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Maximization of Seller's Net Revenue in Discrete Dutch Auctions with Transaction Cost

**Full Paper Submission**

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E-mail: [Jinfeng.Yue@mtsu.edu](mailto:Jinfeng.Yue@mtsu.edu)**ABSTRACT**

This research focuses on the optimal design of discrete Dutch auction to balance the tradeoff between achieving the higher selling price and committing more resources. With consideration of transaction cost incurred by sellers, the objective is to maximize the seller's expected net revenue by determining the optimal number of bid levels and the asking price at each level. The result shows the existence of optimal solutions to the Dutch auction model being considered.

**KEYWORDS:** Dutch auction; Discrete bid level; Revenue maximization; Transaction cost; Salvage value

**INTRODUCTION**

In a Dutch auction, the bid price begins at an extremely high value and consecutively decreases according to a predetermined schedule until a bid is made or the reserve price is reached. In the current discrete Dutch auction literature, the expected selling price generally increases with the number of bid levels set. However, the use of more bid levels results in longer duration and higher transaction cost could occur to seller in the longer auctioning process, such as, office administration cost, personnel expenses, storage and maintenance cost. Obviously, there is a tradeoff between achieving the higher selling price and committing more resources. As such, the transaction cost is an important parameter to evaluate the seller's net revenue. In this paper, we consider a more practical objective, which is to maximize the seller's expected net revenue

in a discrete Dutch auction by determining the optimal number of bid levels to be set and the asking price at each level.

## AUCTION MODEL

We consider a discrete Dutch auction in an independent private value (IPV) setting with symmetric information. Suppose that there are  $n \geq 0$  bidders and  $m \geq 1$  bid levels  $l_1, l_2, \dots, l_m$  with  $l_1 \leq l_2 \leq \dots \leq l_m$ . The auctioneer begins with a high asking price  $l_{m+1}$  and lowers it to each of  $l_m, l_{m-1}, \dots, l_2, l_1$  sequentially. The final selling price is  $l_i$  if and only if  $q \geq 1$  bidders' valuations are in the range  $[l_i, l_{i+1})$ , no one is willing to pay the higher price  $l_{i+1}$  announced previously, and the remaining  $n - q$  bidders' valuations are below  $l_i$ ,  $i \in \{1, 2, \dots, m\}$ . Suppose that there are  $n \geq 0$  bidders and  $m \geq 1$  bid levels  $l_1, l_2, \dots, l_m$  with  $l_1 \leq l_2 \leq \dots \leq l_m$ . The auctioneer begins with a high asking price  $l_{m+1}$  and lowers it to each of  $l_m, l_{m-1}, \dots, l_2, l_1$  sequentially. The final selling price is  $l_i$  if and only if  $q \geq 1$  bidders' valuations are in the range  $[l_i, l_{i+1})$ , no one is willing to pay the higher price  $l_{i+1}$  announced previously, and the remaining  $n - q$  bidders' valuations are below  $l_i$ ,  $i \in \{1, 2, \dots, m\}$ . Let  $u$  be the salvage value, we have  $u \leq l_1 \leq l_2 \leq \dots \leq l_m$ .

As in Li and Kuo (2011, 2013), Cramton et al. (2012), Let each bidder's valuation be drawn from a common uniform distribution  $U(0, \bar{v})$ ,  $l_1 = 0$ ,  $l_{m+1} = \bar{v}$ ,  $F(l_1) = 0$  and  $F(l_{m+1}) = 1$ . Also, let  $P(l_i)$  be the probability that the object is sold at  $l_i$ ,  $i = 1, 2, \dots, m$ , which can be written as

$$\begin{aligned} P(l_i) &= F(l_{i+1})^n - F(l_i)^n \\ &= \left(\frac{l_{i+1}}{\bar{v}}\right)^n - \left(\frac{l_i}{\bar{v}}\right)^n \end{aligned}$$

Let  $c$  be the transaction cost associated with each bid level. Thus, the seller's expected net revenue,  $Z_1$ , can be expressed as the sum of the expected net revenue to auction off the object successfully and the expected net revenue to sell the unsold item in an auction to the strategic shoppers with its salvage value. We have

$$\begin{aligned} Z_1 &= \sum_{i=1}^m [l_i - (m+1-i)c]P(l_i) + (u - mc) \left[1 - \sum_{i=1}^m P(l_i)\right] \\ &= \sum_{i=1}^m l_i \left[ \left(\frac{l_{i+1}}{\bar{v}}\right)^n - \left(\frac{l_i}{\bar{v}}\right)^n \right] - c \sum_{i=2}^{m+1} \left(\frac{l_i}{\bar{v}}\right)^n + u \left(\frac{l_1}{\bar{v}}\right)^n \end{aligned}$$

