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Dynamic marketing-mix policies for subscription services: some theoretical and empirical results

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ABSTRACT

Upon incorporating price and advertising into a well-known diffusion model for subscription services, optimal marketing–mix variables over time are characterized. A monopolistic market is analyzed for which customers' disadoption is allowed. The analytical findings indicate that the marketing – mix policies of the firm could be very different in the presence of customers' disadoption than their counterparts in its absence. Using Canadian Cable TV diffusion data, price is found to mostly affect market potential whereas advertising is found to affect the coefficient of innovation. Managerial implications of the study together with directions for future research are discussed.

KEYWORDS: Price, Advertising, New subscription services, Optimal control theory, Nonlinear regression

INTRODUCTION

With the development of new technologies and the deregulation of several industries, the number of suppliers of subscription services has grown significantly during the last two decades (Libai et al., 2009; Fruchter and Sigué, 2013). The importance of studying subscription services becomes self-evident in light of the fact that, at present, about every household in a western country is involved, in one way or another, in these services. Examples of subscription services include fixed-line phones, cable or satellite television, health clubs and the internet (Mesak and Darrat, 2002; Fruchter and Sigué, 2013).

The extant research focuses on consumer churn occurring when a competitor acquires an existing customer. However, a customer can also disadopt and leave the service category altogether as illustrated in previous research (Reichheld and Schefter, 2000; Meuter et al., 2005). Thus the attrition rate is the sum of the churn plus the disadoption rate (Libai et al., 2009). Three of the major challenging questions that marketing academicians and managers face are (1) How to price and advertise subscription services over time? (2) How the marketing mix policies of the firm differ in the presence and absence of an attrition rate? and (3) How price and advertising affect the service diffusion model? The answers to the above three research questions that have not been fully addressed in the literature constitute the main contributions of the present study.

The above research questions are relevant as managing services is different from managing goods, due to long-recognized differences between the nature of services and the nature of goods. Services are usually highly intangible (cannot be seen, handled, smelled, etc.), heterogeneous (customized making its mass production difficult), perishable and are produced and consumed simultaneously (lack of transportability) (Zeithmal and Bitner, 2003). The differences between goods and services led Rust and Chung (2006, p. 575) in their review article on service and relationships to advocate the use of *optimal control theory* as a viable tool to optimally manage the dynamic relationship with customers.

The analytical modeling effort employed in this study considers the learning cost curve and the discount rate. More specifically, the model incorporates the key marketing-mix variables of price and advertising into the diffusion model of services articulated by Libai et al. (2009) and thus it explicitly considers the customer attrition rate. The related measure of performance is optimized afterwards using optimal control theory. The price variable is represented by the subscription fee. Other charges such as that related to usage are not considered. This situation is relevant to a variety of subscription services such as cable television, health clubs and the internet, to name a few. In addition, subscription services for which advertising is a main source of revenue instead of being an instrument for generating subscriptions such as newspapers, magazines and contemporary electronic media (internet websites) are beyond the scope of the present study. Kumar and Sethi (2009) provide a review of this particular literature. As far as the empirical work is concerned, nonlinear regression methods are used to estimate alternative specifications of service diffusion models.

The second section provides a related literature review. The third section outlines a general dynamic diffusion model for new subscription innovations, formulates the problem and presents the solution method. The fourth section characterizes the optimal marketing-mix policy for the general diffusion model as well as specific diffusion models. The fifth section compares the performance of specific diffusion models that include and exclude marketing-mix variables using Canadian Cable TV diffusion data. The last section summarizes and concludes the study. The derivation of key formulas and proofs of all reported propositions are included in a separate appendix available from the authors upon request.

LITERATURE REVIEW

Notable early examples of diffusion models for subscription services that have used the Bass (1969) include Dodds (1973) who presented an early diffusion model of cable TV services, Kim et al. (1995) who modelled the diffusion of cell phone services and Rai et al. (1998) who studied the diffusion of the internet. More recent articles that aimed at forecasting the service growth in some sectors such as telecommunications include Islam and Fiebig (2001), Lee and Lee (2009). Additionally, studies have modelled diffusion in social networks, wireless voice/data services, and gaming consoles. (Susarla, et.al., 2012; Nicaulescu and Whang, 2012; Altinkemer and Shen, 2008).

In the above references, the models did not account for the role marketing mix variables in the diffusion process. A few researchers (e.g., Fruchter and Rao, 2001; Mesak and Darrat, 2002) have examined specific aspects of subscription service dynamics in relation to usage and/ or access fees whereas Mesak and Clark (1998) considered both optimal dynamic pricing and advertising. The above studies treat subscription services as if they were durable goods. However, services diffusion differs from durable goods diffusion by the presence of two processes, that is the adoption process and the retention process.

