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Effectiveness of Control Charts for Intermittent Demand

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ABSTRACT

Modeling demand for retail items that occur intermittently with possible shifts to periods of high or regular demand and constructing reliable confidence intervals has challenged forecasting researchers. This paper uses a simulation to test the suitability of adapting control charts for intermittent data.

KEYWORDS: Croston's Method, Forecasting, Slow-moving Inventory, Control Chart

INTRODUCTION

Forecasting techniques for slow-moving inventory have been extensively explored broadening the base of knowledge related to forecasting demand for inventory items. (Syntetos and Boylan, 2001, 2005, 2011; Willemain, Smart, and Schwarz, 2004) Many of the characteristics of forecasting items with regular demand are now being applied to items with intermittent or slow-moving demand, including trends, correlated demand, seasonality and other conditions (Altay, Litteral and Rudisill, 2012). Croston's method (1972) has established itself as one of the principal procedures for forecasting items with slow or intermittent demand.

The literature promotes identifying and categorizing demand patterns so the proper forecasting method can be selected (Syntetos, Boylan, and Croston, 2005; Boylan and Syntetos, 2007). In the situation of shifting demand levels, a signal is required to know when to change from one forecasting methodology to another. The literature indicates that control charts can provide a signal in a switching rule. The literature identifies many complex systems that utilize control charts to identify changes and apply the appropriate forecasting techniques (Kadri, Harrou, Chaabane, Sun, and Tahon, 2016; Wang, Liu, and Ji, 2016; and Tan, Lee and Lam, 2015).

This paper utilizes control charts to provide a signal to indicate the switching rule for the proper forecasting technique. A similar process will be applied to control charts as the authors used in previously proposed switching rules. The procedure is tested on concatenated real date to create a demand pattern with multiple shifts in demand.

LITERATURE REVIEW

A motivation of considering control charts for intermittent data are applications such as manufacturing in which small variations occur infrequently. An application can be found in the health care industry. Control charts are useful in monitoring changes in one's health. These changes may be occurring on an intermittent basis (Albers, 2011).

The forecasting methodology proposed by Croston (1972) has evolved into the applied standard for inventory models (Teunter, Syntetos and Babai, 2010; Xu, Wang, & Shi, 2012; Ramaekers and Janssens, 2014). Croston's method has yielded well accepted benefits by Willemain, Smart, Shockor, and DeSautels (1994), Johnston and Boylan (1996) and others. Syntetos and Boylan (2001) provided a modification to correct an identified bias. Studies have compared the bias corrected method with the original without identifying a winner (Eaves & Kingsman, 2004; Syntetos & Boylan, 2005; Teunter & Sani, 2009; and Teunter & Duncan, 2009)

A generalized forecasting method when demand is slow or regular has been suggested, but it potentially introduces more bias than Croston (Teunter & Sani, 2009). Levén and Segerstedt's (2004) proposed methodology introduces a common technique for all demand rates but not for an item with distinct demand rates in different periods. The authors have developed a hybrid technique to adjust for shifts in rates of demand. If demand is not intermittent, a three standard deviation control chart could be constructed assuming that the estimate of the mean demand followed an approximate known distribution.

Croston's Method

See Willemain, et al. (1994) for a concise explanation of Croston's (1972) method. It is not included here to save space. Croston's (1972) recommends small alpha values between 0.1 and 0.2. Since the data sets were designed to have no shifts in the mean and to be consistent with Croston, this work utilizes lower alpha levels.

