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Trading Durations and Realized Volatilities - A Case from Currency Markets

(Full Paper Submission)

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ABSTRACT

A new class of arbitrage bounds derived from the difference in traded time of previously locally traded assets in Geographically Separate Exchanges is being proposed. These assets are identical in terms of underlying, time to expiry, settlement and differ only in trading durations. We study the magnitude of Intraday and Overnight returns in Currency Markets, specifically Indian Rupee Markets and examine how the overnight risks can be managed better by trading in Offshore Rupee Exchanges. The implications of volatility differences are quite significant in terms of competitive behavior of Exchanges, Openness of Economies and Pricing of Options on such assets.

KEYWORDS: Market Risk, Realized Volatility, Intraday Returns, Overnight Risk, Currency Markets, Exchanges, Hedging

INTRODUCTION

Multiple listing of Stocks and Indices has led to fragmentation of trading activity across geographic locations. In such a scenario, multiple arbitrage conditions come into play. However, the difference in trading times also allow for a new class of Arbitrage Bounds. In case, there are differences in trading times, what should be the volatility that ought to be used to price these derivatives? Also how will this affect the competitive nature of exchanges? Will Exchanges be forced to increase trading durations? These are significant questions that need to be addressed.

In this paper, we provide evidence of the difference in realized volatilities for onshore and offshore exchanges taking into consideration the price dynamics of Indian Rupee. In 2008, Rupee trading was allowed on Indian Exchanges in addition to OTC markets. Major currencies trade round the clock; however Indian Rupee trades onshore from 9 AM IST to 5 PM IST for a duration of 8 hours. Several exchanges, most notably Dubai Gold and Commodities Exchange (DGCX) also started trading of INR. INR on DGCX trades for 7 AM DST to 11:30 PM DST (8:30 AM IST to 01:00 AM IST) overlapping the onshore trading duration and exceeding it by 10 hours. The INR futures trading on both Indian and Offshore Exchange expire at official US Dollar reference rate issued by the Reserve Bank of India, based on bank rates in Mumbai at 12:30 PM on the day of trading or earliest available date.

Rupee Options can be hedged using either of these instruments. However, both of these Futures have different realized volatilities. In case, one of the instruments consistently shows
higher realized volatility, then traders would prefer to hedge using that instrument. Then this can have a significant impact on Volumes at Exchanges.

The organization of the paper is as follows. Section 2 describes current literature. Section 3 details the data and empirical methodology. Section 4 presents the main results. Section 5 concludes.

LITERATURE REVIEW

The realized volatility of an asset is the volatility of the actual historical returns of an asset. Thus, unlike implied volatility which is a forward-looking prediction of volatility, realized volatility is backward looking. The basic motivation comes from the observation about which volatility should be used for pricing options of assets which are traded for unequal durations.

The debate for informationally efficient markets has captured the imagination of academicians over the last four decades. The case for information being efficiently assimilated and reflected in markets when they are open has been attempted, however high volumes at Openings and Closing of Market Sessions reflect the uncertainty and assimilation of Overnight Information. Cooper, Cliff and Gulen (2008) show that US equity returns over the years 1996-2006 are solely due to overnight returns i.e. the overnight returns are strongly positive, and intraday returns are close to zero. Lou, Polk and Skouras (2013) similarly show for a period of 20 years that almost all of the abnormal returns in the momentum strategy are generated overnight, rather than intraday. From the perspective of Portfolio Managers, Overnight Risk is quite significant. The near universal acceptance and usage of VaR emphasizes importance that financial institutions put on risk management. Several assets trade in different time zones with different trading durations at separate exchanges. These instances provide financial institutions an opportunity to hedge their overnight risks by continuing to trade even after domestic exchange has closed for the day.

Admati and Pfleiderer (1988) show that variance of price changes and variance of returns follows a U-shaped pattern for intraday trading. Foster and Viswanathan (1990) conclude that variance of price returns reduce over the week and Friday is the only day with concentrated trading.