Libai et al. (2009) were probably the first to incorporate customer attrition into the Bass diffusion model. They showed that customer attrition affects considerably the market growth of a new service. However, their modeling framework did not incorporate marketing-mix variables. Based on the Libai et al. (2009) diffusion model for services, Fruchter and Sigué (2013) model optimal dynamic pricing decisions for subscription services whereas Mesak et al. (2011) model optimal dynamic advertising decisions for such services that consider service cost learning.
Empirical research related to the diffusion of subscription services that incorporate marketing mix variables is scarce. Employing a logistic model for which market potential is price dependent, Bagchi et al. (2008) examine the impact of price decreases on land telephone and cell phone diffusion. Using a modified Bass (1969) that does not incorporate attrition, Mesak and Darrat (2003) show that price affects both the innovation and imitation coefficients. Mesak and Clark (1998) conclude that price affects the coefficient of imitation and advertising affects the coefficient of innovation. Employing a modified Libai et al.(2009) model that includes advertising, Mesak et al. (2011) find out that advertising affects the coefficient of innovation. With the initial intent of empirically examining how price and advertising affect the diffusion of a multi-generational cellular phones service model in Europe that incorporates attrition, Danaher et al. (2001) conclude that advertising has an insignificant effect whereas the interaction in price response across generations is significant.

The above brief literature review suggests that (i) there is a belief that marketing - mix variables do affect the diffusion process of new subscription services, (ii) the substance of these effects is basically an empirical question, (iii) the literature lacks the examination of a general price-advertising dynamic diffusion model for new subscription service innovations, as derived optimal policies may be sensitive to the particular functional form chosen for analysis, and (iv) there is an interest to conduct empirical investigations in conjunction with diffusion models for new subscription services that incorporate both price and advertising to partially fill the gap that currently exists in the literature.

GENERAL MODEL FORMULATION AND SOLUTION METHOD

Let us consider adoptions of a new subscription service in a monopolistic market. A firm manipulates its service charge P_t and advertising expenditure U_t (both assumed to be bounded from above) at each time t over a fixed planning period T, $0 \le t \le T$. The monopoly assumption may seem reasonable in situations in which the firm enjoys a patent protection, a proprietary technology, or a dominant market share. A general service diffusion model is given by

$$dN_t / dt = \dot{N}_t = f\ (N_t, P_t, U_t),\ N_0 \ge 0 \text{ and fixed,} \tag{1}$$

where N_t and $\dot{N}_t$ represent the number of subscribers by time t and the subscription rate at t respectively. Expression (1) suggests that the current subscription rate is related to the current number of subscribers and the current rate of the marketing variables. Function f is assumed to be twice differentiable with the following properties related to the marketing variables where a subscript on a variable denotes partial differentiation with respect to that variable:

$$f \ge 0;\ f_P < 0;\ f_U > 0;\ f_{PP} < 0;\ f_{UU} < 0;\ f_{PU} \le 0 \text{ and } f_{PP}\ f_{UU} - f_{pU}^2 > 0. \tag{2}$$

The inequalities (2) imply that the subscription rate is non-negative (new customers' adoption rate is at least equal to customers' disadoption rate), decreases with an increase in the subscription charge, increases with an increase in advertising and is concave in both marketing variables. Inequality (2) further asserts that price may interact with advertising in affecting the subscription rate *f* and the nature of the interaction is non-positive. The last inequality is stated to ensure that one of the sufficiency conditions of optimality is satisfied (details are found in an Appendix).

We introduce next a cost learning curve by assuming that marginal costs, denoted by C, depend on the number of subscribers such that marginal costs decrease with increasing the number of subscribers (experience) (Chambers and Johnson, 2000; Boone et al., 2008),

$$C_t = C(N_t),\ dC(N_t) / dN_t = C'(N_t) \leq 0. \tag{3}$$

Note that marginal costs could be constant ($C' = 0$). C_t is mainly a function of efforts related to service activation (e.g., installation) and account maintenance (e.g., billing, computer server space, and help provided by the service firm).

