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Renewable Two-Dimensional Pro-Rata Warranty Analysis for Reverse Supply Chain

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ABSTRACT

This paper presents an analysis of renewable two-dimensional Pro-Rata Warranty (PRW) policy for an Advanced Remanufacturing-To-Order system for Sensor-embedded products (SEPs). The goal is to predict a warranty period for end-of-life (EOL) components using the sensor information about age and usage of each and every EOL product on hand to meet product, component and recycled material demands while minimizing the cost associated with warranty and maximizing remanufacturer's profit.

KEYWORDS: Reverse Supply Chain, Simulation, Non-Renewable Warranty Policies, Remanufacturing, Sensor-Embedded Products.

INTRODUCTION

Recently, the number of studies dealing with the end-of-life (EOL) stage of a product has gained much attention from researchers (Gungor and Gupta 1999), (Ilgin, and Gupta 2010b). This is due, on one hand, to environmental factors, government regulations and public demands, and on the other hand, to potential economical profits that could be obtained by implementing reverse logistics and product recycling processes. Manufacturers try to cope with consumer awareness towards environmental issues and stricter environmental legislation by setting up facilities that involve the minimization of the amount of waste sent to landfills by recovering materials and components from returned or EOL products (Gungor and Gupta 2002).The chief concern in such product recovery processes is the uncertainty in quantity and quality of recoverable materials, due primarily to the lack of any information about the condition of components before disassembly. The obvious solution is to test each and every component after disassembling them. This, however, can harm the profitability of a remanufacturer depending on testing time and testing costs. Furthermore, if the test shows that the component is non-functional, then all of the allocated time and resources are wasted.

In product recovery, the disassembly process plays an important role since it allows for selective separation of desired parts and materials. EOL products containing missing and/or nonfunctional components increase the uncertainty associated with the disassembly yield. Sensor-embedded products (SEPs) eliminate a majority of uncertainties involved with EOL management by providing life-cycle information (Gupta and Lambert 2008), (Vadde et al. 2008). This includes information about the content of each product and component conditions that enables the estimation of remaining useful life of the components. Once the data about the product is captured, it is possible to make optimal EOL decisions without any preliminary disassembly or inspection operations (Ilgin, and Gupta 2010a, 2010b, 2010c, 2011). After the components are retrieved, the products can be remanufactured.

The quality of a remanufactured product is usually a suspect for consumers. That is, the consumers are unsure if the remanufactured products will render the expected performance. This ambiguity about a remanufactured product could lead the consumer to decide against buying it. With such apprehension held by consumers, remanufacturers often seek market mechanisms that provide assurance about the durability of the products. One strategy that the remanufacturers often use is to offer warranties on their products (Murthy and Blischke 2006).

Product warranties have three key roles. The first role is insurance and protection, allowing consumers to transfer the risk of product failure to sellers (Heal 1977). Next, product warranties can also signal product reliability to customers (Balachander 2001), (Gal-Or 1989), (Soberman 2003), (Spence 1977). Lastly, the sellers use warranties to extract additional profitability (Lutz and Padmanabhan 1995). There are a few articles and books that consider warranty policies for new products' supply chain management (Blischke 1993, 1995, 2011). However, there are not many papers that consider the warranty for the remanufactured products' reverse and closed-loop supply chain management. Base and extended one-dimensional and two-dimensional warranty can be offered for remanufacturing products using Free Replacement Warrantee (FRW) and Pro-Rata Warranty (PRW) policy (Ammar and Gupta 2015a, 2015b, 2015c, 2016a, 2016b, 2016c). The warranty policy and its effect on consumer behavior has been studied by Liao et al. 2015. Yazdia, et al. 2014 proposed a novel mathematical–statistical model where decisions involve pricing of returned used products (cores), the extent of their remanufacturing, selling price and the warranty period for the final remanufactured products to investigate the joint optimization of remanufacturing, pricing and warranty decision-making for end-of-life products. Kuik et al. 2015 presented mathematical models to examine two types of the proposed extended warranty policies for manufacturers to make the comparisons of their possible gained profit of remanufactured products by manufacturers.

