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# Product Allocation and Pricing Strategies in a Prioritized Multi-Segment Market: A Dynamic Programming Approach

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## ABSTRACT

In this paper, we tackle the problem of optimal product allocation and pricing under uncertain demand in a prioritized multi-segment market for a monopolistic retailer. A mathematical programming model is developed and then solved via dynamic programming to find the optimal allocation scheme and determine the prices correspondingly. Numerical experiments are performed to demonstrate the application of the dynamic programming approach and explore the impacts of consumers' price sensitivity upon the retailer's allocation scheme, pricing strategy, and profitability.

KEYWORDS: Product allocation, Pricing, Optimization, Dynamic programming

## INTRODUCTION

A general problem facing a retailer is how to allocate a fixed quantity of a product for sale and set the price during a single selling season in a multi-segment market to maximize her profit. Since consumers' price sensitivity may substantially vary across segments, a thorough understanding of the dynamic interaction between price and demand is required to find the optimal allocation scheme and set the price accordingly in each segment of the market. This paper intends to tackle the problem for a monopolistic retailer by developing and then numerically solving a mathematical programming model.

In the literature on capacity allocation and pricing, five noteworthy studies are relevant to this paper. Zeng (2013) proposes multiple pricing strategies for a retailer who operates in a two-segment market composed of experienced and inexperienced consumers. Gallego et al. (2011) address the problem of capacity allocation and pricing in a two-segment market where an entrant sells at a lower price to compete against an established incumbent. Their model is designed to study the competitive interactions between the two rivals "under different assumptions about market structure, demand and available capacity." In the work of Deng et al. (2008), demand in each segment of the market is assumed to independently follow a Poisson distribution. A marginal-revenue-based capacity management model is formulated in their study to manage stochastic demand in a three-segment market and develop the policies for the firm to allocate capacity to those segments of higher revenue. Ng and Lee (2008), applying a game theoretic approach, address the issue of optimal capacity allocation in a duopolistic two-segment market where demand is deterministic. A recent paper by Duan et al. (2016) presents a model as a three-stage dynamic game for the allocation of a limited spectrum capacity and the pricing decision in a wireless market with two segments for femtocell and macro-cell services, respectively.

This present paper is significantly different from the five studies cited above in at least four main aspects. First, a general *n*-segment market is taken into account in our modeling framework. Second, the uncertain demand in each segment is modeled as a continuous random variable following a uniform probability distribution. Third, a mathematical programming model is formulated and then solved via dynamic programing (DP) to determine the optimal allocation scheme and the pricing strategy. Fourth, numerical experiments are conducted to demonstrate the application of the DP approach and explore the impacts of consumers' price sensitivity.

In this study, we focus on a profit-maximizing monopolistic retailer facing uncertain demand at the aggregate level. The two research questions that we attempt to address can be specifically stated as follows: (i) What is the optimal scheme for a monopolistic retailer to allocate and price *K* identical units of a product in an *n*-segment market under uniformly distributed uncertain demand during a single selling season so that her expected total profit is maximized? (ii) How does consumers' price sensitivity affect the retailer's optimal allocation scheme, pricing strategy, and profitability? We make the following basic assumptions while addressing the two strategic issues mentioned above:

(i) The price charged by the retailer in each segment is the only independent variable that affects the aggregate demand in that segment.
(ii) Consumers in each segment are well-informed of the price charged in that segment, which is set at the beginning of a selling season.
(iii) The demand in each segment is heterogeneous and independently follows a uniform probability distribution conditioned by the price charged in that segment.
(iv) The allocation decisions are made following a predetermined prioritized sequence.

The addressed problem is envisioned to belong to the class of "inverse newsvendor problems." The classical newsvendor problem is one of optimally choosing a level of capacity to respond to a known demand distribution. The inverse newsvendor problem is one of optimally choosing a demand distribution with fixed capacity. In many firms, capacity is relatively more difficult to adjust than demand because of the time required to install new capacity (Carr and Lovejoy, 2000).