For a firm that aims to find the optimum trajectories P_t^* and U_t^* to maximize the discounted profit stream over the planning period *T*, the problem is formulated as follows for a discount *rate* $r \geq 0$:

$$\underset{P_t,\, U_t}{Max} \int_0^T [e - rt[(P_t - C(N_t)]\, N_t\ - Q\ (U_t)]\ dt \tag{4}$$

subject to $\dot{N}_t = f\ (N_t, P_t, U_t)$, and the initial number of subscribers $N_0 \geq 0$ is fixed and N_T is free. In expression (4), $P_t\ N_t$ represents the total revenue generated from subscribers and $C(N_t)\ N_t$ is the related total variable cost. In expression (4), $Q(U_t)$ is the advertising cost function assumed to be non-negative and convex with respect to its argument with the properties $Q' > 0$ *and* $Q'' \geq 0$ (Piconni and Olson, 1979). As in earlier monopolistic models (Dockner and JØrgensen, 1988a; Thompson and Teng, 1984; Kalish, 1985), we assume no salvage value for the final number of subscribers at time *T*. Dockner and JØrgensen (1988b) assert that this assumption is particularly plausible when the firm is more concerned with its profits stream over the planning period than profits to be made after instant *T*. This assumption also makes results for new subscription services readily comparable with their counterparts related to new products.

The optimal control problem (4) can be solved by applying Pontryagin's maximum principle optimization technique (Pontryagin, 1962). It is advantageous to mention at this point that the optimal control problem (4) is similar to that reported by Xie and Sirbu (1995, p. 914) who only considered the marketing variable of price. The above authors, however, did not optimize their objective function. To apply the maximum principle, we start by forming the current value Hamiltoian (Sethi and Thompson, 2000)

$$H_t\ (P_t, U_t, N_t) = (P_t - C(N_t))\ N_t - Q\ (U_t) + \lambda_t\, f\ (N_t, P_t, U_t), \tag{5}$$

where λ_t is a costate variable that must satisfy the ensuing equation:

$$d\ \lambda_t /dt = r\ \lambda_t - \partial H_t / \partial\ N_t\ ,\ \lambda_T = 0. \tag{6}$$

An economic interpretation of λ_t is found in Sethi and Thompson (2000). Briefly, λ_t has the interpretation of a *shadow price* of the stock of subscribers N_t. In this paper, as in Dockner and JØrgensen (1988a), we consider admissible controls that are twice differentiable in *t* and satisfy

$P_t \geq 0$ and $U_t \geq 0$ for all relevant t. (In what follows, the time argument is eliminated to minimize confusion and improve clarity). Confining our interest to admissible controls, the partial derivatives of the current value Hamiltonian with respect to P and U along the optimal trajectories, as in Feichtinger (1982), must satisfy the following conditions for an interior solution for which $0 \leq \underline{P} \leq P \leq \bar{P}$ and $0 \leq \underline{U} \leq U \leq \bar{U}$:

$$\partial H / \partial P = 0, \partial H / \partial U = 0, \tag{7}$$

where $\underline{P}$ and $\bar{P}$ are the lower and upper bounds of P, $\underline{U}$ and $\bar{U}$ are the lower and upper bounds of U, and

$$\text{Matrix } \mathbf{HM} \text{ is negative definite,} \tag{8}$$

such that **HM** is non-singular and negative semi-definite Hessian matrix of the second partial derivatives of the Hamiltonian H,

$$\boldsymbol{HM} = \begin{bmatrix} \partial^2 H/\partial P^2 & \partial^2 H/\partial P \partial U \\ \partial^2 H/\partial U\, \partial P & \partial^2 H/\partial U^2 \end{bmatrix}. \tag{9}$$

OPTIMAL MARKETING MIX POLICIES FOR NEW SUBSCRIPTION SERVICES

This section starts first by analyzing the situation of the general diffusion model (1) followed by an analysis related to specific diffusion models for subscription services.

Analysis of the General Diffusion Model

Using conditions (7) in conjunction with expressions (5) and (6), we show in the Appendix that the first derivative of the optimal trajectories P^* and U^* with respect to time t, represented by a (2 x 1) column vector ***DER***, are uniquely determined by the equations

$$\begin{bmatrix} dP/dt \\ dU/dt \end{bmatrix} = \boldsymbol{DER} = -\,\boldsymbol{HM}^{-1}\, \boldsymbol{CV}. \tag{10}$$

In (10), $\boldsymbol{HM}^{-1}$ is the inverse of the (2 x 2) matrix ***HM***, and ***CV*** is the (2 x 1) column vector

$$\boldsymbol{CV} = \begin{bmatrix} f + N(f_N - f f_{PN}/f_P) - f_P(P - C - NC_N) + r\lambda f_P \\ -Q'(f_N - f f_{UN}/f_U) - f_U(P - C - NC_N) + r\lambda f_U \end{bmatrix}. \tag{11}$$

Substituting ***HM*** given by (9) and ***CV*** given by (11) in (10) produces

$$\begin{bmatrix} dP/dt \\ dU/dt \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -\partial^2 H/\partial U^2 & \partial^2 H/\partial U \partial P \\ \partial^2 H/\partial P \partial U & -\partial^2 H/\partial P^2 \end{bmatrix} \begin{bmatrix} f + N(f_N - f f_{PN}/f_P) - f_P(P - C - NC_N) + r\lambda f_P \\ -Q'(f_N - f f_{UN}/f_U) - f_U(P - C - NC_N) + r\lambda f_U \end{bmatrix}, \tag{12}$$

where $\Delta = (\partial^2 H/\partial P^2)(\partial^2 H/\partial U^2) - (\partial^2 H/\partial P \partial U)^2 > 0$, given the last condition in (2).

