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New Class of Estimators of Population Mean Utilizing Median of Study Variable

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ABSTRACT

This study deals with the estimation of population mean using the median of the study variable through a new class of estimators. The estimators are tested from the bias and mean squared error (MSE) perspectives up to the first degree of approximation. The optimum value of the constant or characterizing scalar has been obtained. The smallest value of the MSE of the proposed estimators is also obtained for this optimum value of the characterizing scalar.The numerical study shows that the proposed estimators perform better than the competing estimators of the population mean under simple random sampling scheme.

KEYWORDS: Simple random sampling, Population mean, Ratio estimator, Bias, Efficiency.

INTRODUCTION

Sampling is of crucial importance whenever the population is huge as it is costly and time-consuming to collect information on each unit of population. Naturally, it is essential to take the corresponding statistic as an estimator for the estimation of population parameter as it has almost all desirable properties of a good estimator. It has been noted in practice that the corresponding statistic of any parameter has a significant amount of variation around the actual parameter. One of our objectives is to find the estimator for the parameter which has its distribution more and more concentrated around the true parameter. The aim of such estimator is achieved through the use of either additional information on any parameter of the study variable or by the use of the auxiliary variable. Both the variables have a high degree of correlation. Ratio type estimators are used when study and auxiliary variables are highly positively correlated (Cochran, 1940). Product type estimators are used when study and auxiliary variables are highly negatively correlated (Robson, 1957). Thus, in this study, we search for even biased estimator but having sampling distribution very close to true parameter. Although the use of auxiliary variable improves the efficiency of the estimator, it is collected with the additional cost of the survey. Therefore, it is better if we have information on any parameter of the study variable which may be utilized for improved estimation of the parameter without increasing cost of the survey (Subramani, 2016).

In this study, we have utilized the information on the population median of the study variable which is some times easily available in practice. For example, in the surveys regarding the

estimation of average income, average marks, body mass index, etc., where it is assumed that population mean is unknown, but population median is known since it does not require the full information on the population values of the study variable.

LITERATURE REVIEW

Over the years, several estimators of the population mean such as the naive unbiased sample mean estimator, usual ratio estimator and modified ratio type estimators have been proposed in the literature. A summary of the literature review of existing estimators is presented in Table 1.

Table 1: Summary of Literature Review of Existing Estimators

YEAR	REFERENCES	JOURNAL
2016	Yadav et al.	International Journal of Agricultural & Statistical Sciences
2016	Subramani	Statistics in Transition New Series
2015	Yadav and Mishra	Statistika
2015	Malik and Singh	Applied Mathematics & Computation
2015	Rashid et al.	Applied Mathematics & Information Sciences
2014	Onyeka et al.	Open Journal of Statistics
2014	Sanaullah et al.	Applied Mathematics & Computation
2013	Subramani and Kumarapandiyan	Statistics in Transition New Series
2013	Subramani	Journal of Modern Applied Statistical Methods
2013	Yadav and Kadilar	Applied Mathematics & Computation
2013	Onyeka et al.	Global Journal of Science Frontier Research Mathematics & Decision Sciences
2013	Yadav and Kadilar	Hacettepe Journal of Mathematics & Statistics
2012	Subramani and Kumarapandiyan	Journal of Reliability and Statistical Studies
2012	Solanki et al.	ISRN Probability and Statistics
2012	Singh and Solanki	Applied Mathematics and Computation
2011	Tailor et al.	Communications of the Korean Statistical Society
2011	Shabbir and Gupta	Communications in Statistics – Theory & Methods
2010	Koyuncu and Kadilar	Journal of Applied Statistics
2010	Sousa et al.	Journal of Statistical Theory and Practice

PROPOSED ESTIMATORS

Motivated by Subramani (2016), we have proposed the following class of estimators utilizing population median of the study variable for improved estimation of population mean of the study variable as follows:

$$t_P = \bar{y}\left[\alpha\frac{m}{M} + (1-\alpha)\frac{M}{m}\right] \quad (1)$$

Here α is a constant, also known as characterizing scalar, and is determined in such a way that the proposed estimator has least mean squared error.

