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Multi-criteria Stock Portfolio Management – A Distance Function Approach

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ABSTRACT

In this study, we propose a mathematical model for constructing an efficient stock portfolio. It takes into the account maximization of annual return, the minimization of portfolio's risk, and the maximization of dividend. For each objective, the ideal and anti-ideal solutions are calculated separately. The distance function approach is then used to optimally select the portfolio. It minimizes the objective formed by combining the ideal and anti-ideal solutions. To demonstrate the proposed method, a case example is presented based on Dow Jones Industrial Average data. The obtained results have been compared with goal programming techniques considering equal and unequal weights.

KEYWORDS: Portfolio, Multi-criteria Decision Making, Distance Function.

INTRODUCTION

Portfolio management deals with making decisions about creating a mix of investment securities according to the policy that meets the objectives of investment, the allocation of assets for individuals and institutions and balancing the portfolio risk against returns (performance). Thus, portfolio management decisions always require performance measurement which is commonly known as the expected return from the portfolio for a specified level of risk which differs from one investor to another based on the unique goals and circumstances of the investor. The stock markets bounce up and down significantly every week; individual investors clearly should understand the need of a safety net, which is the diversification that prevents the entire portfolio from losing value.

The classic model is the mean-variance optimization model introduced by Markowitz (1952 & 1959). Sharpe (1963) developed a simplified diagonal model for selecting an efficient portfolio. Additionally, the capital asset pricing model (CAPM) was formulated by Sharpe (1964) and Lintner (1965). Markowitz (2008) criticised the relationship explained by Sharpe and Lintner between risk and excess returns, and argued that it is possible to achieve different expected returns from the same risk structure. Markowitz (2008) further emphasized that the mean–variance framework is the cornerstone for assessing the risk of efficient portfolios.

The concept of distance function was proposed by Shephard (1953). Thereafter, different variants of distance function were introduced for solving many business problems (Barros et al., 2011; Färe & Grosskopf, 2000). However, this technique has not been extensively used in the area of portfolio optimization problems to address multi-criteria decision making problems. This motivates us to utilize the distance function concept to solve a multi-criteria stock portfolio optimization problem.

The purpose of the paper is to construct an effective stock portfolio. It is done by utilizing the capability of the distance function approach. This approach ensures the minimization of the difference of the objective value achievable, to the extent possible, from the ideal solution for all the objectives. As a result, for the considered multi-criteria stock portfolio optimization problem, it is possible to select the best combination of stocks.

LITERATURE REVIEW

According to Zelani (1982), distance function is an effective way of solving multi-criteria decision making problems. Romero (1985) established the relationship between the multi-objective programming and distance function approach. The portfolio optimization literature doesn't report much on the application of distance function to solve these problems. A summary of literature review on distance function and portfolio optimization is presented in Table 1.

Table 1: Summary of Literature Review on Distance Function and Portfolio Optimization

YEAR	REFERENCES	JOURNAL
2014	Aouni, Colapinto and Torre	European Journal of Operational Research
2013	Sharma,Jana and Sharma	Journal of Money, Investment and Banking
2011	Barros, Briec and Ratsimbanierana	Journal of Mathematical Finance
2010	Sharma, Sharma and Jana	International Journal of Finance
2009	Sharma, Jana and Sharma	International Research Journal of Finance and Economics
2006	Bilbao-Terol, Perez-Gladish, Arenas-Parra and Rodriguez-Uria	Applied Mathematics and Computation
2000	Färe and Grosskopf	Journal of Productivity Analysis

PORTFOLIO OPTIMIZATION MODEL DEVELOPMENT

In this section, the portfolio optimization model is developed and presented.