Measuring intraday volatility is one of the more difficult tasks facing financial researchers and practitioners. There are a number of efficient methods of estimating volatility. However, the efficiency of volatility estimators decreases as the kurtosis of the return distribution increases, and intraday data are widely known to have thick tails. The distribution of the trading range of a security that follows geometric Brownian motion has been extensively studied. Parkinson (1980) first considered this problem. Using a distribution first derived by Feller (1951), Parkinson found a variance estimator for a security whose log follows a zero-mean diffusion and it is about 5 times as efficient as the conventional close-to-close estimator. Garman and Klass (1980) extend Parkinson’s approach, incorporating the open and close prices and the trading hours of the security. Despite theoretical results suggesting range-based estimators are several times more efficient than classical ones, empirical tests fail to demonstrate their superiority. They use simulation to show that range-based estimators are sensitive to discreteness in price changes, producing downward biased estimates. However, if the two assets are highly correlated then intraday overlapping period volatility is similar.

The possibility of using intraday trade close prices for calculating realized volatility has been established in Garman-Klass (1980) specifically in case of a discontinuous trading scenario. The
data used for the study is traded Open, High, Low, Close prices, which can be significantly different that the High and Low Bid/Ask Quotes for the day. In that sense, analysis should consider using the bid/ask prices for calculation of the realized volatility but using the traded high/low is a close approximation.

The realized variance measures allow for estimates of latent volatility based on high frequency data. In this case, volatility is observable and traditional time series models can be applied.

DATA AND METHODOLOGY

The statistics of the process $P$ sampled at evenly spaced times $i\Delta_n$ for $i = 0, \ldots, \lfloor T/\Delta_n \rfloor$, and $P_{it}$ is the log price of asset. The process $P$ is discontinuous with jumps present in underlying price process. The periods of closure of the markets results in the price process having large price increments that are often recorded from the previous close to the new open price. Consider $m = 1/\Delta = \text{number of sampled observations per trading session};$ and with $T$ number of days in the sample, results in $m*T$ total observations. Returns for intraday $t$, $r_{i,t} = r_{i,t-1} + \Delta + r_{i,t-1} + 2\Delta + \cdots + r_{i,t-1} + m\Delta$. Overnight returns for day $t$, $r_{\text{overnight}} = r_{t,1} - r_{t-1} + m\Delta$, and Realized variance (RV) for process on day $t$,

$$RV_t^m = \sum_{j=1}^{m} r_{t-1+j\Delta}^2 + t = 1, \ldots, T \quad (1)$$

Realized volatility (RVOL) for process on day $t$ is

$$\left(\text{RVOL}\right)_t^{(m)} = \sqrt{RV_t^{(m)}} \quad (2)$$

This process measures the realized sample path variation of the squared return process and is unique and invariant ex-post realized volatility measure that is essentially model free.

The NSE and DGCX INR Futures intraday data has been obtained from Bloomberg for the period between Oct-2014 to Mar-2015. The intraday return is defined as market open and close of the same day and overnight return is calculated from intraday return and close-close return. In this manner, the overnight and intraday return aggregate to the close-close return for a given period.

$$r_{\text{intraday}} = \ln \frac{P_{\text{Close}}}{P_{\text{Open}}} \quad (3)$$

$$r_{\text{overnight}} = \frac{r_{\text{Close}}}{r_{\text{intraday}}} \quad (4)$$
In order to evaluate difference in Realized Volatility to establish bounds for intraday volatility and overnight volatility, the trading day is segmented into smaller time periods. USDINR on NSE runs from 9:00 AM to 5:00 PM IST and on DGCX runs from 8:30 AM to 1:00 AM IST. Appendix I contains the details of the instruments. Time intervals of 15 minutes are used for calculation of Realized Volatility. So there will be 32 periods for USDINR on NSE and 66 periods on DGCX. The 32 periods on NSE are overlapping in DGCX. The Realized Volatility Calculation is thus:

\[ \tau_{Close} = \ln \frac{P_{Close}}{P} \]  

Figure 1: Price vs Time at NSE and DGCX

The overnight effect of “close to open” volatility, INR Futures on NSE can close at price \( P_1 \) and next day open at \( P_2 \). Meanwhile INR Futures on DGCX approximately sees price \( P_1 \) when NSE closes but continues to trade and then closes at Price \( P_3 \) and next day opens before NSE at \( P_4 \). However at time of open of INR Futures at NSE, Price at DGCX shall be approximately \( P_2 \) only. In order to calculate the mathematical bounds of difference in Realized Volatility of the two instruments, it shall reduce to calculating lower bounds of

\[ \left( \ln \frac{P_2}{P_1} \right)^2 - \left[ \left( \ln \frac{P_3}{P_1} \right)^2 + \left( \ln \frac{P_4}{P_3} \right)^2 + \left( \ln \frac{P_2}{P_4} \right)^2 \right] \]  

(7)

The minimum of this would occur when \( \ln(P_3) - \ln(P_1) = \ln(P_4) - \ln(P_3) = \ln(P_2) - \ln(P_4) \) i.e. all partitions are equal. This can be easily proved by the method of Lagrange multipliers, and hence we skip the proof.