SYSTEM DESCRIPTION

The Advanced Remanufacturing-To-Order (ARTO) system deliberated in this study is a product recovery system. A sensor embedded air conditioner (AC) is considered here as an example product. Based on the condition of EOL AC, it goes through a series of recovery operations similar to the one shown in figure 1. Refurbishing and Repairing processes may require reusable components to meet the demand of the product. This requirement satisfies the internal and the external component demand. Both will be satisfied using disassembled components.

EOL ACs arrive at the ARTO system for information retrieval using radio frequency data reader that are stored in the facility's database. Then the ACs go through a six-station disassembly line. Complete disassembly is performed to extract every single component. There are nine components in an AC consisting of, evaporator, control box, blower, air guide, motor, condenser, fan, protector, and compressor. Exponential distributions are used to generate the disassembly times at each station, interarrival times of each component's demand, and interarrival times of EOL AC. All End-Of-Life products (EOLPs) after retrieval of the information are transferred either to station 1 for disassembly or, if EOLP needs only repair for a specific component, to the corresponding station. Two different types of disassembly operations, viz., destructive or nondestructive, are used depending on the component's condition. If the disassembled component is nonfunctional (broken, zero remaining life), then destructive disassembly is used. The unit disassembly cost for a functional component is higher than nonfunctional component. After disassembly, there is no need for component testing due to the availability of information on components' conditions from sensors. It is assumed that the demands and life cycle information for EOLPs are known. It is also assumed that retrieval of information from sensors costs less than actual inspection and testing.

Recovery operations differ for each SEP based on its condition and estimated remaining life. Recovered components are used to meet components and spare parts demands, while recovered products are used for product demands. Also, material demands are met using recycled products and components. Recovered products and components are characterized based on their remaining lifetimes and are placed in different life-bins (e.g. one year, two years, etc.) waiting to be retrieved via a customer demand. Underutilization of any product or component could happen when it is qualified for a higher life-bin and is placed in a lower life bin because the higher life bin is full. Any product, component or material inventory that is greater than the maximum inventory allowed is assumed to be extra and is used for material demand or disposed of.

