ABSTRACT

Increasing supply chain players seek to run business in a responsible way. This work stands from a supplier’s point of view by considering a two-stage supply chain structure in which a responsible supplier produces a responsible component (or raw material, ingredient) through an entrepreneurial process and exploits the business opportunity (e.g., market expansion) to sell it to downstream buyers. Buyers’ market is classified by two: one for price-sensitive and one for responsible-conscious. We develop quantitative models to analyze such a supplier’s production structure, interplaying with the attractiveness to the business opportunity exploitation.

KEYWORDS: Operations management, Dynamic programming, Responsible supply chain

INTRODUCTION

Embedding the responsible behaviors into supply chain draws the interests of academic researchers and business practitioners. Many tragedies have proved that it is a necessity to pay attention to this issue. The collapse of a Bangladeshi garment factory in April 24, 2013, which killed over 1,100 workers, draws the public attentions of how apparels being manufactured irresponsibly/unethically, from which labors are paid with low salaries and expose under poor working conditions. Consequently, the chain effect rapidly spreads to affect several well-known clothing makers who subcontract their products from this factory which is labeled as the sweat factory. This results in the rise of people’s ethical consumerism on boycotting the purchase of apparels from these giant clothing brands.

The above is just one of incidents recently disclosed. As seen, a growing number of consumers seriously make the purchase decision on the basis of how much a company contributes to behave responsibly, from which they are willing to pay a higher premium as reward (Cryer, 1995). Further, the social violations incur a series of their boycott to purchase products from irresponsible brand companies and, through the flourishing social media, the negative effect proliferates worldwide to exacerbate the damage. Moreover, the higher profile a brand company possesses a greater change to be targeted by the publicity to scrutinize each activity under the microscope. The past experiences repeatedly remind us these phenomenon, such as Nike, Coca-Cola, HP, PVH, Wal-mart, and Primark being criticized for social violations (BBC News, 2000; Manik and Greenhouse and Yardley, 2013; Carrigan and Attalla, 2001). As a result, increasing companies start to pay attention to the importance of sourcing responsibly, selecting their suppliers not based on price but rather based on suppliers’ responsible manners, to maintain a good reputation (Roberts, 2003). This emerges an opportunity for a supplier to consider the manner of supplying items responsibly to these brand companies, which gives the edge of differentiating its products from others competitors, pioneering to occupy this growing market. As McWilliams and Siegel (2001) addressed, engaging in the corporate social responsibility as a product differentiation strategy assists a firm to create new demand. More
than this, financially rewarding responsible suppliers, buyers pay a fair price or even more than a fair price (Over 80 million euros Fairtrade premium, on top of the selling price, is paid to producers (2012-13 Fairtrade International Annual Report, 2013). Combining the increasing need and the incentive of being paid a fair price, a supplier is much willing to abnegate the conventional production mode and instead to devote to the production process in a responsible way. Accordingly, a new business opportunity appears for new suppliers to provide a component in a responsible way referred to as entrepreneurial process and for existing suppliers to transform from producing a component through the regular process to producing a component through such the entrepreneurial process.

Since increasing consumers dislike to buy irresponsible products or speaking differently irresponsible activities in any stage of supply chain tarnish the whole chain system (Trudel and Cottele, 2009), more chain players seek to run business in a responsible way by conducting the so-called responsible trading, meaning bilateral actions of downstream players referred as to buyers sourcing responsibly and upstream players referred as to suppliers producing responsible products, to satisfy consumers’ expectations. Fairtrade International Annual Report (2013) indicates that demand and supply of responsible/ethical products grows strongly that consumers spent 4.8 billion euros on Fairtrade certified products in 2012 and the number of Fairtrade producer organizations increases up to 1,149 with approximately 1.35 million farmers and workers in 70 countries, 16% more than that in 2011. Our work is motivated by the surge of this responsible trade. Specifically, little work discuss this issue from the supplier’s point of view and thus we are interested analyzing a supplier’s supplying decision when it devotes an entrepreneurial process to provide responsible components to buyers. From the perspective of entrepreneurial literature, McMullen and Shepherd (2006) develop a conceptual framework of entrepreneurial action, including two key steps: opportunity exploration/recognition and exploitation. We follow this framework to develop parsimonious models by first exploring the potential market expansion served as the opportunity recognition stage, and then determining the production decisions served as the exploitation stage. Model analysis will disclose the structure of a supplier’s production status which is related to the opportunity recognition. Buyers’ market is classified by two: one for price-sensitive which is referred to as established buyers and one for responsible-conscious which is referred to as potential buyers. The supplier can choose supply items either to potential buyers or to established buyers.

This work mainly contributes to examine the effect of opportunity business in the exploration/recognition stage to a suppliers’ production/supply decision. With customers’ high sensitivity of exploiting the business opportunity, a supplier maintains a fixed production amount even though net inventory is high since the marginal loss of revenue from potential buyers for each reduced production unit becomes larger than the marginal cost saving; on the other side, with customers’ low sensitivity of exploiting the business opportunity, a supplier will make no production when net inventory is sufficiently high. For a comparison, in the conventional wisdom of discussing the inventory model for a non-perishable product, we have seen the so call base-stock, list-price optimal replenishment policy. This paper are organized as follows. We first address the relevant literature. Then, we illustrate the general model setting and analyze the supply policy. Last but not the least, the last section contributes to a conclusion.

