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An Examination of Croston's Method with Alternative Time-Between-Demand Distributions

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ABSTRACT

Croston's method (1972) assumes demand is geometrically distributed. We study Croston's method under Symmetric, u-shaped, and uniform distributions for the time between demand occurrences. A Monte Carlo simulation study examines the performance of Croston's method. In certain distributions Croston's technique is superior in performance to the Bias-Corrected Croston technique.

KEYWORDS: Croston's Method, Forecasting, Demand Distributions

INTRODUCTION

Despite shortcomings, Croston's method (1972) has found widespread use and has become the theoretical and practical benchmark for inventory models and forecasts (Syntetos and Boylan, 2005, 2006; Altay, Rudisill and Litteral, 2008; Teunter, Syntetos & Babai, 2010; Xu, Wang, & Shi, 2012; Ramaekers & Janssens, 2014. Croston's (1972) procedure of using separate forecasts for the time between nonzero demands and for the positive demand sizes typically assumes no trend in the data and that the time between nonzero demands and size of demands are independent and assumes that the probability for demand in any period is constant and independent of other demands. In reality the demand is not always independent of other demands. Altay, et al, (2008), Ghobbar and Friend (2003), Lindsey and Pavur (2008) and Snyder (2002) among others have examined violations of the assumption of constant demand due to trends in the demand. Altay, Litteral, and Rudisill (2011) examined the effects of correlation when demand is intermittent. Lindsey and Pavur (2011) considered the effect of seasonality on the probability of demand.

Croston's method assumed that the probability of a demand occurrence at any time unit is constant and that the occurrence is independent of previous occurrences. Although some researchers have acknowledged that occurrences may have a dependency structure, these relationships are difficult to model. However, there may be real world applications in which demands either cluster or in which demands rarely occur in adjacent time periods. An empirical assessment would need to justify the ultimate validity of using a particular distribution. Now, if it can be illustrated that the distribution of the time between demands does not substantially affect the robustness of Croston's procedure, then perhaps the decision to use an approximate distribution or the standard geometric distribution to model an intermittent demand process may be justified. A simulation study cannot examine all distributions for modeling time between demand occurrences but it can assist in understanding how the performance of Croston's technique is affected under a "false" assumption for the distribution of time until a demand occurrence. This paper does not try to justify the use of the distributions selected for this study. This paper examines how alternative distributions affect the robustness of Croston's technique.

LITERATURE REVIEW

The forecasting methodology proposed by Croston (1972) has evolved into one of the most popular techniques to forecast demand for intermittent and slow-moving items. His method has evolved into the applied standard for inventory models (Teunter, Syntetos and Babai, 2010; Xu, Wang, & Shi, 2012; Ramaekers and Janssens, 2014). Croston's method has yielded well accepted benefits by Willemain, Smart, Shockor, and DeSautels (1994), Johnston and Boylan (1996) and others. The procedure creates one forecast for the demand amount and another forecast for the time between demands and pools them into a single forecast. Syntetos and Boylan (2001) provided a modification to correct an identified bias in Croston's method. Several studies have compared the bias corrected method with the original without identifying a winner in the comparisons (Eaves & Kingsman, 2004; Syntetos & Boylan, 2005; Teunter & Sani, 2009; and Teunter & Duncan, 2009). Croston's technique and SES are essentially the same when demand occurs every period. A generalized method when demand is slow or regular has been suggested, but it potentially introduces more bias than Croston (Teunter & Sani, 2009). Levén and Segerstedt's (2004) methodology introduces a common technique for all demand rates.

Croston's Method

See Willemain, et al. (1994) for a concise explanation of Croston's (1972) method. It is not included here to save space. Croston's (1972) recommends small alpha values between 0.1 and 0.2 so to be consistent, this work utilizes lower alpha levels.