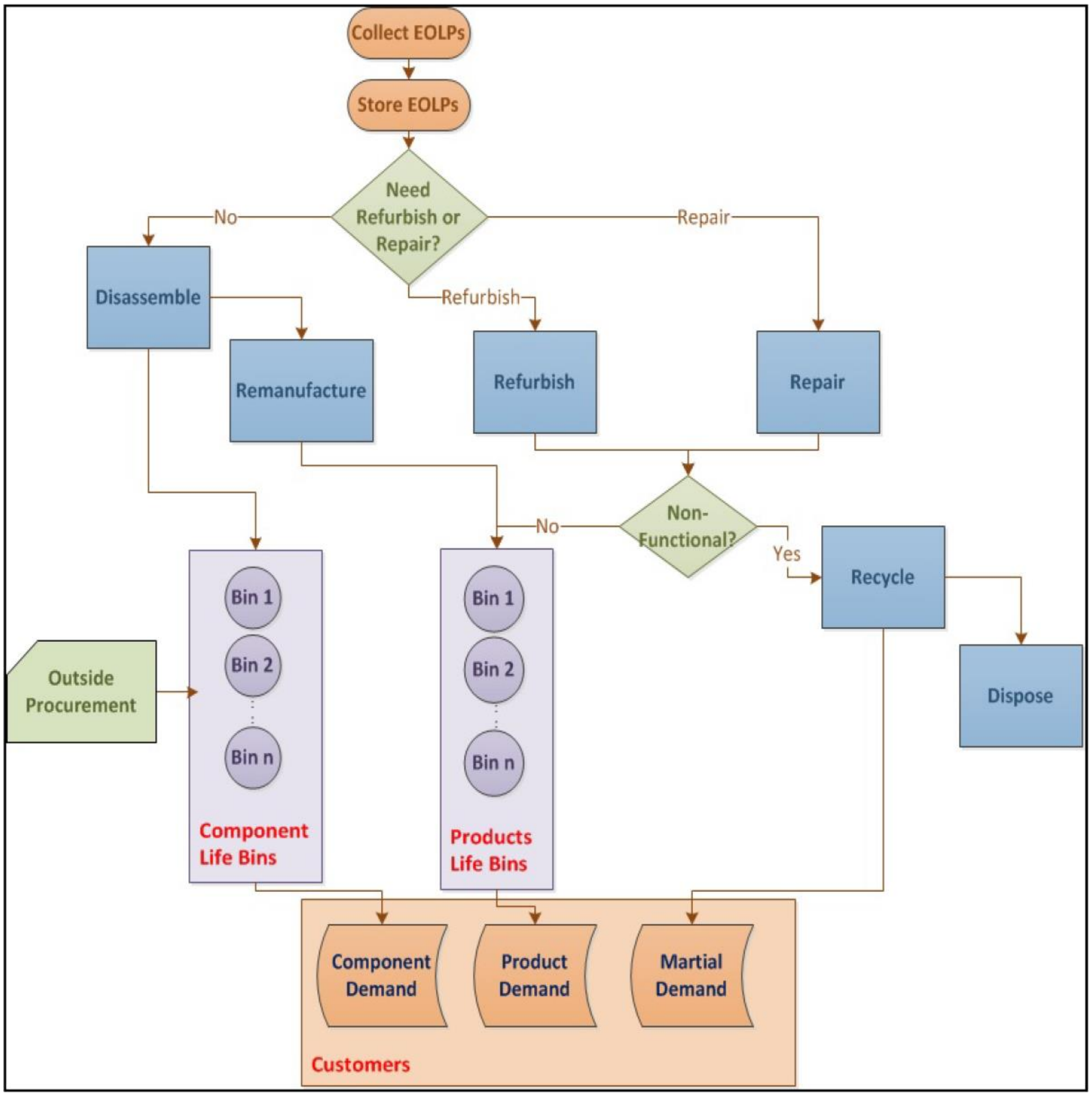

Figure 1: ARTO System's recovery processes

In order to meet the product demand, repair and refurbish options could also be chosen. EOLP may have missing or nonfunctional (broken, zero remaining life) components that need to be replaced or replenished during the repairing or refurbishing process to meet certain remaining life requirement. EOLP may also consist of components having lesser remaining lives than desired, and for that reason might have to be replaced.

TWO-DIMENSIONAL WARRANTY

In a one-dimensional warranty, the policy is valid for an interval called the warranty period, which is defined with respect of a signal variable such as age or usage. Instead, two-dimensional warranties are defined by a region in a two-dimensional plane, typically with one axis representing time or age and the other axis representing the usage. Therefore, many different types of warranties may be defined based on the shape of the warranty coverage region.

Two common possibilities about the region of coverage are shown in figure 2. Figure 2(a), shows the rectangular warranty region with independent limits on time and usage. This region has a tendency to favor the manufacturer because it put bounds on both the maximum time and the maximum usage for the buyer. For a buyer who is a heavy user, the warranty expires before the warranty period W end because usage has reached its maximum limit U. Likewise, for a buyer who is a light user, the warrant expires at time W with total usage below the limit U. In contrast, Figure 2(b) shows the Infinite strips warrant region, the buyer is assured with a minimum time of coverage and a minimum usage. This warranty region is in buyer favor. In this case, a heavy user is covered for a time period W, by which the usage may have exceeded the limit U, and a light user is covered well beyond the time period W, for the policy expires only when the total usage reaches the limit usage U.