It is observed from (12) that dP/dt would possess the same sign as that related to the sign of the first entry of the column vector in (11) and dU/dt would possess the same sign as that related to the sign of the second entry of the column vector in (11) for $\partial^2 H/\partial P \partial U = \partial^2 H/\partial U \partial P = 0$. That is, the characterization of the optimal pricing policy for a model that includes advertising would be similar to that for a model that includes price alone. Similarly, the characterization of the optimal advertising policy for a model that includes price would be similar to that for a model that includes advertising alone. For $\partial^2 H/\partial P \partial U \neq 0$, on the other hand, the sign of dP/dt in the presence of advertising, could be different from its univariate counterpart and the sign of dU/dt

in the presence of price, could be different from its univariate counterpart. In short, for the latter case ($\partial^2 H/\partial P \partial U \neq 0$), the two optimal policies of price and advertising are said to be i*nterdependent*. Policy interdependence occurs when price and advertising interacts in affecting the subscription rate *f*. That is when $f_{PU} \neq 0$ (details are found in an Appendix).

Based on the necessary conditions (7) and the general specification of the subscription rate (1), the following self-explanatory proposition is introduced:

PROPOSITION 1

With subscription rate given by (1), presence of a service cost learning the following relationship holds at any point in time along the optimal trajectories of the marketing variables for new subscription services: The ratio of the advertising elasticity of the subscription rate ($\xi = U f_U / f$) to its price elasticity ($\Lambda = -P f_p / f$) is equal to the ratio of advertising to sales revenue (U/NP), multiplied by the marginal cost of advertising (Q´).

Based on the general expression (12), two propositions are presented below.

PROPOSITION 2

For a considerably large interest rate, optimal service charge decreases over time whereas optimal advertising increases over time in the presence of service cost learning.

PROPOSITION 3

For r =0, presence of cost learning curve and subscription rate given by (1) such that
$f + N\,(f_N - f f_{PN}/f_p) \geq 0,\ \ f_N - f f_{UN}/f_U \geq 0,$
(i) Optimal subscription fee is increasing over time.
(ii)Optimal advertising is decreasing over time.

As the interest rate measures how profits at present are preferred to those earned in the future, a low interest rate (r = 0) could be a sensible approximation when the planning period is short or in a low growth economy that approaches disinflation. Since expression (12) does not clearly characterize the trajectories of the marketing variables P_t^* and U_t^*, several diffusion models with specific functional forms are analyzed next.

Analysis of Specific Diffusion Models

The original Bass model (1969) as modified by Libai et al. (2009) to represent the diffusion of new subscription services in continuous time that explicitly considers customers' disadoption is given by the following expression:

$$d N_t / dt = p\,(M - N_t) + q\,(1 - \delta)\,(N_t / M)\,(M - N_t) - \delta N_t\,, \tag{13}$$

where N_t is the number of subscribers by time *t*, *M* is the market potential, *p* is the coefficient of innovation, *q* is the coefficient of imitation, and δ is the disadoption rate. The Bass model (1969) for the diffusion of new durable goods is obtained from (18) upon putting $\delta = 0$. The above authors assume that only those who did not disadopt spread positive word-of-month communications about the service. Therefore, the level of word-of-mouth promotion by retained customers remains the same (q), but its effective impact is reduced due to disadoption from [$(qN_t)/M$] to [$q(1 - \delta)N_t/M$]. As disadopters consider readoption, the remaining market potential is $M - N_t$ and is not affected by the disadoption process (Libai et al., 2009). Using diffusion data related to cell phones, cable television and online banking in the US, Libai et al. (2009) show that their service diffusion model is empirically appealing for the above service categories. The authors also illustrate that the following relationship among the parameters of their model holds:

$$q(1 - \delta) - p - \delta > 0 \quad . \tag{14}$$

Whenever a parameter or quantity in the Libai et al.(2009) is assumed to depend on one or more of the marketing variables, this parameter or quantity is simply multiplied, as appropriate, by one or more of the functions $w(P)$ which is a pricing response function, $h(U)$ which is an advertising efficiency function, or both. These functions are envisioned to possess the following properties:

$$w > 0;\ h > 0;\ w' < 0;\ h' > 0;\ w'' < 0\ ;\ h'' < 0. \tag{15}$$

To incorporate price and advertising into diffusion models based on the Bass model (1969) when applied to services, Mesak and Clark (1998) show empirically that price affects the coefficient of imitation whereas advertising affects the coefficient of innovation. Upon employing expression (12) in conjunction with an extended Libai et al. (2009) model that incorporates marketing-mix variables, inspired by the findings of Mesak and Clark (1998), two propositions are introduced below.