LITERATURE REVIEW

Our work is relevant to two main streams of literature: adopting dual sourcing in the operational management context which inspires us to discuss a responsible supplier selling components to two types of buyers and the supply chain system in the responsible way.
In the first stream, Zhao and Lau (1992), Lau and Zhao (1993), Lau and Zhao (1994), and Lau and Lau (1994) quantitatively analyzed the cycle inventory under dual or multiple sourcing. Whittemore and Saunders (1977), Tagaras and Vlachos (2001), Groenevelt and Rudi (2003), Sheopuri and Janakiraman and Seshadri (2010), and Plambeck and Ward (2007) extended to explore the dual sourcing with different leadtimes. Veeraraghavan and Scheller-Wolf (2006) further took the capacity into consideration. Tomlin (2006), Tomlin (2009b), Tomlin (2009a), Chiang and Gutierrez (1996), and Vlachos and Tagaras (2001) adopted the periodic review setting to analyze the dual sourcing issue. Differentiated from above work which stands from the buyer’s perspective to discuss the ordering policy, we think from the opposite direction by examining a responsible supplier’s supply policy as the contribution of this work, aiming at adding the research value of supply chain which is done responsibly.

The second stream discusses the topic of embedding the responsibility into the field of supply chain. Many works integrate environmental and social issues in operations and productions. For the environmental concern, work on inventory, such as Hau and Cheng and Wang (2011), Benjaafar and Li and Daskin (2013) and Bonney and Jaber (2011), took the carbon footprint into account when developing quantitative models. For the social concern, we research connecting social responsibility with operations management and incorporating it into quantitative models has only few, such as Ni and Li and Tang (2010), Hsueh and Chang (2008), and Cruz (2008a; 2008b). In our work, we generalize the responsible behaviors to be either social or environmental. Nonetheless, either targeting the environmental or social dimensions, none of these papers considers the growth on expected demand due to the responsible behaviors, which is the highlight of this work by exploring a supplier’s entrepreneurial opportunity through the responsible behavior and then exploiting it through the appropriate production control.

**MODEL DEVELOPMENT**

Consider a two-stage supply chain model in which a upstream player (referred to as supplier) sells a component (or raw material, ingredient) to downstream players (referred to as buyers). This work specifically highlights buyers to be brand companies whose finished goods are directly sold to end customers. Examples of this supplier-buyer supply chain are too numerous to enumerate, such as Starbuck (buyer) who procures coffee beans from growers (supplier) for coffee-related drinks sold in stores. The terms supplier and buyer link the material flow in supply chain from which a supplier manages the product availability (e.g., production quantity) \( Q \) to fulfill buyers’ demand.

Buyers are classified as two types: established buyers and potential buyers. Established buyers are sensitive to the supplier’s unit sales price \( P \). Thus, given \( P \), demand of established buyers adopts a form \( D = d(p) + \xi \), where \( d(p) \) is the price-dependent expected demand and \( \xi \) is a random variable with the support \([r_l, r_u]\) , probability density function \( f(\cdot) \) and cumulative distribution \( F(\cdot) \), which is not necessary normal. To ensure a non-negative demand, in this work, assume that the interval of \( \xi \) is price-dependent and \( d(p) + \xi \) is non-negative (e.g., \( d(p) + \xi \geq 0 \) for any \( P \)). Potential buyers who are generated through a supplier’s entrepreneurial process, instead of price-sensitive, belong to be entrepreneurial-sensitive and are willing to reward the supplier a high price ex ante \( P^* \) if a component is embedded with the opportunity of market exploitation. Demand of potential buyers does not face uncertainty and is quantity-dependent, denoted by \( \gamma(Q) \). \( \gamma \) is also referred to as the demand exploitation ability, representing a responsible supplier’s ability to identify the business
opportunity (e.g., the committed procurement amount from potential buyers). Consider a simple example of $\gamma$:

$$\gamma(q) = \min(q, K) + P(q - K)^0,$$

where $K$ is the guaranteed amount for procurement and $P \in [0, 1]$ is the potential buyers' likelihood of procuring the excessive amount. As seen, following the supplier's entrepreneurial process, the potential buyers will buy a fraction of a supplier's total production amount, meaning that there is a ceiling for potential buyers' procurement amount due to the risk of overstock in the end market. Therefore, a supplier's over-production causes the possibility of units unsold to potential buyers. For an easy model analysis later, without loss of generality, we reasonably impose the assumption that $\gamma(\cdot)$ is non-decreasing concave and is differentiable with

$$0 < \gamma'(q) = \frac{d\gamma}{dq} < 1.$$

Moreover, $\gamma'(\cdot)$ can be viewed as the opportunity attractiveness. In other words, the marginal increase in demand per unit of the production is a proxy of entrepreneurial opportunity attractiveness generated from the commitment of the responsible supplier's resources in production activities. This formalization of the supplier's business opportunity identification into the buyer's demand function has been supported by the recent empirical observation on biotech firms and their partnerships with pharmaceutical companies (Hora and Dutta, 2013). As illustrated aforementioned, the context of the two-stage supply chain derives different models which will be discussed shortly, depending on how buyers' market interact to the supplier's production behavior: regular or entrepreneurial. We proceed to articulate the detail of the context through the sequence of events. First, a supplier decides the production behaviors: regular or entrepreneurial process. The timeline of events is expressed as below.