This paper is organized as follows. In the next section, the retailer's price and profit functions are derived, and a mathematical programming model is formulated to determine the optimal allocation scheme and prices. The third section presents the DP formulation developed to solve the mathematical programming model. Numerical experiments conducted in this study are illustrated in the fourth section to demonstrate the application of the DP approach and explore the impacts of consumers' price sensitivity. Finally, the paper concludes in its fifth section with a summary of its contributions, managerial implications, and directions for future research.

## MODEL DEVELOPMENT

We analyze a model with *n* prioritized customer segments, in which higher priority segments are served before any lower priority segment. If capacity is exhausted by higher priority segments, the lower priority segment goes unfilled. We are not deriving optimal prioritization; rather we assume that priorities are set exogenously. Factors to consider in setting priorities may include willingness to pay higher prices, expected volume of business and expected customer satisfaction (Chao and Wilson, 1987).

Let us consider a monopolistic retailer who has *K* identical units of a product to be allocated in a general *n*-segment market during a single selling season. The *n* segments, denoted as segments *i* ($i$ = 1, 2, …, $n$), are prioritized to receive their allocations of the product in such a way that the allocation decision pertaining to segment *i* is made prior to the allocation decision related to segment $i + 1$. Without loss of generality, we assume that the *K* units of the product are successively allocated to segments *i* ($i$ = 1, 2, …, $n$).
The following notations are used to formulate the retailer's profit and price functions:

| | |
|---|---|
| $K_i$ | the quantity of the product available for sale in segment *i*; |
| $y_i$ | the quantity of the product to be allocated to segment *i* (a decision variable); |
| $P_i$ | the price per unit of the product charged in segment *i* ($P_i > 0$); |
| $C_i$ | the cost per unit of the product allocated to segment *i* ($C_i > 0$); |
| $S_i$ | the salvage value per unit of the product disposed of after the selling season in segment *i* ($S_i > 0$); |
| $d_i$ | the aggregate demand of consumers in segment *i;* |
| $\omega_i$ | the demand parameter in segment *i* ($\omega_i > 0$); |
| $f(x\|\omega_i)$ | the probability density function (p.d.f.) of $d_i$ being a continuous random variable; |
| $E(d_i)$ | the expected value of $d_i$; |
| $\pi_i$ | the retailer's profit yielded from segment *i*; |
| $E(\pi_i)$ | the expected value of $\pi_i$; |
| $\pi$ | the retailer's total profit yielded from the entire *n*-segment market; |
| $E(\pi)$ | the expected value of $\pi$ . |

Since consumers in segment *i* are assumed to be well-informed of the price charged in that segment, they would take the price into consideration while making their purchases. Thus, the uncertain demand in segment *i*, $d_i$ ($i$ = 1, 2, …, $n$) can be modeled as a random variable following a probability distribution conditioned by $P_i$. Following Azoury (1985), we model the aggregate demand, $d_i$, as a continuous random variable following a uniform probability distribution. The p.d.f. of $d_i$ is given by

$$f(x|\omega_i) = \begin{cases} 1/\omega_i\,, & if \quad 0 < x < \omega_i\,, \\ 0\,, & if \quad x \geq \omega_i\,, \end{cases} \tag{1}$$

where $\omega_i > 0$. It is assumed in this study that the demand parameter, $\omega_i$, is a linear function of $P_i$:

$$\omega_i = a_i - \beta_i P_i\,, \tag{2}$$

where $\alpha_i > 0$ and $\beta_i > 0$ ($i$ = 1, 2, …, $n$).

In expression (2), the parameter, $\alpha_i$, reflects the size of segment *i*. A larger value of $\alpha_i$ shows stronger demand in segment *i*. The price coefficient, $\beta_i$, measures the responsiveness of the aggregate demand in segment *i* to the price charged in that segment. According to Huang et al. (2013), "the linear model is extensively used in the literature because it gives rise to explicit results for the optimal solution, and it is relatively easy to estimate its parameters in an empirical study."