Index

s: index for proportion of money invested in security $s \in \{1, 2, \cdots, S\}$

Variables and Parameters

X_s = proportion of money invested in a security s

D_s = dividend on stock *s*

R_s^1 = annual rate of return from the security *s* for one year

R_s^3 = annual rate of return from security *s* for the next three years

R_s^5= expected maximum annual rate of return for five years

U_s = maximum proportion of money invested in security *s*

β_s = risk associated with security *s* PE_s = price earnings ratio of security *s*

$PE_{\max}$ = maximum price earnings ratio

Model Objectives

(i) **Annual return:** The return from securities must be maximized. Three different investment scenarios have been considered. They are one, three and five year returns. Each of them should be maximized and can be expressed as follows:

(a) One year return objective: $$\max: \sum_{s=1}^{S} R_s^1 X_s \quad (1)$$

(b) Three years return: $$\max: \sum_{s=1}^{S} R_s^3 X_s \quad (2)$$

(c) Five years return: $$\max: \sum_{s=1}^{S} R_s^5 X_s\,. \quad (3)$$

(ii) **Portfolio's risk:** The systematic risk of a portfolio is denoted by beta and it is measure of the sensitivity of a security's returns to the market returns which is always minimized. The corresponding objective is expressed as:

$$\min: \sum_{s=1}^{S} \beta_s X_s\,. \quad (4)$$

(iii) **Annual dividend:** The annual dividend income from all the securities must be maximize and the corresponding objective can be expressed as:

$$\max: \sum_{s=1}^{S} D_s X_s\,. \quad (5)$$

Model Constraints

(i) **P/E ratio constraint**: The sum of the price earnings ratio of each security should be less than or equal to the maximum acceptable limit of thecompany's financial performance. The corresponding constraint can be expressed as:

$$\sum_{s=1}^{S} PE_s X_s \le PE_{\max}\,. \quad (6)$$

(ii) **Investment constraint:** The decision maker would like to utilize 100% of his/her fund available for investment and the constraint can be expressed as:

$$\sum_{s=1}^{S} X_s = 1. \quad (7)$$

(iii) **Diversification constraint:** It is always preferable not to invest too much in a specific security. This reduces the risk of the investment. The diversification constraint can thus be constructed as follows:

$$X_s \le U_s, \quad s = 1,2,\cdots,S\,. \quad (8)$$

THE DISTANCE FUNCTION APPROACH

A classical MOP problem can be presented as follows:

$$\min: \{f_1(x), f_2(x), \ldots, f_m(x)\} \quad (9)$$

subject to $g_r(x) \leq 0,\ r = 1,2,\dots,R$ (10)

$x \geq 0,$ (11)

where x is vector of n decision variables; $f_i(x)\ (i = 1,2,\dots,m)$ are individual objectives, $and\ \ g_r(x)\ (r = 1,2,\dots,R)$ are constraint functions.

Let f_i^* be the ideal solution of $f_1(x)$ subject to given set of constraints. Then the auxiliary problem can be formulated as follows (Romero 1985):

$$\min\text{:}\{\sum_{i=1}^{m} w_i\, |f_i^* - f_i(x)|^p\}^{1/p} \quad (12)$$

subject to $g_r(x) \leq 0,\ r = 1,2,\dots,R$ (13)

$x \geq 0.$ (14)

There are different ways of defining the weights w_i. Let f_i^- be the anti-ideal solutions of $f_i(x)$ subject to the given set of constraints. Then according to Hwang and Yoon (1981), the auxiliary problem can be formulated as follows:

$$\min\text{:}\left\{\sum_{i=1}^{m} \frac{|f_i^* - f_i(x)|^p}{|f_i^* - \boldsymbol{f_i^-}|^p}\right\}^{1/p} \quad (15)$$

subject to $g_r(x) \leq 0,\ r = 1,2,\dots,R\ ;\ x \geq 0.$ (16)

The proposed model is illustrated through a portfolio selection problem considering multiple objectives. A sample of 30 stocks is considered from the Dow Jones Industrial Average and is collected from the online source Yahoo Finance. They are presented in Table 2.