When a supplier behaves the entrepreneurial process, it firstly goes through the exploration stage by identifying the entrepreneurial opportunity $\gamma$ and producing each unit with a marginal cost $c^E$ which is noticed to be more expensive than the marginal cost from the regular process $c^R$ (e.g., $c^E > c^R$). Next, it reaches the opportunity exploitation in which a supplier makes the production decision and can sell components not only to potential buyers who pay a high high premium with unit price $p^E$ but also established buyers whose demand is price-dependent (Figure 1). Sometime a responsible supplier can determine who it will sell to, but sometime the sales target can not be selected by itself. For instance, once potential buyers impose the exchange condition of items being sold only to them as their willingness of paying the high premium. In this case, the supply chain is form by a responsible supplier and potential buyers with demand $\gamma(q)$ and we mark this as Model $E$. Suppose the supply chain is formed by a responsible supplier and both potential and established buyers, demand becomes $\gamma(q) + D$ and we mark this as Model $L$. Note that the production quantity occurs before demand realization. On the other side, when a supplier produces items following its regular process at a marginal cost $c^R$, established buyers forms the demand market by the price-dependent magnitude $D$. In other words, a supplier has no chance to exploit the market from potential buyers. We refer to this supply chain as Model $R$. 
Price in this work is the lever to regulate demand of established buyers. As a result, the expected revenue from established buyers for each supply opportunity is $p \cdot d(p)$ which is assumed to be strictly concave in $p$. One example is $d(p) = a_0 - b_0p$, where $a_0 > 0$ is the market size and $b_0 > 0$ is the price sensitivity. In the later model development for the ease of model expression, we substitute $p \cdot d(p)$ with $R(d) = d \times D^{-1}(d)$, where $D^{-1}(d)$ is the corresponding price with respect to $d$.

Our work sheds light on a supplier under the entrepreneurial process. Accordingly, Models $E$ and $L$ are the main supply chain structures analyzed in this work. As for Model $R$ which is the typical one in the conventional operations management domain, in most of time, it will serve as a foil to the research findings from $E$ and $L$. We proceed to develop models $E$ and $L$, articulated as follows.

ANALYSIS

Models follow the standard approach in period-review, dynamic programming with $T$ periods. In each period, the supplier has one opportunity to exploit demand from potential buyers and one production opportunity at the beginning of the period. The setting of finite horizon is based on the contract term between the responsible supplier and the potential buyers wherein the demand exploitation ability $\gamma$ (assumed to be fixed between periods) and unit premium $p^E$ are formalized. In addition, we make the following assumptions toward established buyers.

First, established buyers have a larger tolerate to the backlogging when items are produced in an entrepreneurial way from which we can assume the unit backlogging cost in a period $\pi^E$ is sufficiently small. Second, we assume that established buyers can wait up to $T$ periods for the acquisition of items. In other words, with the limitation on the contract period which is not too long, established buyers will not lose the purchasing willingness within $T$ periods. Third, $\beta^t \cdot \pi^t \cdot \sum_{s=0}^{T} \beta^s \pi^s$ where $\beta$ is the discount factor of the value of each dollar through one period is reasonably assumed through the following explanation. During the contract, each unfulfilled unit can accumulate at most a total cost $\sum_{t=0}^{T} \beta^t \pi^t$. We let the estimation of unit lost-sales cost be approximated by $p^E - c^E$. With above, clearly, $\beta^t \cdot \pi^t \cdot \sum_{s=0}^{T} \beta^s \pi^s$, which plays a weak assumption in the proof listed in Appendix.

**Assumption:**
(a) $T$ and $\pi^E$ are sufficiently small.
(b) Lost-sales from established buyers does not occur when a responsible supplier can fulfill unsatisfied demand in $T$ periods.
(c) $p^t \cdot \beta^t \cdot \pi^t \cdot \sum_{s=0}^{T} \beta^s \pi^s$.
(d) $R(d) = d \times D^{-1}(d)$ is concave in $d$. 

**Figure 1. Timeline of events under entrepreneurial process**

<table>
<thead>
<tr>
<th>Exploration stages</th>
<th>Exploitation stages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify a business opportunity $\gamma$</td>
<td>Deploy the entrepreneurial process with unit cost $c^E$</td>
</tr>
<tr>
<td>Determine the production quantity $q$</td>
<td>Supply potential buyers with premium $p^E$</td>
</tr>
</tbody>
</table>

Time
Throughout the paper, we use non-decreasing/non-increasing in the week sense unless necessarily pointing out as strictly increasing/decreasing. We now develop models, starting from a responsible supplier through an entrepreneurial process and then tracing back to the model following a regular process for comparisons with models under the entrepreneurial process. (Fig. 2 for graphical description).

**Figure 2. Model summary**

![Diagram of models](image)

**Model $E$.** We first develop Model $E$ under which a responsible supplier sells items to potential buyers. As explained aforementioned, the total procurement amount from potential buyers $\gamma(q)$ is associated with a responsible supplier’s supplying availability $q$ so that backlogging will not appear. Further, for over-supplying amount $q - \gamma(q)$ (i.e., items not procured by potential buyers), it loses the value to potential buyers and will be charged with a unit holding cost $h$. Consequently, for each supply opportunity (one period), the expected profit, composed of three terms: revenue, total production cost, and holding cost, becomes

$$p^E \gamma(q) - c^E q - h[q - \gamma(q)].$$

We now consider the accumulated expected profit from multiple supply opportunities. Suppose there is $x$ units leftover with $(T - t)$ periods to go until the end of the period, the optimality equation characterizing the dynamic program to be solved by a responsible supplier in Model $E$ is

$$v_t(x | E) = \max_{q \geq 0} G_t(x, q | E),$$

where

$$G_t(x, q | E) = p^E \gamma(q) - c^E q - h[x + q - \gamma(q)] + \beta v_{t+1}(x + q - \gamma(q) | E)$$

with $v_{T+1}(x | E) = sx$, which is the unit salvage value at the end of the contract. Note that $h > s$ is assumed and this assumption inherits to the rest of this work. Let $q^E_t(x)$ denote the optimal production quantity to optimize $G_t(x, q | E)$ when the initial inventory at the beginning of period $t$ is $x$ unit (e.g., $v_t(x | E) = G_t(x, q^E_t(x) | E)$). Theorem 1 provides an articulated context to the production policy of Model $E$. 
Theorem 1:
(a) $q^E_t(x)$ is irrelevant to $x$.
(b) $q^E_t(\cdot) \leq q^E_{i+1}(\cdot)$.