The expected demand in segment *i*, based on expression (1), is given by

$$E(d_i) = \int_0^{\omega_i} \frac{x}{\omega_i} dx = \frac{\omega_i}{2}\ . \tag{3}$$

Expression (3) reveals that the demand parameter, $\omega_i$, equals twice the expected demand, $E(d_i)$, and thus serves as an indicator of the aggregate demand in segment *i*.

From expressions (2) and (3), the expected demand in segment *i*, can be rewritten as follows:

$$E(d_i) = \frac{a_i}{2} - \frac{\beta_i}{2} P_i\ . \tag{4}$$

Consumers' price sensitivity is referred to as the degree to which the price of a product affects their purchasing behaviors. In practice, price sensitivity is commonly measured by price elasticity of demand (PED). In this study, the price elasticity of demand in segment *i* is derived from (4) as follows (see Huang et al., 2013):

$$PED_i = \frac{-\beta_i P_i}{\alpha_i - \beta_i P_i}\ . \tag{5}$$

Cost-plus pricing is the practice of "adding a standard markup to the cost of the product" (see Kotler and Armstrong, 2004, p.357). In a survey of service firms, Zeithaml et al. (1985) report that 63% of the firms base their prices primarily on costs. Also, there are studies in the literature that incorporate price markup in their modeling frameworks (e.g., Shah and Jha, 1991; Hanson, 1992). In this paper, the unit price charged in segment *i*, $P_i$ ($i$ = 1, 2, …, $n$), is set by a markup over the unit cost; that is, $P_i = \theta_i C_i$ where $\theta_i > 1$. While implementing price discrimination, a retailer usually charges a higher price when the quantity of the product available for sale is relatively scarce and a lower price when that is relatively abundant. We may therefore assume that the price markup factor, $\theta_i$, is a decreasing function of $K_i$:

$$\theta_i = \phi(K_i)\ , \tag{6}$$

where, $d\phi(K_i)/dK_i < 0$. Accordingly, the price charged in segment *i* is a function of $K_i$ :

$$P_i = C_i \phi(K_i)\ . \tag{7}$$

In segment *i* ($i$ = 1, 2, …, $n$), if the demand ($d_i$) exceeds the quantity of the product allocated by the retailer ($y_i$), the profit ($\pi_i$) will equal the profit per unit multiplied by the number of units sold.

On the other hand, if $d_i$ is smaller than $y_i$, a portion of the allocated quantity, $y_i - d_i$, will be unsold and disposed of after the selling season at the unit salvage value, $S_i$. Hence, the retailer's profit yielded from segment $i$ ($i$ = 1, 2, …, $n$) is stated as

$$\pi_i = \begin{cases} P_i d_i - C_i y_i + S_i (y_i - d_i), & d_i \le y_i, \\ (P_i - C_i) y_i, & d_i > y_i. \end{cases} \tag{8}$$

Since $d_i$ is modeled as a continuous random variable, we obtain from (8) the expected value of $\pi_i$:

$$E(\pi_i) = (P_i - S_i) \int_0^{y_i} x f_i(x|\omega_i) dx - (C_i - S_i) y_i \int_0^{y_i} f_i(x|\omega_i) dx + (P_i - C_i) y_i \int_{y_i}^{\infty} f_i(x|\omega_i) dx . \tag{9}$$

Substituting (1) into (9) and carrying out the integrations, we have

$$E(\pi_i) = (P_i - C_i) y_i - \frac{(P_i - S_i)}{2\omega_i} y_i^2, \quad if \quad y_i < \omega_i ; \tag{10}$$

$$E(\pi_i) = \frac{(P_i - S_i)}{2} \omega_i - (C_i - S_i) y_i, \quad if \quad y_i \ge \omega_i . \tag{11}$$

The derivations of expressions (10) and (11) are available from the first author upon request.