Equation 7 would reduce to
Then as $n$ increases, the upper bound of the differences in Realized Variances tends to Overnight Variance at NSE. This is the maximum loss in a strategy.

**Case II**

The Upper side is the difference of the sum of "realized volatility of the non-overlapping period in DGCX and Overnight Volatility at DGCX" and the Overnight Volatility at NSE. This difference in trading duration has never been factored into creating an arbitrage opportunity.

The data of the USDINR contracts trading on NSE and DGCX for the study is taken from Bloomberg. The 15 min data has been taken for both the exchanges from October 2014 contract to March 2015 contract. These securities expire and settle on the Price Fixed by the Reserve Bank of India. So the convergence is guaranteed.

Rupee expires on the two working days prior to the last business day of the expiry month at 12:30 p.m. The November futures would expire but the December Futures would be active. So November Futures Volatility calculation is made only till the 12:15 PM of the Expiry day. This is for all exchanges.

The intraday data for the volatility calculations is trade data. The bid-ask are not used and in illiquid markets, Prices may go move but they might not trade. As a result certain price movements may be missed in these calculations. There are certain days when either exchange is closed, however such days are considered in the analysis.

**RESULTS**

The results of the USDINR listed on NSE and DGCX shows that in all instances except for one month, DGCX USDINR has had a higher realized volatility that NSE USDINR contract. Since they are nearly perfectly correlated and have definitive convergence, this difference in realized volatility can be attributed to longer trading hours of DGCX contract. As a result, it also has more variation in the ranges realized.

<table>
<thead>
<tr>
<th>Month</th>
<th>NSE Monthly RV</th>
<th>DGCX Monthly RV</th>
<th>Outperformance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct-14</td>
<td>1.809%</td>
<td>1.903%</td>
<td>5.201%</td>
</tr>
<tr>
<td>Nov-14</td>
<td>0.803%</td>
<td>1.764%</td>
<td>119.806%</td>
</tr>
<tr>
<td>Dec-14</td>
<td>2.512%</td>
<td>2.473%</td>
<td>-1.545%</td>
</tr>
<tr>
<td>Jan-15</td>
<td>3.065%</td>
<td>3.318%</td>
<td>8.240%</td>
</tr>
</tbody>
</table>
Based on the above data, the estimated Annualized Returns at NSE is 0.073 with 95% confidence interval of (0.052, 0.094). Similarly for DGCX, the estimated Annualized Returns is 0.081 with 95% confidence interval of (0.066, 0.097). The above Annualized Returns estimates have been calculated through bootstrap technique using the monthly data of Realized Volatility for the given 6 months.

Table 2: Intraday and Overnight Realized Volatility at NSE and DGCX for INR Futures.

<table>
<thead>
<tr>
<th></th>
<th>NSE</th>
<th>DGCX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intraday Volatility</td>
<td>3.17%</td>
<td>7.55%</td>
</tr>
<tr>
<td>Overnight Volatility</td>
<td>4.08%</td>
<td>0.69%</td>
</tr>
<tr>
<td>Intraday Volatility in NSE Time</td>
<td>3.39%</td>
<td></td>
</tr>
</tbody>
</table>

The average volatility at DGCX is nearly 13% more than the volatility at NSE for the USDINR contract. The significant outperformance by the longer trading contract allows for the validation that realized volatility differences can be exploited by hedgers. A much better indicator of the realized volatility would have been a price based volatility indicator instead of a time-based indicator.

We also calculate the intraday and overnight realized volatility for NSE and DGCX and realize that Intraday Realized Volatility for DGCX is higher than NSE. Also intraday volatility in DGCX for the period of NSE of 3.39% is close to NSE Intraday Volatility of 3.17% which shows that in this overlapping period the markets are highly correlated. Also the overnight realized volatility at NSE of 4.08% is higher than that of overnight realized volatility at DGCX of 0.69%. Since the overnight realized volatility is lower at DGCX, hence the overnight hedge risk is lower. This shows that DGCX remaining open for a longer duration has a significant impact on risk faced by the traders in INR futures markets.