Thus, the discrete Dutch auction in Model I can be formulated as the following NLP:

$$\begin{aligned} \text{Maximize} \quad & Z_I = \sum_{i=1}^m l_i \left[ \left( \frac{l_{i+1}}{\bar{v}} \right)^n - \left( \frac{l_i}{\bar{v}} \right)^n \right] - c \sum_{i=2}^{m+1} \left( \frac{l_i}{\bar{v}} \right)^n + u \left( \frac{l_1}{\bar{v}} \right)^n \\ \text{subject to:} \quad & l_{i+1} \geq l_i, \quad i = 1, 2, \dots, m \\ & l_1 \geq u \\ & l_{m+1} = \bar{v} \end{aligned}$$

Proposition 1 shows that the objective function is concave, which indicates the existence of optimal solution.

**Proposition 1:**  $Z_I = \sum_{i=1}^m l_i \left[ \left( \frac{l_{i+1}}{\bar{v}} \right)^n - \left( \frac{l_i}{\bar{v}} \right)^n \right] - c \sum_{i=2}^{m+1} \left( \frac{l_i}{\bar{v}} \right)^n + u \left( \frac{l_1}{\bar{v}} \right)^n$  is concave in  $(l_1, l_2, \dots, l_m)$ .

## NUMERICAL EXAMPLES AND ANALYSIS

In this section, we solve a set of problem instances to investigate the impact of the model parameters on the auction outcome. Specifically, it is assumed that the bidder's valuation follows an uniform distribution  $U(0,1)$ , i.e.,  $\bar{v} = 1$ . In addition, let the number of bid levels be  $m \in \{1, 2, \dots, 9\}$ , the number of bidders be  $n \in \{5, 10, 20, 30, 40, 50\}$ , the salvage value be  $u \in \{0, 0.1, 0.2, \dots, 0.8\}$ , and the transaction cost per bid level be  $c \in \{0.05, 0.1, 0.15, 0.2, \dots, 0.4\}$ . The NLPs based on the various combinations of  $m$ ,  $n$  (or  $\lambda$ ),  $u$  and  $c$  with  $\bar{v} = 1$  are set up and solved by running LINGO.

Table 1 shows the seller's expected net revenues with  $u = 0.1$  and  $c = 0.15$ . The highest  $Z_I^*$  among all  $m$  levels for each  $n$  value is highlighted.

n	$Z_{I, m=1}^*$	$Z_{I, m=2}^*$	$Z_{I, m=3}^*$	$Z_{I, m=4}^*$	$Z_{I, m=5}^*$	$Z_{I, m=6}^*$	$Z_{I, m=7}^*$	$Z_{I, m=8}^*$	$Z_{I, m=9}^*$
5	0.4501	0.4979	0.5061	<b>0.5069</b>	0.5068	0.5067	0.5067	0.5067	0.5067
10	0.5749	0.6249	0.6333	0.6346	<b>0.6348</b>	0.6348	0.6348	0.6348	0.6348
20	0.6729	0.7130	0.7182	0.7188	<b>0.7189</b>	0.7189	0.7189	0.7189	0.7189
30	0.7165	0.7490	0.7524	0.7527	<b>0.7528</b>	0.7528	0.7528	0.7528	0.7528
40	0.7417	0.7689	0.7713	<b>0.7716</b>	0.7716	0.7716	0.7716	0.7716	0.7716
50	0.7583	0.7818	0.7836	<b>0.7837</b>	0.7837	0.7837	0.7837	0.7837	0.7837

**Table 1**  $Z_I^*$  vs.  $n$  and  $m$  with  $\bar{v} = 1$ ,  $u = 0.1$  and  $c = 0.15$ <sup>#</sup>.

<sup>#</sup>  $Z_{I, m=a}^*$  denotes the seller's maximum expected net revenue based on Model I with  $a$  bid levels.

## CONCLUSIONS

This research focuses on the optimal design of discrete Dutch auction with consideration of the transaction cost incurred by sellers. The main objective is to maximize the seller's expected net revenue by determining the optimal number of bid levels and the asking price at each level. Although all of the above results make intuitive sense and have managerial implications, there are several limitations in which the present paper may be extended. For one thing, we assume that the risk-averse bidders will bid based on their valuations. It happens in fast Dutch auctions, and bidders have little time to reevaluate their valuations since the price declines very fast. However, in the case of slow Dutch auctions, bidders may not bid truthfully based on their valuations. We are curious about the impact of the auction speed on the bidder's behavior and the optimal Dutch auction design.

(Proof of propositions is available upon requests from the authors.)

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