Theorem 1(a) shows that, in Model $E$, a responsible supplier’s production quantity is not affected by leftover since potential buyers’ commitment on procurement amount only depends on a supplier’s current status of production. Explicitly, high or low demand previously occurred in the buyers’ market has no impact to a supplier’s current production decision. Theorem 1(b) illustrates that a responsible supplier adjusts its production scheme over time. In Model $E$, a supplier is more conservative to its production decision at the beginning of a contract. This lies in that leftover becomes no value, forcing it obsolete until the end of the contract through salvage. This demonstrates that when an item is kept longer, a higher holding cost accumulated to the end of the contract is incurred. With this consideration, a supplier has a decreased production amount when the timing of supply opportunity is farther away from the termination of contract (e.g., $q^E_t(\cdot) \leq q^E_{i+1}(\cdot)$). Further, this finding implies that the length of a contract significantly impacts potential buyers’ ability to supply finished goods to the end market. When a contract is designated with longer periods, potential buyers are possibly exposed to the risk of supply shortage to the end market at the early periods because of a supplier’s unwillingness of producing more.

Except for potential buyers’ adamant request of selling items only to them due to the concerns such as exclusiveness, there is two caveats emerging in Model $E$. Firstly, the obsolete of leftover becomes the eternal waste. Second, the decreased production quantity as mentioned above in the early periods makes a supplier lose the opportunity of exploiting more market from potential buyers. To enhance a responsible supplier’s incidence of pursuing higher profitability, we follow to propose Model $L$ under which a supplier carves a niche by selling items to both potential and established buyers.

**Model $L$**. Model $L$ maintains the advantage of selling items to potential buyers who pay a higher premium and at the same time makes items available for established buyers. Distinguished from Model $E$, Model $L$ creates more value by selling any available item, either left from the previous periods or coming from the current production quantity, to established buyers whose demand is price-dependent. Since selling items to potential buyers creates more profit, from the model perspective, we can regard that items are prioritized to supply potential buyers first and followed to sell to established buyers.

A responsible supplier, given net inventory $x$, produces quantity $q$ which brings potential buyers’ procurement amount $\gamma(q)$. Then, all remaining items, $x + q - \gamma(q)$, can be sold to established buyers with demand $d + \xi$ given the sales price is $D^{-1}(d)$. After fulfilling demand from both types of buyers, leftover $[x + q - \gamma(q) - d - \xi]^+$ is charged with a unit holding cost $h$ and unmet demand (e.g., backlogging) $[d(p) + \xi + \gamma(q) - x - q]^+$ is charged with a unit backlogging cost $\pi^E$. With the flexibility of dual supply, for each supply opportunity (one period), the expected profit composed of five terms, revenue from potential buyers, revenue from established buyers, total production cost, holding cost, and backlogging cost, is

$$p^E \gamma(q) + d D^{-1}(d) - c^E q - h [x + q - \gamma(q) - d - \xi]^+ - \pi^E [\gamma(q) + d + \xi - x - q]^+.$$  

We now consider the accumulated expected profit from multiple supply opportunities. Suppose there is $x$ units leftover (or backlogging) with $(T - t)$ periods to go until the end of the period, the optimality equation characterizing the dynamic program to be solved by a responsible supplier in Model $L$ is

$$v_t(x \mid L) = \max_{q \geq 0} G_t(x, q \mid L),$$
where
\[
G_t(x,q | L) = R(d) + p^E \gamma(q) - c^E q - h E \xi + q - \xi - \gamma(q)]^+ - \pi^E \ [\gamma(q) + d + \xi - x - q]^+ \\
+ \beta E v_{t+1}(x + q - d - \xi - \gamma(q) | L)
\]
with \( v_{t+1}(x | L) = c^E x^+ + sx^+ \). Denote \( \bar{x}^L_t \) to be a threshold of the stock level in which no production starts to take place. Note that \( \bar{x}^L_t \) may not exist. Suppose this happens, we simply assume \( \bar{x}^L_t \rightarrow \infty \). We also let \( q^L_t(x) \) denote the optimal production quantity to optimize \( G_t(x,q | L) \) and \( \Delta^L_t(x) = x + q^L_t(x) - \gamma(q^L_t(x)) \) be product availability for established buyers, then the production policy for a responsible supplier in Model \( L \) is identified, summarized in Theorem 2.

**Theorem 2:**
(a) When \( x < \bar{x}^L_t \), production takes place and \( q^L_t(x) \) is non-increasing in \( x \).
(b) When \( x \geq \bar{x}^L_t \), there is no production.
(c) Further, \( \Delta^L_t(x) \) is non-decreasing in \( x \).