The retailer's expected total profit from the entire $n$-segment market can be expressed as

$$E(\pi) = \sum_{i=1}^{n} E(\pi_i) . \tag{12}$$

Given $K$ identical units of a product, which is to be allocated by the retailer during a single selling season for sale in the $n$-segment market, we aim at finding the optimal quantity to be allocated to segment $i$ ($i$ = 1, 2, …, $n$), $y_i^*$, to maximize the expected total profit. The problem may therefore be formulated as the following mathematical programming model:

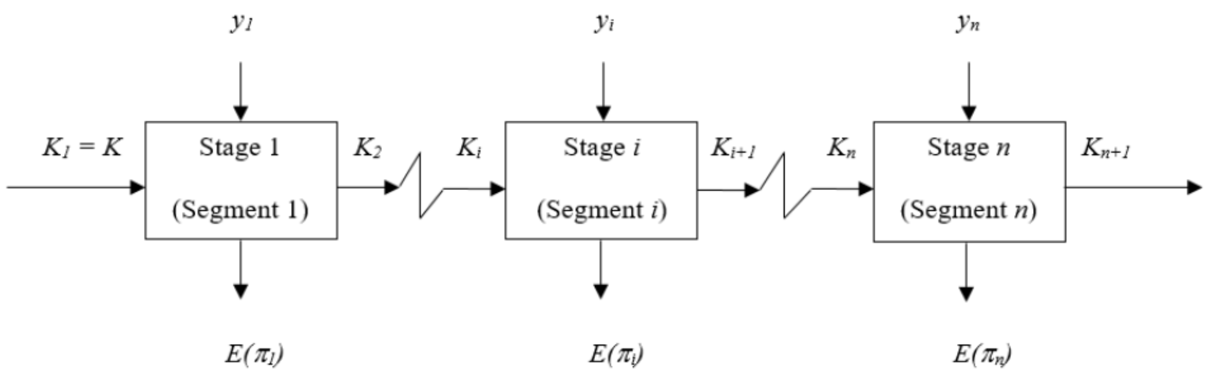
$$\begin{aligned} & \underset{y_1, y_2, \ldots, y_n}{Max} \sum_{i=1}^{n} E(\pi_i) \\ \text{s.t.} \quad & \sum_{i=1}^{n} y_i \le K , \\ & y_i \ge 0 \text{ for } i = 1, 2, \ldots, n. \end{aligned} \tag{13}$$

The model (13) can be cast into a DP formulation that is solved by applying the principle of decomposition. The DP formulation is presented next.

## DYNAMIC PROGRAMING FORMULATION

There are six key components in the DP formulation: (i) the sequence of decision stages, (ii) the input state variable of each stage, (iii) the decision variable of each stage, (iv) the transition function linking the input and output state variables of each stage, (v) the return at each stage, and (vi) the recursive relationship. These components, shown in Figure 1, are identified and discussed below

Figure 1: A dynamic programming representation for the mathematical programming model (13)




Each segment of the market stands for a decision stage to which a certain quantity of the product is allocated. These consecutive stages are indexed corresponding to the indices of the segments defined in the previous section. The mathematical programming model (13) is decomposed into a sequence of smaller and computationally simpler sub-problems, as discussed in the remainder of this section.

The input state variable of stage $i$ is the quantity of the product available for allocation at the beginning of the stage, $K_i$ ($i$ = 1, 2, …, $n$). As shown in Figure 1, each stage's output state variable $K_{i+1}$, serves as the input state variable of the next stage. In particular, $K_{n+1}$ is the output state variable of the last stage, which is the quantity of the product unallocated for the purpose of maximizing the expected total profit. The quantity allocated to stage $i$, $y_i$ ($i$ = 1, 2, …, $n$), is a decision variable. As a whole, there are $n$ decision variables in the DP formulation for the $n$-segment market. The transition function defines the linkage between the input and output state variables of a stage and the decision made for the stage, and may be expressed as

$$K_{i+1} = t_i(K_i, y_i),$$

where,

$K_1 = K$ is given;

$K_{i+1} = K_i - y_i$, $i$ = 1, 2, …, $n$-1;

$t_i(\cdot)$ is the symbol of 'transition function of.'

Expressions (10) and (11) both show that the expected profit yielded from stage $i$ (i.e., the return at stage $i$ ), $E(\pi_i)$, is determined by the decision variable $y_i$, where $y_i \le K_i$ and the price $P_i$. It is shown in expression (7) that $P_i$ is a function of $K_i$. Hence, $E(\pi_i)$ is a function of $K_i$ and $y_i$ and can be expressed as $E(\pi_i(K_i, y_i))$.