Syntetos and Boylan Revised Croston Procedure

Syntetos and Boylan (2001) identified a bias in Croston's (1972) method. The bias increases as the value of the smoothing constant increases. It can be minimized by using a small alpha level, but it is still present. Boylan and Syntetos (2005) correct for the bias using Croston's method by multiplying the demand per period by $1-\alpha/2$, yielding the following new estimator of mean demand:

$$y_t'' = \left(1-\frac{\alpha}{2}\right)\frac{z_t''}{p_t''} \quad (1)$$

When p is the inter demand interval, the bias is expressed as:

$$bias = \left(100\frac{\alpha}{2-\alpha}\right)\left(1-\frac{1}{p}\right) \quad (2)$$

Teunter and Sani (2009) advocate that in some cases when only a limited periods have no demand. Syntetos and Boylan (2005) remind us that Croston's assumptions of stationary, iid of demand sizes and intervals, geometrically distributed inter demand intervals and independence of demand sizes and intervals apply to the new estimator.

Confidence Interval for Mean Demand per Period

A moving average would be the ideal estimate if the time series values were independent and identically distributed (iid). A challenge with Croston's method is that a confidence interval would need to be constructed for the ratio of two random variables – demand and time between demand. Syntetos and Boylan (2011) note that assuming independence of the demand size and intervals between demand, that the following relationships hold

$$E = \left(\frac{Z'_t}{p'_t}\right) = E(Z'_t) * E\left[\frac{1}{p'_t}\right] \quad (3)$$

and that

$$E = \left(\frac{Z'_t}{p'_t}\right) \neq E(Z'_t) * \left[\frac{1}{E(p'_t)}\right] \quad (4)$$

In this paper, confidence intervals are constructed as follows.

$$\frac{Z'_t}{p'_t} \pm Z_{\alpha/2}(\widehat{V}ar\left(\frac{Z'_t}{p'_t}\right))^{1/2} \quad (5)$$

$$\frac{\left(1-\frac{\alpha}{2}\right)Z'_t}{p'_t - \frac{\alpha}{2}} \pm Z_{\alpha/2}(\widehat{V}ar\left(\frac{\left(1-\frac{\alpha}{2}\right)Z'_t}{p'_t - \frac{\alpha}{2}}\right))^{1/2} \quad (6)$$

An approximation to the estimate of the variance supplied by Croston (1972) for the demand estimate per time period is provided by Syntetos and Boylan (2011) and is shown below:

$$V\left(\frac{Z'_t}{p'_t}\right) \approx \left[\frac{\alpha}{2-\alpha}\right]\left[\frac{(p-1)}{p3}\left(\mu^2 + \frac{\alpha}{2-\alpha}\sigma^2\right) + \frac{\sigma^2}{p^2}\right] + \left[\frac{\alpha^4}{1-(1-\alpha)^4}\frac{\mu^2}{p^4}\left(1-\frac{1}{p}\right)\right]\left[9\left(1-\frac{1}{p}\right)p^2+1\right] \quad (7)$$

However, they go on to show that the variance for Croston's method can be "safely" approximated as:

$$Var\left(\frac{Z'_t}{p'_t}\right) \approx \left[\frac{\alpha}{2-\alpha}\right]\left[\frac{(p-1)}{p3}\left(\mu^2 + \frac{\alpha}{2-\alpha}\sigma^2\right) + \frac{\sigma^2}{p^2}\right] \quad (8)$$

Furthermore, Syntetos and Boylan (2011) provide the variance for the bias corrected Croston, which they denoted as "SY Croston" and will be adopted here, formulation as:

$$Var\left[\frac{\left(1-\frac{\alpha}{2}\right)Z'_t}{p'_t - \frac{\alpha}{2}}\right] \approx \left[\frac{\alpha(2-\alpha}{4}\right]\left[\frac{\left(p-\frac{\alpha}{2}\right)^2\sigma^2 + p(p-1)\mu^2 + \frac{\alpha}{2-\alpha}p(p-1)\sigma^2}{\left(p-\frac{\alpha}{2}\right)^4}\right] \quad (9)$$

To utilize each of the provided formulations for the variance a standard deviation is required. Determining the standard deviation of the population can be problematic. It would be rare to know the actual population variance. A sample variance is probably more likely to be available.