Theorem 2 leads to crucial managerial insights, some identical to conventional inventory model in operations management (OM) realm while some quite interestingly fresh. We first mention the identical part. A responsible supplier in Model \( L \) launches production when observing insufficient inventory (e.g., high demand in the previous period), whereas production is terminated when there are too much leftover (e.g., low demand in the previous period). The surprising finding from Theorem 2 lies in the likelihood of different structures of optimal production policy which is associated to how a supplier’s demand exploitation ability \( \gamma(\cdot) \) affects the interplay between the inventory level and the production amount. We first consider the following example that suppose a responsible supplier fails to take advantage of the opportunity exploitation by gaining demand (i.e., \( \gamma(q) = 0 \)), the optimal production amount is decreasing in the inventory level. Explicitly, given one more inventory kept, a supplier decreases the needed production amount precisely one unit, as shown in Fig. 3(a). In this case, product availability for established buyers \( \Delta^L_t(x) \) before stock level reaching the threshold \( \bar{x}^L_t \) is a fixed amount. This result is not surprising to be consistent with papers considering the ordering policy of the inventory model for a non-perishable products.

Figure 3: Optimal quantity as a function of net inventory with respect to

\[ T = 3, t = 2, h = 1, \pi = 2, c = 5, s = 1.5, D = (30 - p + \xi), \beta = 0.95, \xi \in N(0,10) \text{ truncated at } \pm 10, \ p^E = 30.\]
What surprises us lies in the positive effect from the demand exploitation ability (e.g., $\gamma(q) > 0$). To elaborate the detail, we follow to enumerate two scenarios. The first scenario considers that the demand exploitation ability displays a non-decreasing concave to the production quantity with high marginal gaining on demand when production amount is small. Here, we adopt a form $\gamma(q) = -0.015q^2 + \zeta q$ for $q \leq \frac{\xi}{0.03}$ and $\gamma(q) = -0.015\left(\frac{\xi}{0.03}\right)^2 + \frac{\zeta}{0.03}$ for $q > \frac{\xi}{0.03}$ where $\xi \to 1$. Fig. 3(b) indicates that the optimal production amount drops with the increased net inventory, nonetheless, the structure is not similar to the case of $\gamma(q) = 0$. We clearly see the appearance of zigzag decrease when net inventory is in the range between $-5$ to $12$, implying that the structure of dual supply results in a responsible supplier not necessarily reducing the production quantity exactly one unit when there is one more inventory. Further, a responsible supplier in Model $L$ is incentivized to
possibly reduce less production amount by being willing to bear more overstock risk (e.g.,
\[
\frac{\partial q^L_t(x) - y(q^L_t(x))}{\partial x} \in (-1,0),
\]
leading to product availability for established buyers \(\Delta^L(x)\) to be non-decreasing in leftover. A more surprising finding is how the production quantity reacts when net inventory is sufficiently large. Intuitively, less amount should be produced to alleviate the risk of overstock. Against the conventional wisdom, we counterintuitively observe that a responsible supplier maintains a fixed production amount even net inventory is high, resulting in more overall product availability for buyers. The rationality behind the increased total product availability lies in that, for each one unit reduction on production, the marginal loss of revenue from potential buyers for each reduced production unit becomes larger than the marginal cost saving (i.e., \(\frac{-\partial p}{\partial q} y(q) > -\frac{\partial}{\partial q} [c^R q - h(x^q - y(q) - d - \xi)]\)). Hence, a supplier would rather not decrease the production quantity when it is lower than a certain level. In other words, by taking advantage of gaining a higher marginal revenue from the potential buyers, a supplier maintains to produce a fixed amount once net inventory exceeds a certain level. On the other side that suppose the marginal loss of revenue from the potential buyers under the production reduction is less than the marginal cost saving (see Fig. 3(c) for example in which a low demand exploitation ability with a linear form \(y(q) = 0.1 \cdot q\) is adopted), the production quantity reduces with the increased net inventory until no production takes place. That is, \(\bar{x}^L_t\) exists and once again we observe that the marginal decrease is in the interval \([-1, 0]\).

To distinguish the evolution of the structure of production policy when a supplier transforms from operating the regular process to behaving the entrepreneurial process, we next briefly discuss the model for a regular supplier, labeled as Model \(R\).

**Model \(R\).** In Model \(R\) (Figure 2), given net inventory \(x\), a regular supplier produces quantity \(q\) with the unit cost \(c^R\) and sells all available items \(x+q\) to established buyers with demand \(d+\xi\). Leftover \([x+q-d-\xi]^+\) is charged with a unit holding cost \(h\) and unmet demand (e.g., backlogging) \([d(p)+\xi-x-q]^+\) is charged with a unit backlogging cost \(\pi^R\). Consequently, for each supply opportunity (one period), the expected profit, composed of four terms: revenue from established buyers, total production cost, holding cost, and backlogging cost becomes

\[pd(p) - c^R q - hE[x+q-d-\xi]^+ - \pi^R E[d+\xi-x-q]^+\]

We now consider the accumulated expected profit from multiple supply opportunities. Suppose there is \(x\) units leftover(or backlogging) with \((T-t)\) periods to go until the end of the period, the optimality equation characterizing the dynamic program to be solved by a regular supplier in Model \(R\) is

\[v_t(x|R) = \max_{q\geq 0} G_t(x,q|R),\]

where

\[G_t(x,q|R) = R(d) - c^R q - hE[x+q-d-\xi]^+ - \pi^R E[d+\xi-x-q]^+
+ \beta E v_{t+1}(x+q-d-\xi|R)\]

with \(v_{T+1}(x|R) = c^R x^+ + sx^+\). Denote \(\bar{x}^L_t\) be the threshold where no production takes place. Let \(q^R_t(x)\) denote the optimal production quantity to optimize \(G_t(x,q|R)\) and \(\Delta^R_t(x) = x + q^R_t(x)\) be product availability for established buyers, then the production policy for a regular supplier in Model \(R\) is formalized in Theorem 3.