Table 2: Required financial data

Company	Beta	Dividend	Yield	Earnings/ Share	Forward P/E	P/E	Return on Equity	Year 1	Year 3	Year 5
AA	2.17	0.12	0.82%	-1.28	12.56	-13.87	-8.22	58.57%	-24.39%	-11.21%
AXP	2.11	0.72	1.73%	1.53	12.87	27.08	13.78	67.18%	-10.13%	-0.53%
BA	1.28	1.68	2.30%	1.8	16.04	39.03	320.14	87.72%	-5.47%	6.36%
BAC	2.4	0.04	0.22%	0.03	9.02	-62.21	-1.32	100.34%	-26.56%	-13.38%
CAT	1.8	1.68	0.26%	1.41	15.92	44.75	12.07	81.39%	-1.96%	9.97%
CSCO	1.22	0	0.00%	1.04	16.88	24.84	15.5	34.73%	-0.89%	8.55%
CVX	0.63	2.72	3.55%	7.81	154.02	14.64	11.74	19.15%	2.44%	11.37%
DD	1.41	1.64	4.33%	1.92	14.09	19.74	25.17	40.53%	-4.70%	-0.69%
DIS	1.15	0.35	0.99%	1.76	15.52	20.31	9.61	61.25%	1.17%	6.95%
GE	1.59	0.04	2.18%	1.01	14.78	17.8	9.81	47.73%	-17.44%	-9.51%
HD	0.7	0.95	2.92%	1.57	15.06	20.89	14.1	27.06%	-1.83%	0.89%
HPQ	1.02	0.32	0.60%	3.31	10.95	16.04	19.83	48.75%	8.85%	22.06%
IBM	0.76	2.2	1.72%	10.01	10.7	12.88	74.37	26.62%	9.76%	12.63%
INTC	1.18	0.63	2.81%	0.78	12.4	29.14	10.75	45.78%	4.03%	1.30%
JNJ	0.57	1.96	2.98%	4.4	12.3	14.95	26.35	28.62%	3.52%	1.64%
JPM	1.13	0.2	0.44%	2.26	9.45	20.17	5.95	36.11%	-2.72%	7.62%
KFT	0.59	1.16	3.82%	2.03	13.04	14.97	12.54	34.82%	0.49%	2.15%
KO	0.6	1.76	3.18%	2.93	14.91	18.88	30.15	31.89%	4.83%	7.89%
MCD	0.62	2.2	3.26%	4.11	14.11	16.27	33.2	29.53%	15.06%	21.28%
MMM	0.77	2.1	2.50%	4.51	14.75	18.57	28.2	49.37%	3.19%	4.50%
MRK	0.84	1.52	4.03%	5.37	9.56	6.68	33.03	61.41%	-6.18%	6.45%
MSFT	0.97	0.52	1.78%	1.82	13.17	16.07	41.28	47.63%	1.15%	4.70%
PFE	0.7	0.72	4.22%	1.36	7.35	13.81	11.69	33.67%	-8.72%	-4.48%
PG	0.58	1.76	2.78%	4.22	15.65	17.12	16.71	30.91%	1.84%	5.53%
T	0.69	1.68	6.43%	2.12	11.01	12.36	12.65	5.73%	-8.36%	6.59%
TRV	0.65	1.32	2.45%	6.39	9.08	8.42	13.67	34.51%	2.49%	11.20%
UTX	0.98	1.7	2.29%	4.12	14.07	18.17	21.37	54.71%	5.49%	9.87%
VZ	0.64	1.9	6.07%	1.29	12.58	24.38	8.76	7.04%	-1.97%	2.75%
WMT	0.23	1.21	2.19%	3.7	12.67	14.95	21.19	12.71%	7.15%	5.18%
XOM	0.42	1.68	2.49%	3.98	9.4	17.07	17.25	2.94%	-3.56%	5.35%

RESULTS AND ANALYSIS

The following analysis is based on the assumption that the investor will restrict his/her investment proportion to any particular security to 5%. Accordingly, the ideal and anti-ideal solutions for each objective are obtained using LINGO 10.0 and are presented in Table 3.

Table 3: Ideal and anti-ideal solutions

Objective	Ideal Objective Value	Anti-ideal Objective Value
Risk	0.7110	1.2435
Dividend	1.6437	0.8250
Return: 1 year	0.5290	0.2933
Return: 3 year	0.0298	– 0.0589
Return: 5 year	0.0867	0.0103

Three investment scenarios have been considered. They are 1-year, 3-year and 5-year returns. They are presented in the following tables.