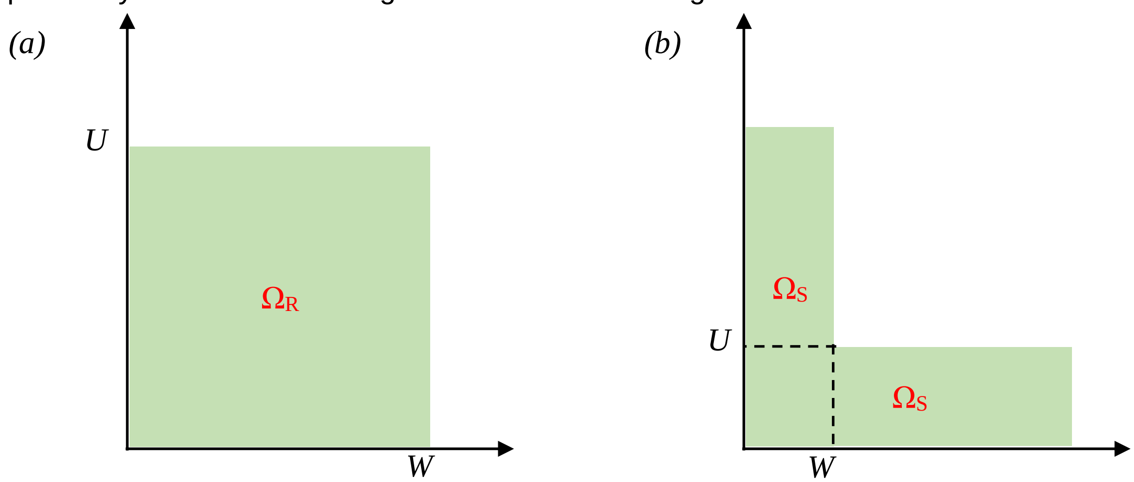

Figure 2: Two-dimensional warranty regions for usage versus time

RENEWING PRO-RATA WARRANTY COST ANALYSIS

In the process of deciding to purchase merchandise, the buyers usually compare features of a product with other competing brands that are selling the same product. In some cases, the competing brands make similar products to each other with similar features such as cost, special characteristics, quality and credibility of the product and even insurance from the provider. In these cases, after sale factors come into effect such as discount, warranty, availability of parts, repairs, and other additional services. These factors are very significant to the buyer in such situation and specially the warranty since it further assures the buyer of the reliability of the product.

A warranty is an agreement that requires the manufacturer to correct any product failures or compensate the buyer for any problems that occurs with the product during the warranty period in relevance to its sale. The objective of the warranty is to promote the products quality and guarantee its performance to assure production for both the manufacturer and the buyer.

There are many different available two-dimensional warrantee policies that most products are sold with. This paper considers the basic Pro-Rata Warrantee (PRW). The basic PRW is when the seller agrees to refund a fraction of the original sales price. The amount refunded depend on the age and usage of the remanufactured item at failure. In the case of unconditional refund, the buyer receives a cash refund and has no obligation to buy a replacement. In the case of conditional refund, the refund is tied to a replacement purchase.

NOMENCLATURE

In renewing policies, the warranty period begins anew with each replacement or repair. While in non-renewing policies, the repaired or replacement item is covered under warranty for the time remaining in the original warranty period. Under renewing PRW policy, a remanufactured product is replaced by a new one if a failure occurs under warranty. The new item comes with a new warranty. Since every failure results in replacement with a new one, the time between failures under warranty are a sequence of independent and identically distributed random variables.

Variables	***Definition***
W	Warranty period;
A	Warranty age limit;
U	Warranty usage limit;
S	Sales price;
C_s	Cost of each replacement;
C_r	Cost of each repair;
L	Remaining Life;
R	Usage rate;
r	Failure rate;
θ	Distribution parameters;
τ_r	Time at which warranty ceases conditional on R = r;
F(|)	Conditional distribution function;
$H(\tilde{\theta}\|r)$	Conditional probability of an item failing under warranty;
N	Number of replacements under warranty;
N_r	Stopping time for the renewal process;
$N(\tilde{\theta}\|r)$	Conditional renewal counting process;
Q	The total refund per item;
Q_r	The total refund per item conditional on R = r;
E[.]	Expected Value;
G(R)	Density function of a random variable r;

PROBLEM FORMULATION

Rectangular Region (Ω_R):

The probability that the remanufactured product fails under warranty is given by:

$$H(\tilde{\theta}|r) = \begin{cases} F(A|r) & if\ r < r_1 \\ F(\tau_r|r) & if\ r \geq r_1 \end{cases} \quad (1)$$