**Theorem 3:**
(a) When \( x < \bar{x}_i^R \), \( q_i^R(x) \) decreases in \( x \) with \( q_i^R(x) = -1 \).

(b) When \( x \geq \bar{x}_i(R) \), \( q_i^R(x) = 0 \).

(c) \( \Delta^R(x) \) is fixed for \( x \leq \bar{x}_i^R \).

Theorem 3 (a)(b)(c) fully present how a regular supplier leverages his production amount by observing its product availability which are identical to the structure in Model \( L \) with \( \gamma(\cdot) = 0 \) shown in Fig. 2. A regular supplier prepares the stock level up to a fixed level since leftover always can be re-sold to establish buyers in the following periods, leaving a regular supplier without incentive to adjust this stock level. In most of OM research, it is referred to as order-up-to level. To sum up, by comparing the production policy of a supplier evolving from the regular process to entrepreneurial process, the market exploitation to potential buyers results in that a supplier, by taking advantage of potential buyers paying a high premium, does not necessarily need to have the marginal production quantity equal to \(-1\).

CONCLUSION

An increasing number of players in supply chain embed the responsible behaviors as part of their competitiveness, aiming at not just satisfying current market demand but further exploiting the business opportunity to gain more market demand. This work develops three quantitative models to analyze a supplier’s production structure when an entrepreneurial process is devoted. The result reveals that such a structure is not consistent with the conventional order-up-to level replenishment policy. Further, the numerical results reveal that with customers’ high sensitivity of exploiting the business opportunity, a supplier maintains a fixed production amount even though net inventory is high; on the other side, with customers’ low sensitivity of exploiting the business opportunity, a supplier will make no production when net inventory is sufficiently high.

With this motivation and current results, we now discuss some extensions to enrich the finding in this research area. As seen in Models \( L \) and \( R \), demand market incorporates established buyers. For profit-maximizing, in addition to creating value of leftover by re-selling it to established buyers, we can extend to consider the mechanism of dynamic pricing. In other words, a supplier considers an endogenous price charged to established buyers for each supply opportunity with the aim of gaining better profitability. Another extension is to consider the production risk during the entrepreneurial process, depicted by the reliability level \( \rho \). That is, a smaller \( \rho \) refers to a higher risk. Such risk is commonly seen in many business practices when a firm launches a new process and the possible causes may impute to the immaturity of skills which are still undergoing the learning curve. With this, albeit producing \( q \) units at the beginning, a supplier can only supply a total amount \( \rho q \) to its buyers.

To summarize, this work pioneers to discuss a supplier production control when the entrepreneurial process is adopted. In essence, we have opened a door to a new corridor that we hope others will follow.
REFERENCES


**APPENDIX**

When proving Theorem 1 and 2, for an easy exposition, we drop superscript $E$ from the unit production cost and backlogging cost.

**Proof of Theorem 1.** We will prove the results by induction on period $t$. We firstly show them in period $T$. For a given $x$, the concavity of $G_T(x, q | E)$ in $q$ is seen by taking the first and second derivatives that

$$
\frac{\partial G_T(x, q | E)}{\partial q} = (p^E + h - \beta s)\gamma'(q) - c - h + \beta s
$$

$$
\frac{\partial^2 G_T(x, q | E)}{\partial q^2} = (p^E + h - \beta s)\gamma''(q) \leq 0.
$$

Setting $\frac{\partial G_T(x, q | E)}{\partial q} \bigg|_{q=q^*_T(x)} = 0$ results in

$$
\gamma'(q^*_T(x)) = \frac{c + h - \beta s}{p^E + h - \beta s},
$$

which is irrelevant to $x$. Thus, let $q^*_T$ denote the optimal production amount in period $T$.

Further,

$$
v_T'(x | E) = \left. \frac{\partial G_T(x, q | E)}{\partial x} \right|_{q=q^*_T} = -h + \beta s.
$$

Next, suppose $v_k(x | E)$ is concave in $x$ with $v_k(x | E) = \beta^{r-k+1} - \sum_{m=k}^{T} \beta^{T-m} h$ (a negative, $x$- independent value) for $k = t+1, t+2, \ldots, T$ and consider $k = t$. We show that $G_t(x, q | E)$ is concave in $q$ by taking the first and second derivatives with respect to $q$.

$$
\frac{\partial G_t(x, q | E)}{\partial q} = (p^E + h)\gamma'(q) - c - h + [1 - \gamma'(q)]v_{t+1}'(x + q - \gamma(q) | E)
$$

$$
= (p^E + h - v_{t+1}'(x + q - \gamma(q) | E))\gamma'(q) - c - h + v_{t+1}'(x + q - \gamma(q) | E)
$$

$$
\frac{\partial^2 G_t(x, q | E)}{\partial q^2} = (p^E + h - v_{t+1}'(x + q - \gamma(q) | E))\gamma''(q) \leq 0.
$$

Setting $\frac{\partial G_t(x, q | E)}{\partial q} \bigg|_{q=q^*_t(x)} = 0$ derives
$$\gamma'(q^E_t(x)) = \frac{c + h - v'_{i+1}(x + q - \gamma(q) | E)}{p^E + h - v'_{i+1}(x + q - \gamma(q) | E)},$$

which once again is irrelevant to $x$. Thus, let $q^E_t$ denote the optimal production amount in period $t$. Furthermore, $q^E_t \leq q^E_{t+1}$ due to the non-decreasing property of $\gamma(x)$.