The following assumptions are made in studying the sampling properties of the proposed class of estimators,

$\bar{y} = \bar{Y}(1+e_0)$ and $m = M(1+e_1)$ such that $E(e_0) = 0$, $E(e_1) = \frac{\overline{M} - M}{M} = \frac{Bias(m)}{M}$ and

$E(e_0^2) = \frac{1-f}{n} C_y^2$, $E(e_1^2) = \frac{1-f}{n} C_m^2$, $E(e_0 e_1) = \frac{1-f}{n} C_{ym}$.

where, $\overline{M} = \frac{1}{n}\sum_{i=1}^{n} m_i$

Up to the approximation of degree one, the mathematical expressions for bias and MSE of proposed class of estimators respectively are,

$$B(t_p) = \frac{1-f}{n}\bar{Y}[(1-\alpha)C_m^2 - (1-2\alpha)C_{ym} - (1-2\alpha)\frac{B(m)}{M}] \quad (2)$$

and

$$MSE(t_p) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \alpha_1^2 C_m^2 - 2\alpha_1 C_{ym}] \quad (3)$$

The optimum value of α_1 or equivalently α is obtained by partially differentiating (3) with respect to α_1 which minimizes the mean squared error of the proposed estimator as,

$$\alpha_1 = \frac{C_{ym}}{C_m^2}$$

The expression for the smallest MSE for proposed estimator is obtained by substituting the value of α_1 in (3) as follows,

$$MSE_{\min}(t_p) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 - \frac{C_{ym}^2}{C_m^2}\right] \quad (4)$$

EFFICIENCY COMPARISONS

The proposed estimators are compared with the mentioned competing estimators; mean per unit estimator, Cochran (1940) usual ratio estimator, Watson (1937) usual regression estimator, Bahl and Tuteja (1991) and Subramani (2016) estimators. The respective conditions under which it is better than above estimators are given below.

$V(t_0) - MSE_{\min}(t_p) = \frac{C_{ym}^2}{C_m^2} > 0$,

$MSE(t_1) - MSE_{\min}(t_p) = C_x^2 - 2C_{yx} + \frac{C_{ym}^2}{C_m^2} > 0$,

$MSE(t_2) - MSE_{\min}(t_p) = \frac{C_{ym}^2}{C_m^2} - C_y^2\rho_{yx}^2 > 0$,

$$MSE(t_3) - MSE_{\min}(t_p) = \frac{C_x^2}{4} - C_{yx} + \frac{C_{ym}^2}{C_m^2} > 0,$$

$$MSE(t_4) - MSE_{\min}(t_p) = R_4^2 C_m^2 - 2R_4 C_{ym} + \frac{C_{ym}^2}{C_m^2} > 0,$$

NUMERICAL STUDY

For the judgment regarding the properties of the proposed and the existing estimators of population mean, we have considered the population given by Subramani (2016). The tables given below named Tables 2 to 5 show the parameters of the population under consideration, biases of different estimators under study including proposed estimators, variances and mean squared errors of existing and proposed estimators and percentage relative efficiencies of the proposed estimators over other existing estimators of population mean respectively.

Table 2. Various Parameters of three natural populations

Parameter	Popln 1
N	34
n	5
${}^{N}C_{n}$	278256
$\bar{Y}$	856.4118
$\bar{M}$	736.9811
M	767.5
$\bar{X}$	208.8824
R_1	4.0999
R_5	1.1158
C_y^2	0.125014
C_x^2	0.088563
C_m^2	0.100833
C_{ym}	0.07314
C_{yx}	0.047257
ρ_{yx}	0.4491

Table 3. Bias of the competing and proposed estimators

Estimator	Popln 1
t_1	35.3748
t_3	1.39995
t_4	57.7705
t_p	4.8130

Table 4. Variance and MSE of the competing and proposed estimators

Estimator	Popln 1
t_0	15640.97
t_1	14895.27
t_2	12486.75
t_3	12498.01
t_4	10926.53
t_p	9002.22

Table 5. PRE of the proposed estimator t_p w.r.t. competing estimators

Estimator	Popln 1
t_0	173.7457
t_1	165.4622
t_2	138.7075
t_3	138.8325
t_4	121.3759

CONCLUSION

In this study, we have proposed a class of estimators of the population mean utilizing the population median of study variables. We have derived the MSE and bias of the proposed estimators estimating the approximation upto the first degree. The least value of the MSE of the proposed estimators is also achieved to the optimum value of the characterizing scalar. The proposed estimators are theoretically compared with the competing estimators of the population mean. Finally, it has been shown that the proposed estimators are better than estimators mentioned above since they have the least MSEs.

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