Control Chart Construction

A 3 standard deviation control was constructed to track the mean demand per period. As long as the mean stays within three standard deviations of the long term average, the series will be considered as "not changing" and "in control". If the mean deviates by more than three standard deviations it will signal that the mean shifted and a new forecasting model should be utilized.

METHODS

Five hundred simulations were conducted for 32 conditions using 3 standard error control charts. The process was then repeated using 3.4 standard error control charts. Four scenarios were simulated for each forecasting methodology. Each scenario represents an item with a given demand between 1.1 periods between demands to 10 periods between demands. The occurrence of demands is assumed to follow a time-dependent Bernoulli process. The first scenario represented items with an average mean time between demands (MTBD) of 1.1, then 2, 5 and 10 periods between demand. The demand was simulated from a population with a normal distribution having a demand of 200 and a standard deviation of 10. However, the occurrence of nonzero demand is assumed to follow a geometric distribution with probability of an occurrence being less than 1. The simulations are repeated for a 3.4 standard deviation control chart based on a t distribution with 20 degrees of freedom. This value produces a lower percentage of time when the control chart signals a problem.

RESULTS

Table 1 illustrates the percent of time that the control chart indicates the process is out of control, when it is actually in control decreases as the mean time between demands increases.

Table 1: Percentage of Observations outside of Control Limits using 3 standard deviations.

		Scenario1	Scenario2	Scenario3	Scenario4
MTBD		1.1	2	5	10
	Estimator				
Alpha = 0.05	**Croston (sample variance)**	3.18	.321	.297	.231
	Croston (pop variance)	3.26	.321	.297	.231
	SY Croston (sample variance)	3.03	.388	.557	.436
	SY Croston (pop variance)	2.96	.394	.555	.436
Alpha = 0.3	**Croston (sample variance)**	19.15	1.91	.804	.857
	Croston (pop variance)	22.40	1.92	.811	.856
	SY Croston (sample variance)	17.75	2.97	3.94	4.13
	SY Croston (pop variance)	20.71	2.98	3.95	4.14

To decrease the out of control signal, the simulation study was repeated with broader control limits. A t distribution with 20 degrees of freedom is approximately 3.4 standard deviations. The value of 3.4 standard deviations is used in the control chart to produce a lower number of out-of-control signals when the system is actually "in control." Table 2 shows that for the "3.4" value, the probability of an out of control signal is now lower than that expected for a traditional control chart when a smoothing value of .05 is used and when the data are truly intermittent (MTBD is at least 2). Using the wider control chart based on 3.4 standard deviations, did achieve the goal of lowering the percent of time that an observation signals "out of control".

Table 2: Percentage of Observations outside of Control Limits using 3.4 standard errors.

		Scenario1	**Scenario2**	**Scenario3**	**Scenario4**
MTBD		1.1	2	5	10
Estimator					
Alpha = 0.05	**Croston (sample variance)**	2.24	.078	.104	.083
	Croston (pop variance)	2.29	.078	.104	.083
	SY Croston (sample variance)	2.07	.122	.208	.183
	SY Croston (pop variance)	2.10	.122	.208	.183
Alpha = 0.3	**Croston (sample variance)**	13.84	1.18	.415	.498
	Croston (pop variance)	17.14	1.19	.416	.498
	SY Croston (sample variance)	12.76	1.81	2.48	2.81
	SY Croston (pop variance)	15.77	1.82	2.49	2.82

DISCUSSION AND CONCLUSIONS

As demand changes managers need guidance when to modify forecasting methodology. The use of control charts is quite common to monitor a process and signal changes in the process. The application of control charts may allow medical professionals to understand when a health related process has changed. This paper addresses these needs. The performance of Croston's method and the bias corrected (SY Croston) methodology are compared. The performance of the models with different smoothing constants is investigated. This paper adds to the increasing body of work related to Croston's method.

REFERENCES

Albers, W. (2011). Risk-Adjusted Control Charts for Health Care Monitoring, *International Journal of Mathematics and Mathematical Sciences*, doi:10.1155/2011/895273.