PROPOSITION 4

For a low interest rate, presence of service cost learning, and a service diffusion model $dN_t/dt = ph(M - N_t) + qw(N_t/M)(M - N_t)$,
(i)Optimal subscription fee is increasing over time.
(ii)Optimal advertising is decreasing over time.

PROPOSITION 5

For a low interest rate, presence of service cost learning, and a service diffusion model $dN_t/dt = ph(M - N_t) + qw(1 - \delta)(N_t/M)(M - N_t) - \delta N_t$,
(i)Optimal subscription fee is increasing over time (or increasing first then decreasing later).
(ii)Optimal advertising is decreasing over time (or decreasing first then increasing later).

Proposition 5 implies that the optimal pricing and advertising policies of the service firm in the presence of customers' disadoption could be different from their counterparts in the absence of the same (Proposition 4). Note that for the service diffusion models depicted in Propositions 4 and 5, price does not interact with advertising in affecting the subscription rate f (that is $f_{UN} = 0$). The results obtained upon analyzing other service diffusion models for which price interacts with advertising in affecting the subscription rate f, where $f_{UN} < 0$, are introduced in Propositions 6 and 7.

PROPOSITION 6

For a low interest rate, presence of service cost learning, and a service diffusion model
$d N_t / dt = ph (wM – N_t) + q (1 – \delta) (N_t/wM) (wM – N_t) - \delta N_t$,
(ai)For δ = 0, optimal subscription fee is increasing over time.
(aii) For δ = 0, optimal advertising is decreasing over time.
(bi) For 0 < δ <1, optimal subscription fee is increasing (or increasing first then decreasing later).
(bii) For 0 < δ <1, optimal advertising is decreasing (or decreasing first then increasing later).

PROPOSITION 7

For a low interest rate, presence of service cost learning, and a service diffusion model
$d N_t / dt = phw (M – N_t) + q (1 – \delta) (N_t/M)w (M – N_t) - \delta N_t$,
(ai)For δ = 0, optimal subscription fee is increasing over time.
(aii) For δ = 0, optimal advertising is decreasing over time.
(bi) For 0 < δ <1, optimal subscription fee is increasing (or increasing first then decreasing later).
(bii) For 0 < δ <1, optimal advertising is decreasing (or decreasing first then increasing later).

The service diffusion model employed in Proposition 6 assumes that advertising affects the coefficient of innovation whereas price affects market potential. The service model employed in Proposition 7 assumes that advertising affects the coefficient of innovation whereas price affects the untapped market *(M-N)*. Propositions 6 and 7 demonstrates again that the optimal pricing and advertising policies of the service firm in the presence of customers' disadoption [parts (bi) and (bii)] could be different from their counterparts in the absence of the same [parts (ai) and (aii)].

EMPIRICAL ANALYSIS

In the previous section, a variety of new subscription service diffusion models were analytically examined. In this section, several of such models will be empirically investigated with the following two main questions born in mind:
(1)Does the performance of the Libai et al. (2009) service model improve upon the inclusion of the marketing –mix variables of price and advertising?
(2)How would price and advertising be incorporated in the service model?

Description of Data

Since the empirical analysis involves the estimation of several diffusion parameters, this required a comprehensive set of historical subscription data along with service charges and advertising expenses. We were also interested in an industry where disadoption plays a significant role while the market is monopolistic at the same time. A plausible example of an industry with these attributes is Cable TV. A cable TV service provider assumes a monopoly in the area in which the provider operates. The provider assembles selected cable channels into certain packages and makes them available to potential subscribers for a subscription fee. The selection of channels offered by a provider generally starts with a "basic" package for a fixed monthly charge. This package typically includes local and several distant broadcast stations. The "premium" package such as HBO, Cinemax etc. are offered as optional packages for additional monthly charges. However, since the subscriptions to these premium packages require subscription to the basic service first, the empirical analysis reported herein is confined