A backward induction process is employed to formulate the recursive relationship, which links the optimal decision for a stage to the optimal decisions made in previously considered stages. Starting from stage *n*, the recursive relationship is given by expressions (14) and (15).

For stage *n*,

$$F_n^*(K_n) = \max_{\forall y_n \le K_n} E(\pi_n(K_n, y_n)). \tag{14}$$

For stage *i* = 1, 2, …, *n*-1,

$$F_i^*(K_i) = \underset{\forall y_i \le K_i}{Max} \{E(\pi_i(K_i, y_i)) + F_{i+1}^*(K_{i+1})\}, \tag{15}$$

where $K_{i+1} = K_i - y_i$.

The optimal solution to the DP model formulated above, $y_i^*$ (*i* = 1, 2, …, *n*), is a function of the input state variable $K_i$ and hence can be expressed as $y_i^*(K_i)$. The recursive optimization is carried out in a backward manner until the first stage is reached. At stage 1, the maximum total return (i.e., the expected total profit), $F_1^*(K_1)$, and the corresponding optimal quantity to be allocated to stage 1, $y_1^* = y_1^*(K_1)$, are both determined. $y_1^*(K_1)$ is a unique value because $K_1 = K$ is a given constant. Then, it is possible to backtrack from the first stage through the succeeding stages to obtain the optimal quantity to be allocated to each of the other stages in the following manner:

At stage 2, compute the optimal input value $K_2^* = K_1 - y_1^*$ and then determine the optimal quantity to be allocated to stage 2 through the function $y_2^* = y_2^*(K_2^*)$. Afterwards, for stage *i* (*i* = 3, 4, …, *n*), compute the optimal input state value $K_i^* = K_{i-1}^* - y_{i-1}^*$ and then determine the optimal quantity to be allocated to stage *i* through the function $y_i^* = y_i^*(K_i^*)$. Using the optimal input state value $K_i^*$, the optimal price $P_i^*$ (*i* = 1, 2… *n*) is determined through the price function (7).

## NUMERICAL EXPERIMENTS

In this section, several numerical experiments are presented to illustrate the application of the DP approach described in the previous section and explore the impacts of consumers' price sensitivity upon the retailer's optimal allocation scheme, pricing strategy, and profitability. A market composed of *n* = 4 segments is chosen for these experiments. The price markup factor (6) in segment *i*, $\theta_i$, is specified to take the following functional form:

$$\theta_i = \phi(K_i) = \frac{b_1 + K_i}{b_2 + K_i}, \tag{16}$$

where, $b_1 > b_2 > 0$. It can be easily shown that $d\phi(K_i)/dK_i < 0$ and $d^2\phi(K_i)/dK_i^2 > 0$, implying that $\phi(\cdot)$ is a convex decreasing function of $K_i$.

For illustrative purposes, the following six cases of the price-coefficient vector ($\beta_1, \beta_2, \beta_3, \beta_4$) associated with the four-segment market are considered in the experiments:

Case 1a: ($\beta_1, \beta_2, \beta_3, \beta_4$) = ($8.0\times10^{-5}$, $6.0\times10^{-5}$, $4.0\times10^{-5}$, $2.0\times10^{-5}$);
Case 1b: ($\beta_1, \beta_2, \beta_3, \beta_4$) = ($2.0\times10^{-5}$, $4.0\times10^{-5}$, $6.0\times10^{-5}$, $8.0\times10^{-5}$);
Case 2a: ($\beta_1, \beta_2, \beta_3, \beta_4$) = ($8.0\times10^{-5}$, $7.5\times10^{-5}$, $7.0\times10^{-5}$, $6.5\times10^{-5}$);
Case 2b: ($\beta_1, \beta_2, \beta_3, \beta_4$) = ($6.5\times10^{-5}$, $7.0\times10^{-5}$, $7.5\times10^{-5}$, $8.0\times10^{-5}$);
Case 3a: ($\beta_1, \beta_2, \beta_3, \beta_4$) = ($3.5\times10^{-5}$, $3.0\times10^{-5}$, $2.5\times10^{-5}$, $2.0\times10^{-5}$);
Case 3b: ($\beta_1, \beta_2, \beta_3, \beta_4$) = ($2.0\times10^{-5}$, $2.5\times10^{-5}$, $3.0\times10^{-5}$, $3.5\times10^{-5}$).