As a result, N_r is distributed according to a geometric distribution function with probability mass function given by:

$$Prob.\{N_r = n\} = H(\tilde{\theta}|r)^{n-1}\,[1 - H(\tilde{\theta}|r)] \quad (2)$$

Hence, the expected value of N_r is given by:

$$E[N_r] = \frac{1}{[1-H(\tilde{\theta}|r)]} \tag{3}$$

The total refund resulting from all replacement remanufactured products is given by:

$$Q_r = \sum_{i=1}^{N_r} R(T_i, rT_i|r) \tag{4}$$

The expected value of Q_r given by:

$$E[Q_r] = E[N_r]\, E[R(T_i, rT_i|r)] \tag{5}$$

The expected warranty cost (i.e. the expected total refund) per unit sale is given by:

$$EC(\tilde{\theta}) = E[Q] = \int_0^{\infty} E[\,Q_r]\, dG(r) = S\{\int_0^{r_1} [1/(1-F(A|r))\int_0^{K} \left(1-\frac{t_1}{A}\right)\left(1-\frac{rt_1}{U}\right) dF(t_1|r)]\, dG(r)$$
$$+ \int_{r_1}^{\infty}[1/(1-F(\tau_r|r))\int_0^{\tau_r}\left(1-\frac{t_1}{A}\right)\left(1-\frac{rt_1}{U}\right) dF(t_1|r)]\, dG(r)\,\} \tag{6}$$

Infinite strips Region (Ω_S):

Here the approach to obtaining the expected warranty cost per item is similar to that for PRW policy with rectangular region except that *R (t, rt)* is given by

$$R(t,rt) = \begin{cases} \left[1 - Min\left\{\frac{t}{A}, \frac{rt}{U}\right\}\right] S & if\ (t,rt) \in \Omega \\ 0 & if\ (t,rt) \notin \Omega \end{cases} \tag{7}$$

and $H(\tilde{\theta}|r)$ is given by

$$H(\tilde{\theta}|r) = \begin{cases} F(\tau_r|r) & if\ r < r_1 \\ F(A|r) & if\ r \geq r_1 \end{cases} \tag{8}$$

As a result, the expected warranty cost per item, $EC(\tilde{\theta})$ is given by

$$EC(\tilde{\theta}) = S\{\int_0^{r_1} [1/(1-F(\tau_r|r))\int_0^{\tau_r} \left[1 - Min\left(\frac{t}{A}, \frac{rt}{U}\right)\right] dF(t|r)]\, dG(r)$$
$$+ \int_{r_1}^{\infty}[1/(1-F(A|r))\int_0^{k}\left[1 - Min\left(\frac{t}{A}, \frac{rt}{U}\right)\right] dF(t|r)]\, dG(r)\} \tag{9}$$

NUMERICAL EXAMPLE

The example considers renewing Two-dimensional Pro-Rata Warrantee (PRW) policy for the remanufactured AC's components and products with three different remaining lives (1 year, 2 years and 3 years) with three different warranty periods (1 month, 2 months and 3 months) and usage (100 hours, 200 hours and 300 hours). Under this warranty, all failed items are repaired or replaced at no cost to the buyer as long as the failure occurs within the warranty period and usage. The warranty expires when either the age or usage limit is reached. The usage could be tracked and recorded using an embedded sensor. Suppose that there are three set of user population (Light, Medium and Heavy users) with different mean usage rates E(R) as stated in Table 1. The other data used for implementation of the model are shown in Table 2:

Table 1: Users' uniform limits and expected usage rates

Type of Users	***Uniform distribution limits***		***E[R]***
	Lower Rate Limit	***Upper Rate Limit***	
Light (L)	0.10	0.90	0.50
Medium (M)	0.70	1.30	1.00
Heavy (H)	1.10	2.90	2.00

Table 2: Operation costs (disassembly, assembly), sale price and repair cost for AC components