Syntetos and Boylan Revised Croston Procedure

Syntetos and Boylan (2001) identified a bias in Croston's (1972) method. The bias is minimized by using small alpha levels. The bias is corrected by multiplying the demand per period by 1−α/2, yielding the following new estimator of mean demand:

$$y_t'' = \left(1 - \frac{\alpha}{2}\right)\frac{z_t''}{p_t''} \qquad (1)$$

When p is the inter demand interval, the bias is expressed as:

$$bias = \left(100\frac{\alpha}{2-\alpha}\right)\left(1 - \frac{1}{p}\right) \qquad (2)$$

Teunter and Sani (2009) advocate that in some cases when limited periods have no demand, Croston's method excels and when most periods have no demand the use of the Syntetos and Boylan's correction is better. Syntetos and Boylan (2005) remind us that Croston's assumptions of stationary i.i.d. of demand sizes and intervals, geometrically distributed inter demand intervals and independence of demand sizes and intervals apply to the new estimator as well.

METHODS

Five hundred replications were performed for each set of four conditions. A smoothing constant of 0.10 is used. The mean average time between demands is set at an average of 5 periods. RMSE is used to compare the accuracy of the forecast. Each simulation scenario examines the performance of Croston's method, the bias corrected form of Croston's method, and SES. Three sets of six simulations were completed but only one set is reported here. The first set considered the discrete distributions in Table 1 for the time-between-occurrences distribution. The second and third groups were based on allowing the distribution of the time-between-occurrences distribution to change or alternate.

To perform a comparative analysis several discrete distributions were selected for the time until a positive demand occurs. Croston's method is usually assessed assuming independent likelihood of a demand in each period, which does not change. If the probability of each period's demand is p, then the distribution of the length of the period until the next positive demand follows the Geometric distribution. This typically assumed distribution is replaced with an alternative distribution, which, in fact, will violate the assumption of independent probabilities of a demand per period. The following table presents the four distributions reported in this study. The expected value of each discrete distribution is 5. The variance of the first four distributions is approximately 2. The variance of the U shaped distribution is approximately 20. The Geometric distribution has this same variance for $p = 0.2$.

Table 1: Distributions

	Normal-Shaped Discrete	Uniform Discrete 5 Values	U Shaped Discrete
1	0.02	0	0.55
2	0.03	0	0
3	0.05	0.2	0
4	0.2	0.2	0
5	0.4	0.2	0
6	0.2	0.2	0
7	0.05	0.2	0
8	0.03	0	0
9	0.02	0	0
10	0	0	0.45

RESULTS

The results of the 500 simulations are displayed in Table 2. The RMSE is shown in the first column using Croston's method and the Bias Corrected Croston's method in the second. The RMSE based on SES is in the third column. The fourth column has the RMSE based on actual values using SES. The fifth column has the RMSE based on actual values using Croston's method. The sixth column shows the RMSE based on actual values using the Bias Corrected Croston's Method. The last two columns show the percent reduction in the RMSE for each method. The row shows what distribution is utilized for the mean time between demands. Table 2 illustrates that Croston's method and the bias Corrected Croston's method both perform well in estimating the true mean demand per period. For the first two distributions Croston's method had an advantage. In the last two distributions, the bias corrected method was better.

Table 2: Distributions Analysis

	Average Root Mean Square Error (RMSE) in Estimating True Mean Demand per period.			Average Root Mean Square Error (RMSE) in Estimating Observed Demand per period.			Percent Reduction in RMSE in Estimating True Mean Demand per period. (Percentage)	
DISTRIBUTION	Croston	Bias Corrected Croston	SES	SES	Croston	Bias Corrected Croston	Croston	Bias Corrected Croston
1 Normal-Discrete	2.59	2.79	8.37	84.29	80.47	80.49	69.07	66.70
2 Uniform Discrete	2.61	2.81	8.40	84.28	80.47	80.48	68.97	66.60
3 U shaped Distribution	9.18	8.81	20.09	81.87	80.88	80.83	54.29	56.14
4 Geometric Distribution p=.2	8.31	7.93	18.20	82.15	80.53	80.48	54.31	56.41

DISCUSSION AND CONCLUSIONS

Many studies have investigated the performance of Croston's method in forecasting demand for slow moving items and concluded that it is a robust methodology. It is generally assumed that the geometric distribution is appropriate for the time between the demands when using Croston's method or the bias corrected Croston's method. This paper examines several conditions that violate the assumption of the geometric distribution. While the authors have little expectation that another distribution is more appropriate than the geometric, it is interesting to know how violations to this assumption will impact forecasts. At least compared to SES, it appears that Croston's method is a viable tool even when other distributions are present. This should provide the manager with increased confidence in forecasts developed using Croston's method (or the bias corrected version) even when little is known about the underlying distribution for the time between demands.