This completes the proof.

**Proof of Theorem 2.** We will prove the results by induction on period $t$. We firstly show them in period $T$. For a given $x$, the concavity of $G_T(x, q | L)$ in $q$ is seen by taking the first and second derivatives that

$$\frac{\partial G_T(x, q | L)}{\partial q} = (p^E - \pi - \beta c)\gamma(q) - c + \pi + \beta c$$

$$- (h + \pi - \beta s + \beta c)(1 - \gamma'(q))F(x + q - d - \gamma(q))$$

$$\frac{\partial^2 G_T(x, q | L)}{\partial q^2} = (p^E - \pi - \beta c)\gamma(q)' - (h + \pi - \beta s + \beta c)(1 - \gamma'(q))^2$$

$$\cdot f(x + q - d - \gamma(q)) - \gamma(q)' F(x + q - d - \gamma(q)) \leq 0$$

The above justification shows the concavity. To show the existence of $x^L_T$, such that $q^L_T(x) > 0$ when $x < x^L_T$ and $q^L_T(x) = 0$ when $x \geq x^L_T$, we relax $q^L_T(x)$ to temporarily neglect the restriction $q \geq 0$ and, through the Implicit Function Theorem, Eq. (1) derives

$$q^L_T(x) = - \frac{[1 - \gamma'(q^L_T(x))]x^L_T(x)}{[1 - \gamma'(q^L_T(x))]x^L_T(x) + \psi^L_T(x)},$$

where

$$x^L_T(x) = \left[-h - \pi + \beta s - \beta c\right]f(x + q - d - \gamma(q))] \leq 0$$

$$\psi^L_T(x) = \left[(p^E - \pi - \beta c)\gamma(q) + (h + \pi - \beta s + \beta c)\gamma(q)' F(x + q - d - \gamma(q))ight] \leq 0.$$ 

Explicitly, $q^L_T(x) \leq 0$, implying the existence of $x^L_T$. We also can express Eq. (2) as the form

$$q^L_T(x)[1 - \gamma'(q^L_T(x))] = - \frac{[1 - \gamma'(q^L_T(x))]x^L_T(x)}{[1 - \gamma'(q^L_T(x))]x^L_T(x) + \psi^L_T(x)} \in [-1, 0].$$

Now consider the restriction $q \geq 0$. The production policy is discussed based on either $x < x^L_T$ or $x \geq x^L_T$.

(i) When $x < x^L_T$, clearly, $q^L_T(x)$ derived by setting $\frac{\partial G_T(x, q | L)}{\partial q}_{q=q^L_T(x)} = 0$ does not violate the restriction $q \geq 0$. Thus, $v_T(x | L) = G_T(x, q^L_T(x) | L)$. We next show that $v_T(x | L)$ is concave in $x$. 

Chen Production structure
\[ v'_r(x | L) = \frac{\partial G_r(x, q | L)}{\partial x} \bigg|_{q=q^L_r(x)} + \frac{\partial G_r(x, q | L)}{\partial q} \bigg|_{q=q^L_r(x)} q^L_r(x) \]

\[ = (-h - \pi + \beta s - \beta c) F(x + q^L_r(x) - d - \gamma(q^L_r(x))) + \beta c + \pi \]

and

\[ v^0_r(x | L) = (-h - \pi + \beta s - \beta c)[1 + q^L_r(x)](1 - \gamma'(q^L_r(x))] f(x + q^L_r(x) - d - \gamma(q^L_r(x))) \leq 0 \]

by referring to Eq. (3). Therefore, the concavity of \( v_r(x | L) \) is seen. In addition, Eq. (3) indicates that

\[ \frac{\partial}{\partial x} [x + q^L_r(x) - \gamma(q^L_r(x))] \in [0,1]. \]

This shows that product availability for regular buyers at the beginning of period \( T \),

\( \Delta^L_T = x + q^L_r(x) - \gamma(q^L_r(x)) \), is non-decreasing in \( x \).

(ii) When \( x > \overline{x}_r^L \), \( q^L_r(x) \) derived by setting \( \frac{\partial G_r(x, q | L)}{\partial q} \bigg|_{q=q^L_r(x)} = 0 \) is a negative value which violates the restriction \( q \geq 0 \). Due to the concavity of \( G_r(x, q | L) \) in \( q \), the optimal solution is to set \( q^L_r(x) = 0 \) and

\[ v_{r, \{1\}} = G_r(1,0 | L) = R_d(1\cdot q^{L}_r + \beta c) \int_{x}^{\gamma(q^L_r(x))} f(x - d \cdot l(x)) dx. \]

With this, we proceed to show that \( v_r(x | L) \) is concave in \( x \). By taking the first and second derivatives,

\[ v'_r(x | L) = (-h - \pi + \beta s - \beta c) F(x - d) + \beta c \]

\[ v^0_r(x | L) = (-h - \pi + \beta s - \beta c) f(x - d) \leq 0. \]

To sum up (i) and (ii), \( q^L_r(x | L) > 0 \) when \( x < \overline{x}_r^L \) and \( q^L_r(x | L) = 0 \) when \( x \geq \overline{x}_r^L \).

Further, \( v_r(x | L) \) is concave in \( x \) with \( v'_r(x | L) \leq \beta c + \pi \).