Components	*Cs* = Operation costs ($/unit)	*S* = Sale Price ($/unit)			*Cr* = Repair costs ($/unit)
		L = 1 Year	*L* = 2 Years	*L* = 3 Years	
Evaporator	$4.00	$10	$15	$35	$8.00
Control Box	$4.00	$20	$30	$15	$8.00
Blower	$2.80	$5	$12	$15	$5.60
Air Guide	$1.20	$5	$12	$60	$2.40
Motor	$4.00	$45	$55	$25	$8.00
Condenser	$1.66	$15	$18	$20	$3.32
Fan	$2.34	$15	$18	$20	$4.68
Protector	$0.60	$15	$20	$65	$1.20
Compressor	$3.40	$50	$60	$35	$6.80
AC	$55.00	$180	$240	$310	$85.00

RESULTS

It is assumed that $W = U$; these correspond to warranties of 1 month/100 hours, 2 months/200 hours and 3 months/300 hours. Table 3 and Table 4 give the expected warranty cost for the remanufactured components as a fraction of the sale price (i.e.,$EC(\tilde{\theta})/S$) under rectangular region (Ω_R) and Infinite strips region (Ω_S) PRW warranty for the three cases for W = U = 1, 2 and 3 respectively. Table 5 gives Remanufactured AC expected warranty cost as a fraction of the sales price for Rectangular and Infinite strips Region (Ω_S) PRW. This fraction can be used to predict warranty costs by multiplying the table value by sales price S. Suppose, for example, for the case of rectangular region (Ω_R) that a three years remaining life AC is sold at $310 with a 2 months/200 hours warranty and that on average warranty cost $310(0.0551) = $17.08 or 5.51% of the sale price of the AC for light users and this changes to $310(0. 0667) = $20.68 or 6.67% of the sales price of the AC for medium users and to $310(0.0727) = $22.54 or 7.27% of the sales price for the heavy users. In Infinite strips region (Ω_S) case, the average warranty cost increases to $310(0.5916) = $183.40 or 59.16% of the sales price, $310(0.3501) = $108.53 or 35.01% of the sales price and $310(0. 5574) = $172.79 or 55.74% of the sales price for light, medium and heavy users respectively.

CONCLUSION

The renewing Two-dimensional Pro-Rate warranty (PRW) cost for remanufactured products and components was evaluated in this paper for different periods and usages. The main objective was to introduce the idea of providing a two-dimensional PRW for a remanufactured products and how to predict a warranty period for an end-of-life (EOL) components using the sensor information about the age and usage of each and every EOL components and remanufactured products on hand to meet product, component and recycled material demands while minimizing the cost associated with warranty and maximizing manufacturer's profit. A simulation model was used to optimize the system and predict the warranty period that should be assigned to each and every disassembled component and remanufactured product.

Table 3: AC Components expected warranty cost as a fraction of the sales price for Rectangular Region (Ω_R) PRW