REFERENCES

Altay, N., Rudisill, F., and Litteral, L. A., (2008). Adapting Wright's modification of Holt's method to forecasting intermittent demand. *IJPE*, *111*(2), 389-408.

Altay, N., Litteral, L. A., and Rudisill, F., (2011). Effects of correlation on intermittent demand forecasting and stock control. *International Journal of Production Economics, 135*(1) 275-283.

Boylan, J. E. and Syntetos, A. A., (2007). The accuracy of a modified Croston procedure. *International Journal of Production Economics*, *107*(2), 511-517

Croston, J. D., (1972). Forecasting and stock control for intermittent demands. *Operational Research Quarterly*, 23(3), 289-303.

Eaves, A. H. C., & Kingsman, B. G. (2004). Forecasting for the ordering and stock-holding of spare parts. *Journal of the Operational Research Society*, 55(4), 431-437.

Ghobbar, A. A., & Friend, C. H. (2002). Sources of intermittent demand for aircraft spare parts within airline operations. *Journal of Air Transport Management, 8*(4), 221-231.

Johnston, F.R. & Boylan, J.E. (1996). Forecasting for items with intermittent demand, *Journal of the Operational Research Society, 47*, 113-121.

Levén, E. & Segerstedt, A., (2004). Inventory control with a modified Croston procedure and Erlang distribution. *International Journal of Production Economics, 90*(3), 361-367.

Lindsey, M. D. & Pavur R. (2008). A Comparison of Methods for Forecasting Intermittent Demand with Increasing or Decreasing Probability of Demand Occurrences. *Advances in Business and Management Forecasting, 5*, 115-132.

Lindsey, M.D. & Pavur, R., (2011). Forecasting Intermittent Demand with Seasonality: A simulation study. Proceedings of the Southwest Decision Sciences.

Ramaekers, K., & Janssens, G.K. (2014). Optimal policies for demand forecasting and inventory management of goods with intermittent demand. *Journal of Applied OR, 6*(2), 111-123.

Snyder, R. D., (2002). Forecasting sales of slow and fast moving inventories. *European Journal of Operational Research, 140*(3), 684-699.

Syntetos, A. A., & Boylan, J. E. (2001). On the bias of intermittent demand estimates. *International Journal of Production Economics, 71*(1/3), 457-466.

Syntetos, A. A., & Boylan, J. E. (2005). The accuracy of intermittent demand estimates. *International Journal of Forecasting, 21*(2), 303-314.

Syntetos, A. A., & Boylan, J. E. 2006. On the stock-control performance of intermittent demand estimators. *International Journal of Production Economics, 103*(1), 36-47.

Teunter, R. H., & Duncan, L., (2009). Forecasting intermittent demand, a comparative study. *The Journal of the Operational Research Society, 60*(3), p321-329.

Teunter, R.H., & Sani, B., (2009). On the bias of Croston's forecasting method, *European Journal of Operational Research, 194*(1), 177-183.

Teunter, R., Syntetos, A.A., Babai, M.Z., (2010). Determining Order-Up-To Levels under Periodic Review for Compound Binomial (Intermittent) Demand. *European Journal of Operational Research*, *203* (3), 619-624.

Willemain, T. R., Smart, C. N., & Schwarz, H. F. (2004). A new approach to forecasting intermittent demand for service parts inventories. *IJF, 20*, 375-387.

Willemain, T. R., Smart, C. N., Shockor, J. H. and DeSautels, P. A., (1994). Forecasting intermittent demand in manufacturing: A comparative evaluation of Croston's method. *International Journal of Forecasting, 10*(4), 529-538.

Xu, Q., Wang, N., & Shi, H. (2012). Review of Croston's method for intermittent demand forecasting. In *Fuzzy Systems and Knowledge Discovery (FSKD), 2012 9th International Conference* on (pp. 1456-1460). IEEE.