Altay, N., Litteral, L. A., and Rudisill, F., (2012). Effects of correlation on intermittent demand forecasting and stock control. *International Journal of Production Economics, 135*(1) 275-283.

Boylan, J. E. & Syntetos, A. A., (2007). The accuracy of a modified Croston procedure. *International Journal of Production Economics*, *107*(2), 511-517.

Croston, J. D., (1972). Forecasting and stock control for intermittent demands. *Operational Research Quarterly*, 23(3), 289-303.

Eaves, A. H. C., & Kingsman, B. G. 2004. Forecasting for the ordering and stock-holding of spare parts. *Journal of the Operational Research Society*, 55(4), 431-437.

Herbst, N. R., Huber, N., Kounev, S., & Amrehn, E. (2014). Self-adaptive workload classification and for. for proactive resource provisioning. *Concurrency and Computation*, *26*(12), 2053-2078.

Johnston, F.R. & Boylan, J.E. (1996). Fore. for items with interm. demand, *JORS, 47*, 113-121.

Kadri, F., Harrou, F., Chaabane, S., Sun, Y., & Tahon, C. (2016). Seasonal ARMA-based SPC charts for anomaly detection. *Neurocomputing*, *173*, 2102-2114

Levén, E. & Segerstedt, A., (2004). Inventory control with a modified Croston procedure and Erlang distribution. *International Journal of Production Economics, 90*(3), 361-367.

Ramaekers, K., & Janssens, G. (2014). Optimal policies for demand forecasting and inventory management of goods with intermittent demand.

Syntetos, A. A., & Boylan, J. E. (2001). On the bias of intermittent demand estimates. *International Journal of Production Economics, 71*(1/3), 457-466.

Syntetos, A. A., & Boylan, J. E. (2005). The accuracy of intermittent demand estimates. *International Journal of Forecasting, 21*(2), 303-314.

Syntetos, A. A., & Boylan, J. E. (2011). Intermittent demand: estimation and statistical properties. In *Service Parts Management* (pp. 1-30). Springer London.

Syntetos, A. A., Boylan, J. E. and Croston, J.D (2005). On the categorization of demand patterns. *Journal of the Operational Research Society, 56*, 495-503.

Tan, Y.C., Lee, M. H. and Lam, W. W. W. (2015) "An improved switching rule in variable sampling interval Hotelling's T2 control chart," *Industrial Engineering and Engineering Management (IEEM), IEEE International Conference on*, Singapore, 1412-1416.

Teunter, R. H., & Duncan, L., (2009). Forecasting intermittent demand, a comparative study. *The Journal of the Operational Research Society, 60*(3), p321-329.

Teunter, R.H., & Sani, B., (2009). On the bias of Croston's forecasting method, *European Journal of Operational Research, 194*(1), 177-183.

Teunter, R., Syntetos, A.A., Babai, M.Z., (2010). Determining Order-Up-To Levels under Periodic Review for Compound Binomial (Intermittent) Demand. *EJOR, 203* (3), 619-624.

Wang, Q., Liu, C. S., & Ji, W. Y. (2016). Self-Adjusting ANN Air Pollution Index Forecasting Models Based on Quality Control Charts—A Case Study. *EESED 2015*.

Willemain, T. R., Smart, C. N., & Schwarz, H. F. (2004). A new approach to forecasting intermittent demand for service parts inventories. *Intern. Journal of Forecasting, 20*, 375-387.

Willemain, T. R., Smart, C. N., Shockor, J. H. and DeSautels, P. A., (1994). Forecasting intermittent demand in manufacturing: A comparative evaluation of Croston's method. *International Journal of Forecasting, 10*(4), 529-538.

Xu, Q., Wang, N., & Shi, H. (2012, May). Review of Croston's method for intermittent demand forecasting. In *Fuzzy Systems and Knowledge Discovery (FSKD), 2012* (pp. 1456-1460). IEEE.