The mean and standard deviation of the data set {$\beta_1, \beta_2, \beta_3, \beta_4$} for each of the six cases are shown in Table 1. The values of the other model parameters are selected as follows:

$K$ = 30 units; $b_1$ = 40, $b_2$ = 10;
$\alpha_i$ = 20, $C_i$ = \$100000, $S_i$ = \$30000, $i$ = 1, 2, 3, 4.

To improve exposition, we here confine the decision variables $y_i$ to take on nonnegative integer values. Given $K$ units available for sale in the entire market, the domain of the decision variable $y_i$ ($i$ = 1, 2, 3, 4) is uniformly discretized to take on the $K$+1 values, 0, 1, 2, …, $K$. The mathematical programming model (13) is solved through the DP formulation for all the six cases mentioned above. A computer program is developed by coding in C++ the recursive relationship characterized by expressions (14) and (15) and the backtracking procedure described in the third section. A personal computer that runs at 2.50 GHz and 8 GB of RAM was used to obtain the optimal solution. It took less than a second to solve the DP model for each of the six cases.

Table 1: Descriptive statistics of the data set {$\beta_1, \beta_2, \beta_3, \beta_4$}

| | Mean | Standard Deviation |
|---|---|---|
| Case 1a | $5.00\times10^{-5}$ | $2.58199\times10^{-5}$ |
| Case 1b | $5.00\times10^{-5}$ | $2.58199\times10^{-5}$ |
| Case 2a | $7.25\times10^{-5}$ | $6.45497\times10^{-6}$ |
| Case 2b | $7.25\times10^{-5}$ | $6.45497\times10^{-6}$ |
| Case 3a | $2.75\times10^{-5}$ | $6.45497\times10^{-6}$ |
| Case 3b | $2.75\times10^{-5}$ | $6.45497\times10^{-6}$ |

Table 2 reports the DP optimal allocation scheme {$y_i^*$}, together with the optimal prices {$P_i^*$} and expected total profit $E^*(\pi)$ for each case. For example, given the price-coefficient vector ($\beta_1, \beta_2, \beta_3, \beta_4$) = ($8.0\times10^{-5}$, $6.0\times10^{-5}$, $4.0\times10^{-5}$, $2.0\times10^{-5}$) and the chosen values of the other model parameters shown above, the retailer should allocate 5, 7, 9 and 9 units of the product to segment $i$ ($i$ = 1, 2, 3, 4) and set the unit price at \$175000, \$185714, \$207143, and \$257895 in segment $i$ ($i$ = 1, 2, 3, 4), respectively; the expected total profit would be \$1,393,228.50.

Table 2: Optimal allocation schemes and prices in the six cases of ($\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$)