to the basic service only. In addition to the availability of relevant data, basic cable TV service was chosen in the empirical analysis as it represents an example of a service innovation that comprises solely the line of business of each service provider and is acquired only once per household (subscriber).
Aggregate annual data are available from Canada at the industry level (National and provincial) in terms of number of subscribers (N_t) , service charge (P_t) and advertising expense (U_t). Following a similar mathematical approach as that employed in Mesak and Darrat (2003), it can be shown that if the consumer adoption process in each geographic area is represented by a Libai et al. (2009) model, the consumer adoption process at the aggregate sub-national and national levels for the cable TV industry can be also represented by a Libai et al. (2009) model. The data related to the cable industry in Canada and its provincial regions (Quebec, Ontario and Nova Scotia & New Brunswick) were available from *Statistics Canada*. The annual data related to Canada and its three provincial regions covered the period 1976-1994 as the record indicated that the data were available as early as 1976. In 1995, a change in the definition of number of subscribers affected the count of subscribers and therefore, data from 1995 and beyond were not comparable with previous periods.

Estimation and Model Selection Procedure

The discrete analogues of the Libai et al. (2009) model depicted in (13) that does not incorporate marketing mix variables together with its two modified versions that incorporate marketing mix variables analytically analyzed in Propositions 5 and 6 together with a third variant are provided, upon minor arrangement of terms, by expressions (16), (17), (18) and (19).

$$N_t = p\,(M_t - N_{t-1}) + q\,r\,(N_{t-1}/M_t)\,(M_t - N_{t-1}) + r\,N_{t-1}. \tag{16}$$
$$N_t = p\,\ln(U_t)(M_t - N_{t-1}) + q\cos(\theta P_t)\,r\,(N_{t-1}/M_t)(M_t - N_{t-1}) + r\,N_{t-1}. \tag{17}$$
$$N_t = p\,\ln(U_t)(M_t\cos(\theta P_t) - N_{t-1}) + q\,r\,(N_{t-1}/M_t\cos(\theta P_t))(M_t\cos(\theta P_t) - N_{t-1}) + r\,N_{t-1}. \tag{18}$$
$$N_t = p\,\ln(U_t)(M_t - N_{t-1})\cos(\theta P_t) + q\,r\,(N_{t-1}/M_t)(M_t - N_{t-1})\cos(\theta P_t) + r\,N_{t-1}. \tag{19}$$

In models (17) through (19), as in the study of Horsky and Simon (1983), an advertising efficiency function of the form $h(U_t) = ln(U_t)$ is employed. In model (17) the coefficient of imitation q is multiplied by a pricing response function $w(P_t) = cos\,(\theta P_t)$. While in model (18) the market potential M_t is assumed to be affected by the charged price, model (19) assumes that the untapped market, or the remaining market potential ($M_t - N_{t-1}$) is affected by the same. In all models (16) through (19), the retention rate $r = 1 - \delta$ and M_t represents the number of households wired for cable access in year *t*. For model (16), three parameters (*p*, *q* and *r*) need to be estimated. It is shown in the next section that model (16) does not perform well. In models (17) through (19) four parameters (*p*, *q*, *r* and θ) need to be estimated. The models (16) through (19) are nonlinear in the parameters. SAS nonlinear estimation procedure MARQUARDT was used to estimate such models. Although additional models were empirically estimated, models (17) through (19) were the only ones proven to be of empirical appeal.

The empirical literature suggests that a reasonable metric for judging rival models is their out-of-sample forecastability (Zanias, 1994; Pindyck and Rubinfeld, 1998). Bottomley and Fildes (1998) contend that goodness of fit raises issues regarding the robustness of the findings when models are estimated and “fitted” using much shorter data series as typifies new product/service forecasting applications. In addition, evidence in the literature (Rao, 1985; Young, 1993) demonstrates the lack of correlation between a model’s fit and its forecasting performance.

Therefore, the selection among models (17) through (19) is carried out according to a two-stage screening process. Only models of reasonable explanatory power (significance of price and advertising together with coefficients of the correct signs and magnitudes) are considered eligible candidates for the second stage. In the second stage, eligible models are compared based on their predictive power (measured by Theil's coefficient *UII*).

In computing coefficient *UII* for a given model and a certain region, the related data set is partitioned into two parts. The first large part (13 observations) is used to estimate the model, whereas the second smaller part (5 observations) of the data set (holdout sample) is used to compare actual with the out-of-sample predicted values. Using one-step ahead forecasting process, the model is first fitted for 13 observations, then a new subscribers' forecast N_{14} is obtained. The model is re-estimated afterwards using 14 observations and a forecast N_{15} is obtained and so on. Price and advertising during a certain year are used to obtain new subscribers' forecast for the same year, as appropriate. Coefficient *UII* (Theil, 1965, p. 28) is computed using the expression given below.

$$UII = \frac{\left[\sum_{i=1}^{T}\left(A_i - PR_i\right)^2\right]^{1/2}}{\left[\sum_{i=1}^{T} A_i^{\,2}\right]^{1/2}}, \qquad (20)$$

where (A_i, PR_i) stand for a pair of an actual and predicted N values for year *i*, and *T* represents the number of periods in the holdout sample (five years in this case). Finally, the selected service diffusion model is that eligible model with the smallest *UII* coefficient.