Case I: 1-Year return

Table 4a: 1-Year return

X3	X4	X5	X7	X8	X9	X11	X12	X13	X15	X17
0.05	0.02	0.05	0.03	0.05	0.05	0.04	0.05	0.05	0.05	0.05

Table 4b: 1-Year return

X18	X19	X20	X21	X22	X23	X24	X26	X27	X28	X29
0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.01	0.05

Table 5: Objective values for 1-Year return

Objective	Compromised Objective Value
Risk	0.8750
Dividend	1.4332
Return: 1 year	0.4344

The minimized objective value is obtained as 0.5678.

Case II: 3-Year return

Table 6a: 3-Year return

X3	X7	X8	X11	X12	X13	X14	X15	X17	X18	X19	X20
0.05	0.03	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

Table 6b: 3-Year return

X21	X22	X24	X25	X26	X27	X28	X29	X30
0.05	0.03	0.05	0.05	0.05	0.05	0.05	0.05	0.013

Table 7: Objective values for 3-Year return

Objective	Compromised Objective Value
Risk	0.7573
Dividend	1.5517
Return: 3 year	0.3457

The minimized objective value is obtained as 0.1862.

Case III: 5-Year return

Table 8a: 5-Year return

X3	X7	X8	X11	X12	X13	X15	X16	X17	X18	X19	X20
0.05	0.03	0.03	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

Table 8b: 5-Year return

X21	X22	X24	X25	X26	X27	X28	X29	X30
0.05	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.013

Table 9: Objective values for 5-Year return

Objective	Compromised Objective Value
Risk	0.7511
Dividend	1.5243
Return: 5 year	0.3410

The minimized objective value is obtained as 0.2504.

COMPARISON OF RESULTS

The results obtained have been compared with that of the Goal Programming (GP) technique with equal and different priorities (Sharma et al., 2013) and are presented as follows:

Table 10: 1-Year Return using GP and Distance Function

	GP with Equal Priority	GP with Different Priority	Distance Function
Risk	0.7503	0.7971	0.8750
Dividend	1.6023	1.4718	1.4332
1-Year Return	0.3433	0.3999	0.4344

Table 11: 3-Year Return using GP and Distance Function

	GP with Equal Priority	GP with Different Priority	Distance Function
Risk	0.8140	0.7971	0.7573
Dividend	1.3556	1.4718	1.5517
3-Year Return	0.3581	0.3999	0.3457

Table 12: 5-Year Return using GP and Distance Function

	GP with Equal Priority	GP with Different Priority	Distance Function
Risk	0.8283	0.8062	0.7511
Dividend	1.4692	1.4536	1.5243
5-Year Return	0.3753	0.3583	0.3410

The analysis of the results reveals that the proposed model did yield the expected results for the one, three and five year investment. We believed that the risk (beta) and return can be managed better with the proposed model compare to the GP techniques. The above cases are also shown in Figures 1-4.