DISCUSSION AND CONCLUSIONS

This is the first time, trading durations have been used a basis for Volatility Arbitrage bounds. The significant difference in the realized volatility of the same asset because of different trading durations shall have major implications for the trading community. The reduction in Market Risk due to longer trading hours, such exchanges would have a competitive advantage over other exchanges. The overnight Gap Risk is also reduced as the information generated in the world markets are assimilated with lower risk if markets are open for longer durations. It would also be due to entities trading on shorter trading time exchanges would be overpricing the volatility as compared to trading on longer trading time. It would not be possible to make the two instruments non-arbitragable and in the long term, exchanges would have to have uniform trading times.

The second major implication would be the regulatory cost borne by exchanges can be estimated as a function of the difference between the realized volatilities. The third and most
important implication would be for the volatility pricing. Now the traders would have to find a suitable proxy for the multiple realized volatilities possible for the same asset and this shall have major impact on the derivative pricing.

The estimator used for the realized volatility calculation is based on a time scale; however the volatility should be calculated on the basis of the price scale. In order to do that, a new estimator based on trade price movement to calculate realized volatility at tick levels is needed.

### APPENDIX

**NSE USDINR Contract Specifications**

<table>
<thead>
<tr>
<th>Trading</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>USDINR</td>
</tr>
<tr>
<td>Contract Size</td>
<td>USD 1000</td>
</tr>
<tr>
<td>Delivery Months</td>
<td>Monthly contracts for twelve months forward</td>
</tr>
<tr>
<td>Last Trading Day</td>
<td>Two Business Days prior to the last working day of the contract month</td>
</tr>
<tr>
<td>Settlement Day</td>
<td>The Business Day immediately following the last day of expiring contract</td>
</tr>
<tr>
<td>New Contract Listing</td>
<td>Business day immediately following the last trading day</td>
</tr>
<tr>
<td>Price Quote</td>
<td>INR quoted in paise per 100 Indian Rupees (e.g. 61.52/61.53 Indian Rupees per US Dollars)</td>
</tr>
<tr>
<td>Tick Size</td>
<td>INR0.0025 per tick</td>
</tr>
<tr>
<td>Trading Days</td>
<td>Monday through to Friday</td>
</tr>
<tr>
<td>Trading Hours</td>
<td>09:00 - 17:00 Hours Indian Standard time (GMT+530)</td>
</tr>
<tr>
<td>Cash Settlement Price</td>
<td>The DSP would be based on the official US Dollar reference rate issued by the Reserve Bank of India, based on bank rates in Mumbai at 12 noon on the day of trading or earliest available date</td>
</tr>
</tbody>
</table>

**DGCX USDINR Contract Specifications**

<table>
<thead>
<tr>
<th>Trading</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>DINR</td>
</tr>
<tr>
<td>Contract Size</td>
<td>INR 2,000,000</td>
</tr>
<tr>
<td>Delivery Months</td>
<td>Monthly contracts for twelve months forward</td>
</tr>
<tr>
<td>Last Trading Day</td>
<td>Two Business Days prior to the last working day of the contract</td>
</tr>
</tbody>
</table>
Settlement Day
The Business Day immediately following the last day of expiring contract

New Contract Listing
Business day immediately following the last trading day

Price Quote
US$ quoted in Cents per 100 Indian Rupees (e.g. 209.56 /209.62 US Cents per 100 Indian Rupees)

Tick Size
US$ 0.000001 per INR or US$ 2 per tick

Trading Days
Monday through to Friday

Trading Hours
07:00 - 23:30 Hours Dubai time (GMT+4)

Cash Settlement Price
Open Positions at expiry of contract shall be settled in US Dollars as per the Daily Settlement Price (DSP) declared by the Exchange. The DSP would be based on the official US Dollar reference rate issued by the Reserve Bank of India, based on bank rates in Mumbai at 12 noon on the day of trading or earliest available date

REFERENCES


Cooper, M. J., Cliff, M. T., & Gulen, H. (2008). Return differences between trading and non-trading hours: Like night and day. *Available at SSRN 1004081.*