Estimation Results

The variables N_t, P_t, U_t and M_t were operationalized as shown below.

N_t = Number of cable subscribers to the basic service by year *t* (in thousands).
$P_t = Pr_t / Pr_1$, where Pr_t is the service charge of cable TV (in Canadian dollars) divided by consumer price index (*CPI*) in year *t* and Pr_1 is the service charge of cable TV (in Canadian dollars) divided by *CPI* in the first year of a given time series, so that $P_1 = 1$.
$U_t = A_t/A_1$, where A_t is advertising expenditure of cable TV (in thousands of Canadian dollars) divided by consumer price index (*CPI*) in year *t* and A_1 is the advertising expenditure of cable TV (in thousands of Canadian dollars) divided by *CPI* in the first year of a given time series, so that $U_1 = 1$.
M_t = Number of households wired for cable access in year *t* (in thousands).

After adding error terms to expressions (16) through (19), Tables 1 to 4 summarize the estimation results upon using SAS nonlinear estimation procedure MARQUARDT.

Table 1: Estimation of parameters of Model (16) - Diffusion is not affected by marketing-mix variables

Region	Year	*p*	*q*	*r*	*SSE (in millions)*	*UII*

Canada	1976-1994	- 0.3908 (0.4658)	1.4697 (1.2549)	0.8540** (0.0830)	47.410	---
Quebec	As above	-0.1521 (0.2521)	0.9699 (1.0868)	0.8395** (0.1389)	14.536	---
Ontario	As above	-1.3978 (1.1439)	3.7260 (3.1716)	0.7877** (0.1249)	12.666	---
Nova Scotia & New Brunswick	As above	0.0073 (0.2539)	0.5963 (0.5632)	0.9430** (0.0388)	0.458	---

Notes: standard errors are in parentheses; **indicates significance at the 0.05 level

Table 2: Estimation of parameters of Model (17) - Advertising affects p and price affects q

Region	Year	*p*	*q*	*r*	*θ*	*SSE (in millions)*	*UII*
Canada	1976-1994	0.154** (0.036)	0.7154** (0.0824)	0.9106** (0.0172)	0.7960** (0.0678)	14.800	0.0257
Quebec	As above	0.141** (0.048)	0.7235** (0.2542)	0.8377** (0.0884)	0.7282** (0.2123)	7.682	0.0783
Ontario	As above	0.189** (0.069)	0.631** (0.118)	0.9402** (0.0226)	0.8279** (0.1205)	6.814	0.0268
Nova Scotia & New Brunswick	As above	0.271** (0.099)	1.0117** (0.1856)	0.9486** (0.0153)	1.0143** (0.1642)	0.295	0.0157

Notes: standard errors are in parentheses; **indicates significance at the 0.05 level

Table 3: Estimation of parameters of Model (18) - Advertising affects p and price affects market potential M_t

Region	Year	*p*	*q*	*r*	*θ*	*SSE (in millions)*	*UII*
Canada	1976-1994	0.148** (0.047)	0.4179** (0.0759)	0.9567** (0.0204)	0.3921** (0.0412)	19.131	0.0147
Quebec	As above	0.143** (0.052)	0.5009 (0.2876)	0.8675** (0.1011)	0.3510** (0.1085)	8.451	0.0578
Ontario	As above	0.216* (0.107)	0.3841** (0.1184)	0.9734** (0.0261)	0.3839** (0.0657)	7.830	0.0159
Nova Scotia & New Brunswick	As above	0.229 (0.203)	0.5472** (0.1277)	0.9841** (0.0452)	0.3919** (0.1814)	0.362	---

Notes: standard errors are in parentheses; **indicates significance at the 0.05 level; *indicates significance at the 0.10 level

Table 4: Estimation of parameters of Model (19) - Advertising affects p and price affects untapped market ($M_t - N_{t-1}$)

Region	Year	*p*	*q*	*r*	*θ*	*SSE (in millions)*	*UII*
Canada	1976-1994	0.187** (0.046)	0.6571** (0.0746)	0.9149** (0.0172)	0.7279** (0.0593)	14.980	0.0192
Quebec	As above	0.166** (0.052)	0.6591** (0.263)	0.8381** (0.0881)	0.5986** (0.1606)	7.615	0.0651
Ontario	As above	0.242** (0.101)	0.5907** (0.1170)	0.9426** (0.0233)	0.7735** (0.1125)	7.028	0.0205
Nova Scotia & New Brunswick	As above	0.409 (0.244)	0.9947** (0.2067)	0.9488** (0.0164)	0.9971** (0.1961)	0.309	---