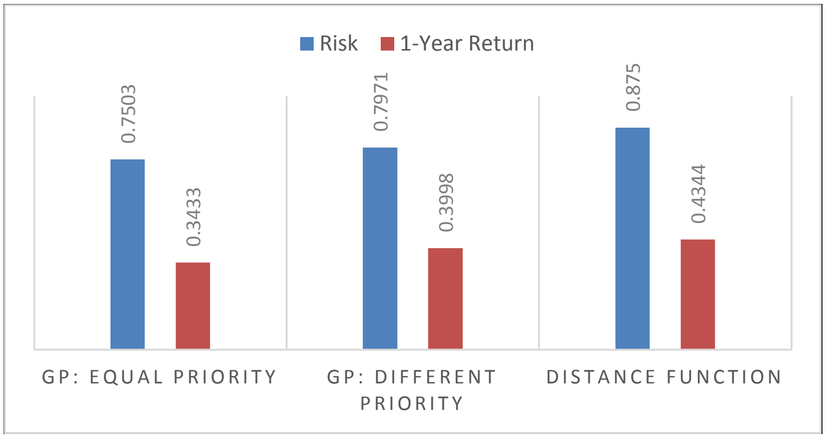

Figure 1: Risk v 1-Year Return

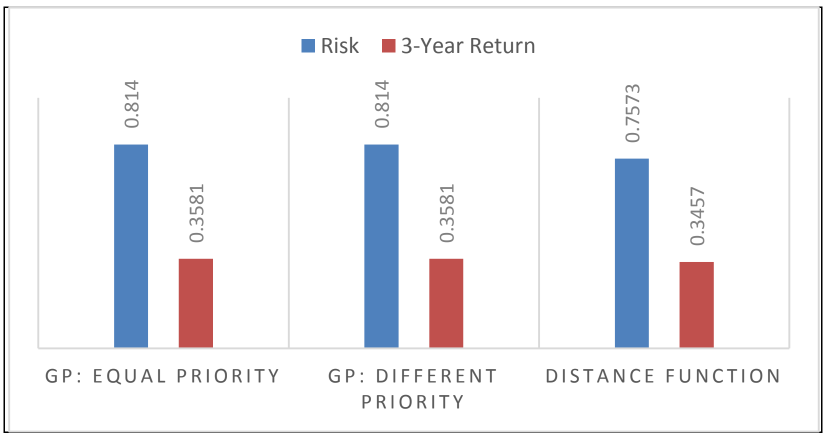

Figure 2: Risk v 3-Year Return

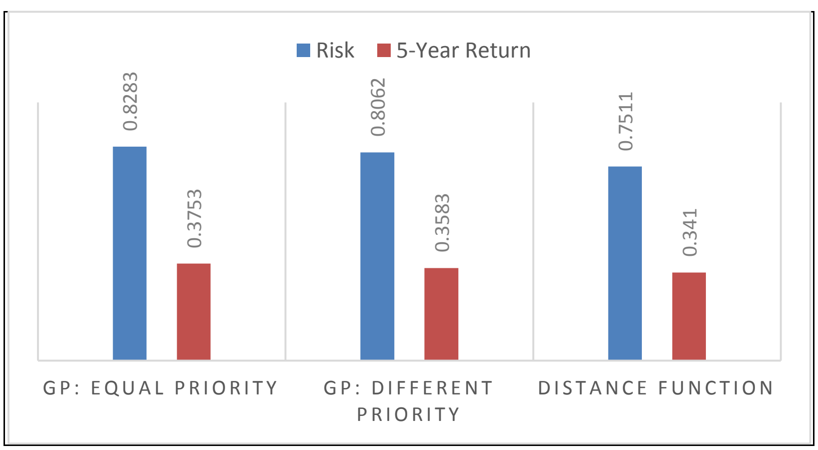

Figure 3: Risk v 5-Year Return

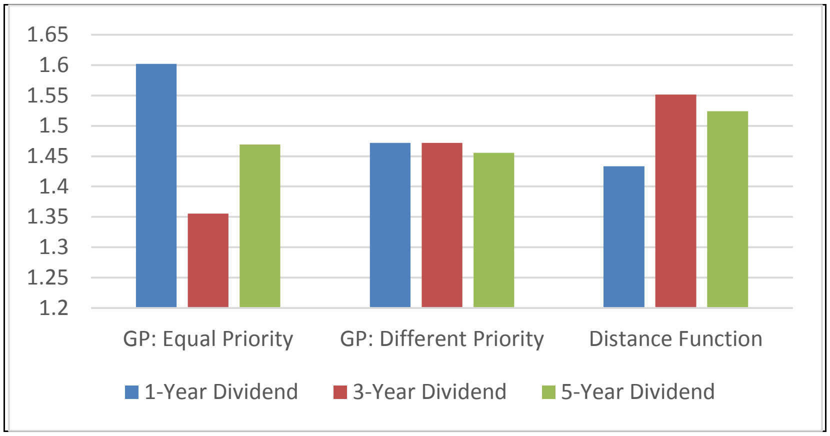

Figure 4: Comparison of Dividend

CONCLUSIONS

This paper proposes a mathematical model of a multi-criteria stock portfolio problem. A distance function based solution technique is then proposed to solve the proposed problem. The proposed model is flexible in nature. It is possible for the decision maker to modify the model. New criteria can be accommodated easily. Therefore, the proposed model can be used as a reference model for any decision maker. The proposed approach is tested with the Dow Jones Industrial Average data. The results obtained have been compared with the GP technique considering equal and unequal weights. It has been observed that the proposed approach provides better results.

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