Items	*A* = U	(Expected Warranty Cost)/(Sales Price)			*Items*	*A* = U	Expected Warranty Cost)/(Sales Price)			*Items*	*A* = U	Expected Warranty Cost)/(Sales Price)			Users
		W=1.0	W=2.0	W=3.0			W=1.0	W=2.0	W=3.0			W=1.0	W=2.0	W=3.0	
		0.0261	0.0423	0.0580			0.0243	0.0393	0.0538			0.0252	0.0408	0.0559	Light
	1.00	0.0312	0.0395	0.0436		1.00	0.0289	0.0367	0.0405		1.00	0.0300	0.0381	0.0420	Medium
		0.0314	0.0348	0.0366			0.0292	0.0324	0.0340			0.0302	0.0336	0.0353	Heavy
		0.0290	0.0544	0.0905			0.0270	0.0505	0.0840			0.0280	0.0524	0.0872	Light
Evaporator	2.00	0.0392	0.0659	0.0836	**Air Guide**	2.00	0.0364	0.0612	0.0777	**Condenser**	2.00	0.0378	0.0635	0.0806	Medium
		0.0509	0.0659	0.0730			0.0473	0.0612	0.0679			0.0491	0.0635	0.0704	Heavy
		0.0305	0.0604	0.0896			0.0283	0.0561	0.0832			0.0294	0.0582	0.0863	Light
	3.00	0.0432	0.0831	0.1192		3.00	0.0402	0.0772	0.1107		3.00	0.0417	0.0801	0.1149	Medium
		0.0653	0.1110	0.1340			0.0607	0.1031	0.1245			0.0630	0.1070	0.1292	Heavy
		0.0237	0.0384	0.0525			0.0246	0.0398	0.0545			0.0255	0.0413	0.0566	Light
	1.00	0.0282	0.0358	0.0395		1.00	0.0293	0.0372	0.0410		1.00	0.0304	0.0386	0.0425	Medium
		0.0284	0.0316	0.0332			0.0295	0.0328	0.0344			0.0306	0.0340	0.0357	Heavy
		0.0263	0.0493	0.0820			0.0273	0.0511	0.0851			0.0283	0.0531	0.0883	Light
Control Box	2.00	0.0355	0.0597	0.0758	**Motor**	2.00	0.0369	0.0620	0.0787	**Protector**	2.00	0.0382	0.0643	0.0816	Medium
		0.0462	0.0597	0.0662			0.0479	0.0620	0.0687			0.0497	0.0643	0.0713	Heavy
		0.0276	0.0548	0.0812			0.0287	0.0568	0.0842			0.0297	0.0590	0.0874	Light
	3.00	0.0392	0.0753	0.1080		3.00	0.0406	0.0781	0.1121		3.00	0.0422	0.0811	0.1163	Medium
		0.0592	0.1006	0.1215			0.0614	0.1044	0.1260			0.0637	0.1083	0.1308	Heavy
		0.0240	0.0388	0.0532			0.0249	0.0403	0.0552			0.0258	0.0418	0.0573	Light
	1.00	0.0286	0.0363	0.0400		1.00	0.0297	0.0376	0.0415		1.00	0.0308	0.0391	0.0430	Medium
		0.0288	0.0320	0.0336			0.0299	0.0332	0.0349			0.0310	0.0344	0.0362	Heavy
		0.0266	0.0499	0.0830			0.0276	0.0518	0.0861			0.0287	0.0537	0.0894	Light
Blower	2.00	0.0360	0.0605	0.0768	**Fan**	2.00	0.0373	0.0627	0.0796	**Compressor**	2.00	0.0387	0.0651	0.0826	Medium
		0.0467	0.0605	0.0670			0.0485	0.0627	0.0695			0.0503	0.0651	0.0721	Heavy
		0.0280	0.0554	0.0822			0.0290	0.0575	0.0853			0.0301	0.0597	0.0885	Light
	3.00	0.0397	0.0762	0.1094		3.00	0.0411	0.0791	0.1135		3.00	0.0427	0.0821	0.1177	Medium
		0.0600	0.1019	0.1230			0.0622	0.1057	0.1276			0.0645	0.1097	0.1324	Heavy

Table 4: AC Components expected warranty cost as a fraction of the sales price for Infinite Strips Region (Ω_S) PRW