Notes: standard errors are in parentheses; **indicates significance at the 0.05 level

The results depicted in Tables 1- 4 reveal the following:

(1) The Libai et al (2009) service model does not perform well for all the geographical regions considered (Table 1), except upon the inclusion of marketing-mix variables.
(2) For all geographical regions, advertising affects the coefficient of innovation *p* (Tables 2, 3 and 4).
(3) For three geographical regions (Canada, Quebec, Ontario), price affects the market potential M_t (table 3).
(4) For the fourth geographical region (Nova Scotia & New Brunswick), price affects the coefficient of imitation *q* (Table 2).

SUMMARY AND CONCLUSIONS

This section summarizes the main theoretical and empirical findings of the study, highlights managerial implications, and proposes directions for future research. The diffusion models analytically explored in this article represent a unique attempt in the literature that aim at deriving optimal pricing and advertising policies for new subscription service innovations together with empirically validating the structure of such models.

Our approach considers demand dynamics, learning curve, and discounting that are managerially relevant. Demand dynamics are reflected in the differential equation of the diffusion model through incorporating saturation, word of mouth and disadoption effects. In introducing new subscription service innovations, the decision maker may not be interested in setting the pricing and advertising policies in isolation, but rather he or she would be more satisfied by considering the interactive influence of such policies upon the subscription rate. This study addresses such concern by recognizing the simultaneous impact of the studied two marketing variables on the diffusion process. The analytical findings of the study summarized in Propositions 3 through 7 generally argue in favor of increasing the subscription fee for new subscriber services overtime, perhaps followed by a period in which the fee may be decreasing when disadoption is significant, and decreasing the advertising expenditure, perhaps followed by a period in which advertising is increasing when disadoption is significant. The empirical research findings reported in the last section suggest that for the studied new subscription service innovations, service charge and advertising expenditure affect the diffusion process in ways to generate optimal pricing and advertising policies overtime consistent with Propositions 5 and 6. More specifically, price is found to mostly affect market potential whereas advertising is found to affect the coefficient of innovation.

The modeling effort developed here is exploratory revealing many possibilities for future research. First, given that the current empirical research estimates and compares several models of diffusion of innovations related to one subscription service (Cable TV) over a span of a certain time period, the study meets the exploratory objectives of this research. Nevertheless, future research should examine a wider range of subscription service innovations particularly those related to mobile commerce (Balasubraminian et al., 2002; Kleijnen et al., 2005) and different time spans to assess the robustness of the drawn conclusions. Furthermore, studying the sensitivity of the optimal marketing mix policies to changes in the disadoption rate would be advantageous.

A second direction for future research is to consider optimal pricing and advertising of a “secondary” subscription service that is contingent on the subscription of a “primary” subscription service (e.g., a customer cannot subscribe to a secondary pay service such as HBO unless he or she is a subscriber to the primary basic cable service). The study of Fruchter and Rao (2001) is useful in this respect.

A plausible third direction for future research is to incorporate service bundling in the modeling effort (Guiltman, 1987). For example, a cable TV provider may expand the business by providing internet access services together with telephone services. Whether to use a mixing bundling strategy or a pure bundling strategy (i.e., selling the bundle only) is an interesting research topic (Ansari et al., 1996).

Fourth, other marketing mix variables such as service quality (Rust, et al., 1995; Van Mieghem, 2000), service guarantees (Fruchter and Gerslner, 1999; Kumar et al., 1997) and distribution (Jones and Ritz, 1991; Bronnenberg et al., 2000) could be considered endogenously to enrich the modeling effort.

Fifth, this article assumes that the market has only one service provider operating (as in the case of cable TV). An interesting direction for future research is to characterize optimal pricing and advertising policies under the threat of competitive entry (Gupta and Di Benedetto, 2006). Alternatively, competition could be modeled in a (non-cooperative) game- theoretic framework within the same industry (Thompson and Teng, 1984; Dockner and JØrgensen, 1988b), or between industries such as cable TV and satellite dish (Ross, 1999).

A sixth direction for future research is to extend the modeling effort to consider different generations of the new service innovation. The notable studies of Danaher et al. (2001) and Danaher (2002) should be particularly informative as they provide lucid reviews of research undertaken in this area. Addressing the above research issues, and perhaps several others, should be beneficial to both academicians and practitioners in the service sector.

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