| | $y_1^*$ | $y_2^*$ | $y_3^*$ | $y_4^*$ | $K-\sum y_i^*$ |
|---|---|---|---|---|---|
| Case 1a | 5 | 7 | 9 | 9 | 0 |
| Case 1b | 8 | 6 | 3 | 1 | 12 |
| Case 2a | 3 | 4 | 4 | 4 | 15 |
| Case 2b | 4 | 4 | 3 | 2 | 17 |
| Case 3a | 9 | 8 | 7 | 6 | 0 |
| Case 3b | 10 | 8 | 7 | 5 | 0 |
| | $P_1^*$($) | $P_2^*$($) | $P_3^*$($) | $P_4^*$($) | $E^*(\pi)$ ($) |
| Case 1a | 175000 | 185714 | 207143 | 257895 | 1393228.50 |
| Case 1b | 175000 | 193750 | 215385 | 230435 | 935356.05 |
| Case 2a | 175000 | 181081 | 190909 | 203448 | 630952.74 |
| Case 2b | 175000 | 183333 | 193750 | 203448 | 587933.34 |
| Case 3a | 175000 | 196774 | 230435 | 287500 | 2015279.20 |
| Case 3b | 175000 | 200000 | 236364 | 300000 | 1955566.80 |
| | $E(d_1)$ | $E(d_2)$ | $E(d_3)$ | $E(d_4)$ | |
| Case 1a | 3.000 | 4.429 | 5.857 | 7.421 | |
| Case 1b | 8.250 | 6.125 | 3.538 | 0.783 | |
| Case 2a | 3.000 | 3.209 | 3.318 | 3.388 | |
| Case 2b | 4.313 | 3.583 | 2.734 | 1.862 | |
| Case 3a | 6.938 | 7.048 | 7.120 | 7.125 | |
| Case 3b | 8.250 | 7.500 | 6.455 | 4.750 | |
| | $\lvert PED_1\rvert$ | $\lvert PED_2\rvert$ | $\lvert PED_3\rvert$ | $\lvert PED_4\rvert$ | |
| Case 1a | 2.333 | 1.258 | 0.707 | 0.348 | |
| Case 1b | 0.212 | 0.633 | 1.826 | 11.78 | |
| Case 2a | 2.333 | 2.116 | 2.014 | 1.952 | |
| Case 2b | 1.319 | 1.791 | 2.657 | 4.370 | |
| Case 3a | 0.441 | 0.419 | 0.405 | 0.404 | |
| Case 3b | 0.212 | 0.333 | 0.549 | 1.105 | |

The unallocated quantity (i.e., $K - \sum y_i^*$), the expected demand $\{E(d_i)\}$, and the absolute values of price elasticity of demand $\{|PED_i|\}$ are also reported in Table 2.

As shown in Table 2, the expected total profit yielded from a market with smaller absolute values of PED on average tends to be higher (see Cases 3a and 3b). Everything else being equal, the retailer allocates more units of the product to those segments with smaller absolute values of PED to enhance her profitability. In addition, a pricing strategy of charging higher prices in those less price-elastic segments helps the retailer increase the expected total profit. Table 2 also shows that dictated by the optimal allocation schemes, the total available quantity of the product is not completely allocated in three of the six cases. In other words, having excess capacity could be a dominant strategy for the retailer to maximize her profits.

## SUMMARY AND CONCLUSIONS

In this paper, we tackle the problem of optimal product allocation and pricing for a profit-maximizing monopolistic retailer in a multi-segment market. A linear demand function is employed to describe the relationship between the price of the product and the expected demand in each segment. A mathematical programming model is formulated and then numerically solved via dynamic programming to find the optimal allocation scheme and determine the corresponding pricing strategy in each of the six cases associated with a four-segment market.

The presented model is not only a valuable decision tool for making pricing and allocation decisions but also provides valuable insights on the implications of consumers' price sensitivity. The impacts of consumers' price sensitivity on the retailer's optimal allocation scheme, pricing strategy, and profitability are examined in the numerical experiments. Based on the chosen values of the model parameters and domains of the decision variables, notable managerial implications from the experiments are summarized below.

(i) In general, higher prices are charged in those segments where demand is less price-elastic.
(ii) Larger quantities of a product are allocated to those segments where demand is less price-elastic.
(iii) To boost profits, the total available quantity of a product may not be completely allocated to a market where demand is highly price-elastic.

This exploratory study reveals some possibilities for future research. First, we have addressed the issue of optimal product allocation and pricing for a monopolistic retailer. An interesting research direction would be to solve the problem in a competitive environment. Second, although the findings reported in this paper are appealing, their generalizability should be considered with care. A plausible direction would be to enlarge the scope of the numerical experiments by relaxing the prioritization assumption and obtain the best DP solution among solutions related to an exhaustive list of segments' prioritizations. Third, the mathematical programming model developed in this study is based on the uniformly distributed uncertain demand. Consumer demand following other probability distributions may be incorporated in the modeling framework in future research.

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