Items	*A* = U	(Expected Warranty Cost)/(Sales Price)			*Items*	*A* = U	Expected Warranty Cost)/(Sales Price)			*Items*	*A* = U	Expected Warranty Cost)/(Sales Price)			Users
		1.00	2.00	3.00			1.00	2.00	3.00			1.00	2.00	3.00	
Evaporator	1.00	0.0827	0.0944	0.1551	Air Guide	1.00	0.1172	0.1338	0.2200	Condenser	1.00	0.1663	0.1897	0.3120	Light
		0.0516	0.1002	0.2238			0.0731	0.1421	0.3173			0.1037	0.2015	0.4500	Medium
		0.0793	0.1740	0.4199			0.1125	0.2468	0.5955			0.1595	0.3499	0.8444	Heavy
	2.00	0.1847	0.1847	0.2119		2.00	0.2619	0.2619	0.3004		2.00	0.3714	0.3714	0.4260	Light
		0.1013	0.1093	0.2238			0.1437	0.1550	0.3173			0.2038	0.2198	0.4500	Medium
		0.0895	0.1740	0.4199			0.1270	0.2468	0.5955			0.1800	0.3499	0.8444	Heavy
	3.00	0.3109	0.3203	0.3340		3.00	0.4408	0.4542	0.4736		3.00	0.6250	0.6440	0.6716	Light
		0.1607	0.1719	0.2244			0.2278	0.2437	0.3181			0.3230	0.3456	0.4511	Medium
		0.1267	0.1885	0.4199			0.1797	0.2673	0.5955			0.2548	0.3791	0.8444	Heavy
Control Box	1.00	0.0929	0.1060	0.1743	Motor	1.00	0.1317	0.1503	0.2472	Protector	1.00	0.1868	0.2132	0.3505	Light
		0.0579	0.1126	0.2514			0.0821	0.1596	0.3565			0.1165	0.2264	0.5055	Medium
		0.0891	0.1955	0.4718			0.1264	0.2772	0.6690			0.1792	0.3931	0.9486	Heavy
	2.00	0.2075	0.2075	0.2380		2.00	0.2942	0.2942	0.3375		2.00	0.4172	0.4172	0.4786	Light
		0.1138	0.1228	0.2514			0.1614	0.1741	0.3565			0.2289	0.2469	0.5055	Medium
		0.1006	0.1955	0.4718			0.1426	0.2772	0.6690			0.2023	0.3931	0.9486	Heavy
	3.00	0.3492	0.3598	0.3752		3.00	0.4952	0.5102	0.5321		3.00	0.7022	0.7235	0.7545	Light
		0.1805	0.1931	0.2521			0.2559	0.2738	0.3574			0.3629	0.3883	0.5068	Medium
		0.1424	0.2118	0.4718			0.2019	0.3003	0.6690			0.2863	0.4259	0.9486	Heavy
Blower	1.00	0.1044	0.1191	0.1958	Fan	1.00	0.1480	0.1689	0.2777	Compressor	1.00	0.2098	0.2395	0.3937	Light
		0.0651	0.1265	0.2825			0.0923	0.1794	0.4005			0.1309	0.2543	0.5679	Medium
		0.1001	0.2196	0.5300			0.1420	0.3115	0.7516			0.2013	0.4416	1.0657	Heavy
	2.00	0.2331	0.2331	0.2674		2.00	0.3306	0.3306	0.3792		2.00	0.4687	0.4687	0.5377	Light
		0.1279	0.1380	0.2825			0.1814	0.1956	0.4005			0.2572	0.2774	0.5679	Medium
		0.1130	0.2196	0.5300			0.1602	0.3115	0.7516			0.2272	0.4416	1.0657	Heavy
	3.00	0.3924	0.4043	0.4216		3.00	0.5563	0.5732	0.5978		3.00	0.7889	0.8128	0.8476	Light
		0.2028	0.2170	0.2832			0.2875	0.3076	0.4015			0.4077	0.4362	0.5694	Medium
		0.1599	0.2380	0.5300			0.2268	0.3374	0.7516			0.3216	0.4785	1.0657	Heavy

Table 5: Remanufactured AC expected warranty cost as a fraction of the sales price for Rectangular and Infinite strips Region PRW

Components	***A* = U**	**Expected Warranty Cost)/(Sales Price) for Rectangular Region (Ω_R)**			**Expected Warranty Cost)/(Sales Price) for Infinite strips Region (Ω_S)**			**Users**
		W=1.0	**W=2.0**	**W=3.0**	**W=1.0**	**W=2.0**	**W=3.0**	
AC	**1.00**	0.0265	0.0428	0.0587	0.2648	0.3023	0.4970	Light
		0.0315	0.0400	0.0441	0.1652	0.3210	0.7168	Medium
		0.0318	0.0353	0.0371	0.2541	0.5574	1.3451	Heavy
	2.00	0.0294	0.0551	0.0916	0.5916	0.5916	0.6787	Light
		0.0397	0.0667	0.0847	0.3246	0.3501	0.7168	Medium
		0.0516	0.0727	0.0839	0.2868	0.5574	1.3451	Heavy
	3.00	0.0309	0.0612	0.0907	0.9957	1.0259	1.0698	Light
		0.0438	0.0841	0.1206	0.5146	0.5506	0.7186	Medium
		0.0661	0.1124	0.1357	0.4059	0.6039	1.3451	Heavy

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