Next, suppose \( v_s(x | L) \) is concave in \( x \) with \( \sum_{k=1}^{t+1} \sum_{e} e_{s,e}^k \) for \( k = t+1, t+2, \ldots, T \) and consider \( k = t \). We firstly show that \( G_r(x, q | L) \) is concave in \( q \). By taking the first and second derivatives with respect to \( q \),

\[ \frac{\partial G_r(x, q | L)}{\partial q} = (p_F - \pi) \gamma'(q) - c + \pi - \sum_{k=1}^{t+1} \sum_{e} e_{s,e}^k \gamma'(q) F(x + q - d - \gamma(q)) \]

\[ + \beta (1 - \gamma'(q)) \int v_{s,e}(x + q - d - \gamma(q) | L) f(\xi) d\xi \]

and

\[ \frac{\partial^2 G_r(x, q | L)}{\partial q^2} = (p_F - \pi) \gamma''(q) - (h + \pi)(1 - \gamma'(q))^2 f(x + q - d - \gamma(q)) \]

\[ - \gamma'(q) F(x + q - d - \gamma(q)) + \beta \int \gamma'^2(q) v_{s,e}(x + q - d - \gamma(q) | L) \]

\[ - \gamma(q) v_{r+1}(x + q - d - \gamma(q) | L) f(\xi) d\xi. \]
Since \( \psi(x|L) \) by induction, \( p^x - \pi - \beta \int v_i(x + q - d - \xi - \gamma(q)|L) f(\xi)d\xi \geq 0 \) indicates that
\[
\frac{\partial^2 G_i(x,q|L)}{\partial q^2} \leq 0.
\]

The above justification shows the concavity. To show the existence of \( \lambda_i^L \) such that
\( q_i^L(x) > 0 \) when \( x < \lambda_i^L \) and \( q_i^L(x) = 0 \) when \( x = \lambda_i^L \), we relax \( q_i^L(x) \) to temporarily neglect the restriction \( q \geq 0 \) and, through the Implicit Function Theorem, Eq. (4) derives
\[
q_i^L(x) = -\frac{\chi_i^L(x)}{\omega_i^L(x) + \psi_i^L(x)},
\]
where
\[
\chi_i^L(x) = -(h+\pi)[1 - \gamma'(q_i^L(x))] f(x + q_i^L(x) - d - \gamma(q_i^L(x))]
\]
\[
+\beta \int [1 - \gamma'(q_i^L(x))] v_i(x + q_i^L(x) - d - \xi - \gamma(q_i^L(x))|L) f(\xi)d\xi \geq 0.
\]

Explicitly, \( q_i^L(x) \leq 0 \), showing the existence of \( \lambda_i^L \). Eq. (5) can be expressed as
\[
q_i^L(x)(1 - \gamma'(q_i^L(x))) = -\frac{\chi_i^L(x)}{1 - \gamma'(q_i^L(x))} \in [-1,0].
\]

We now consider the restriction \( q \geq 0 \). Similar to the case in period \( T \), the production policy is discussed based on either \( x < \lambda_i^L \) or \( x \geq \lambda_i^L \).

(i) When \( x < \lambda_i^L \), clearly, \( q_i^L(x) \) derived by setting
\[
\frac{\partial G_i(x,q|L)}{\partial q}_{q=q_i^L(x)} = 0
\]
does not violate the restriction \( q \geq 0 \). Thus, \( v_i(x|L) = G_i(x,q_i^L(x)|L) \) and the concavity in \( x \) is shown below.
\[
v_i(x|L) = \frac{\partial G_i(x,q|L)}{\partial x}_{q=q_i^L(x)} + \frac{\partial G_i(x,q|L)}{\partial q}_{q=q_i^L(x)} q_i^L(x)
\]
\[
= -(h+\pi) f(x + q_i^L(x) - d - \gamma(q_i^L(x))]
\]
\[
+\beta \int v_i(x + q_i^L(x) - d - \xi - \gamma(q_i^L(x)))|L) f(\xi)d\xi + \pi
\]
\[
\leq [1 + \gamma'(q_i^L(x))] \left(1 - \gamma'(q_i^L(x))\right) \left[ -(h+\pi) f(x + \rho q_i^L(x) - d - \gamma(q_i^L(x))]
\]
\[
+\beta \int v_i(x + \rho q_i^L(x) - d - \xi - \gamma(q_i^L(x)))|L) f(\xi)d\xi \right] \leq 0
\]
by referring to Eq. (6) and assuming \( v_i^\psi(\cdot|L) < 0 \) by induction. Therefore, \( v_i(x|L) \) shows the concavity. Eq. (6) also indicates that
\[
\frac{\partial [x + q_i^L(x) - \gamma'(q_i^L(x))]}{\partial x} \in [0,1],
\]
implying that product availability for regular buyers at the beginning of period $t$, 
$\Delta_i^t = x + q_i^L(x) - \gamma q_i^L(x)$, is non-decreasing in $x$.

(ii) When $x > \bar{x}_i^L$, $q_i^L(x)$ derived by setting $\partial G_i(x, q | L) \bigg|_{q=q_i^L(x)} = 0$ is a negative value which violates the restriction $q \geq 0$. Due to the concavity of $G_i(x, q | L)$ in $q$, the optimal decision is to set $q_i^L(x) = 0$ from which

$$v_i(x | L) = -(k + \beta) f(x + d) + \beta \int v_{i,0}(x + d - v | L) f(v) dv + x,$$

by assuming $v_{ij}^n (\cdot | L) \leq 0$ through induction.

Once again to sum up (i) and (ii), $q_i^L(x) > 0$ when $x < \bar{x}_i^L$ and $q_i^L(x) = 0$ when $x \geq \bar{x}_i^L$.

Further, $v_i(x | L)$ is concave in $x$ with $v_{ij}^n (\cdot | L) \leq 0$.

This completes the proof.