimality implies that

\[
\frac{\partial \pi}{\partial P_1} = \alpha_1 - 2\beta_1 P_1 + (\beta_{12} + \beta_{21}) P_2 = 0, \tag{A.1}
\]

\[
\frac{\partial \pi}{\partial P_2} = \alpha_2 - 2\beta_2 P_2 + (\beta_{12} + \beta_{21}) P_1 = 0. \tag{A.2}
\]

Solving equations (A.1) and (A.2) simultaneously, we obtain

\[
P_1^* = \frac{\alpha_2 (\beta_{12} + \beta_{21}) + 2\alpha_1 \beta_2}{4\beta_1 \beta_2 - (\beta_{12} + \beta_{21})^2}, \tag{A.3}
\]

and

\[
P_2^* = \frac{\alpha_1 (\beta_{12} + \beta_{21}) + 2\alpha_2 \beta_1}{4\beta_1 \beta_2 - (\beta_{12} + \beta_{21})^2}. \tag{A.4}
\]

Since \(\frac{\partial^2 \pi}{\partial P_1^2} = -2\beta_1 < 0\), \(\frac{\partial^2 \pi}{\partial P_2^2} = -2\beta_2 < 0\), \(\frac{\partial^2 \pi}{\partial P_1 \partial P_2} = \beta_{12} + \beta_{21}\), and \(\frac{\partial^2 \pi}{\partial P_2 \partial P_1} = \beta_{12} + \beta_{21}\), \((\frac{\partial^2 \pi}{\partial P_1^2})(\frac{\partial^2 \pi}{\partial P_2^2}) - (\frac{\partial^2 \pi}{\partial P_1 \partial P_2})(\frac{\partial^2 \pi}{\partial P_2 \partial P_1}) = 4\beta_1 \beta_2 - (\beta_{12} + \beta_{21})^2 > 0\). This indicates that the total profit, \(\pi\), reaches its maximal level at \(P_1^*\) and \(P_2^*\). ■

Proof of Proposition 2

Consider \(P_1^* = \frac{\alpha_2 (\beta_{12} + \beta_{21}) + 2\alpha_1 \beta_2}{4\beta_1 \beta_2 - (\beta_{12} + \beta_{21})^2}\) (see Proposition 1). We obtain the first partial derivative with respect to \(\beta_1\) and \(\beta_2\), respectively:
\[
\frac{\partial R^*}{\partial \beta_1} = \frac{(-4)[\alpha_1(\beta_1 + \beta_2) + 2\alpha_2 \beta_1]\beta_1}{[4\beta_1 \beta_2 - (\beta_1 + \beta_2)^2]^2},
\]

(A.5)

and

\[
\frac{\partial P^*_1}{\partial \beta_2} = \frac{(-2)(\beta_2 + \beta_1)[\alpha_1(\beta_1 + \beta_2) + 2\alpha_2 \beta_1]}{[4\beta_1 \beta_2 - (\beta_1 + \beta_2)^2]^2}.
\]

(A.6)

Since \(4\beta_1 \beta_2 - (\beta_1 + \beta_2)^2 > 0\) and \(\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_{12}, \) and \(\beta_{21}\) all take positive values, \(\partial P^*_1 / \partial \beta_1 < 0\) and \(\partial P^*_1 / \partial \beta_2 < 0\). Hence, \(P^*_1\) is monotonically decreasing in \(\beta_1\) and \(\beta_2\), respectively. ■

**Proof of Proposition 3**

Consider \(P^*_1 = \frac{\alpha_1(\beta_1 + \beta_2) + 2\alpha_2 \beta_1}{4\beta_1 \beta_2 - (\beta_1 + \beta_2)^2}\) (see Proposition 1). We obtain the first partial derivative with respect to \(\beta_{12}\) and \(\beta_{21}\), respectively:

\[
\frac{\partial P^*_1}{\partial \beta_{12}} = \frac{(\beta_1 + \beta_2)[\alpha_1(\beta_1 + \beta_2) + 2\alpha_2 \beta_1] + 4\alpha_2 \beta_1 \beta_2}{[4\beta_1 \beta_2 - (\beta_1 + \beta_2)^2]^2}.
\]

(A.7)

Since \(4\beta_1 \beta_2 - (\beta_1 + \beta_2)^2 > 0\), and \(\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_{12}, \) and \(\beta_{21}\) all take positive values, \(\partial P^*_1 / \partial \beta_{12} = \partial P^*_1 / \partial \beta_{21} > 0\). Hence, \(P^*_1\) is monotonically increasing in \(\beta_{12}\) and \(\beta_{21}\), respectively. ■

**Proof of Proposition 4**

Consider \(P^*_2 = \frac{\alpha_1(\beta_1 + \beta_2) + 2\alpha_2 \beta_1}{4\beta_1 \beta_2 - (\beta_1 + \beta_2)^2}\) (see Proposition 1). We obtain the first partial derivative with respect to \(\beta_1\) and \(\beta_2\), respectively:

\[
\frac{\partial P^*_2}{\partial \beta_1} = \frac{(-2)(\beta_1 + \beta_2)[2\alpha_1(\beta_1 + \beta_2) + 2\alpha_2 \beta_1]}{[4\beta_1 \beta_2 - (\beta_1 + \beta_2)^2]^2},
\]

(A.8)

and

\[
\frac{\partial P^*_2}{\partial \beta_2} = \frac{(-4)[\alpha_1(\beta_1 + \beta_2) + 2\alpha_2 \beta_1]\beta_1}{[4\beta_1 \beta_2 - (\beta_1 + \beta_2)^2]^2}.
\]

(A.9)

Since \(4\beta_1 \beta_2 - (\beta_1 + \beta_2)^2 > 0\), and \(\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_{12}, \) and \(\beta_{21}\) all take positive values, \(\partial P^*_2 / \partial \beta_1 < 0\) and \(\partial P^*_2 / \partial \beta_2 < 0\). Hence, \(P^*_2\) is monotonically decreasing in \(\beta_1\) and \(\beta_2\), respectively. ■
Proof of Proposition 5

Consider \( P_2^* = \frac{\alpha_1 (\beta_{12} + \beta_{21}) + 2\alpha_2\beta_2}{4\beta_1\beta_2 - (\beta_{12} + \beta_{21})^2} \) (see Proposition 1). We obtain the first partial derivative with respect to \( \beta_{12} \) and \( \beta_{21} \), respectively:

\[
\frac{\partial P_2^*}{\partial \beta_{12}} = \frac{\partial P_2^*}{\partial \beta_{21}} = \frac{\alpha_1 (\beta_{12} + \beta_{21}) + 4\alpha_2\beta_2}{[4\beta_1\beta_2 - (\beta_{12} + \beta_{21})^2]^2}.
\]

(A.10)

Since \( 4\beta_1\beta_2 - (\beta_{12} + \beta_{21})^2 > 0 \), and \( \alpha_1, \alpha_2, \beta_1, \beta_2, \beta_{12}, \) and \( \beta_{21} \) all take positive values, \( \frac{\partial P_2^*}{\partial \beta_{12}} = \frac{\partial P_2^*}{\partial \beta_{21}} > 0 \). Hence, \( P_2^* \) is monotonically increasing in \( \beta_{12} \) and \( \beta_{21} \), respectively. ■

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