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Impacts of Multi-Period Planning Methods on Supply Chain Financial Performance

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ABSTRACT

Solutions to deterministic optimizing models for supply chains can be very sensitive to the formulation of the objective function and the length of the planning horizon. Synthesis of optimization and simulation in this research reveals how multi-period optimizing models may be counterproductive if traditional accounting of revenue and costs is performed and planning occurs with too short a planning horizon. We propose a "value added" complement to traditional financial accounting that allows planning to occur with shorter horizons than previously thought necessary. The robustness of this "value added" approach is tested by introducing supply chain disruptions downstream.

KEYWORDS: Supply Chain Multi-Period Planning, Simulation, Supply Chain Financial Accounting and Supply Chain Disruptions.

INTRODUCTION

Organizations plan based on expectations, often with a rolling horizon. They implement decisions according to plans early in the planning horizon, experience events that differ from expectations, and revise the plans as new information becomes available. In long international supply chains, multi-period supply chain (SC) planning problems pose enormous challenges to managers (Ghadge et al., 2012; Tang et al., 2012; Wagner and Bode, 2006; Blackhurst et al., 2005). When striving to optimize SC performance with a rolling horizon, organizations must incorporate slack to allow for risk and impose boundary constraints to ensure that they will be positioned well for ongoing operations. They must account for revenues and costs that occur outside the planning horizon and consider how long the planning horizon should be. These issues are ignored in most published studies that employ optimization models for tactical and operational decisions in SC management (e.g., Ciarallo et al., 1994; Wang and Gerchak, 1996; Sabri and Beamon 2000; Lee and Kim, 2002; Leung et al., 2006; Lin and Chen, 2009; You et al., 2009; Cardoso et al., 2015 etc.).

Theoretically, the planning horizon of a SC optimizing model should embrace the longest lead time upstream, the production cycle time, and the longest lead time downstream. This may result in a model that requires excessive time to solve and support. If the planning horizon is too short, however, solutions may be counterproductive – especially if revenues and costs are recognized in the model when they would actually occur. To resolve this dilemma, we propose a "value added" approach in the optimizing model's objective function and test the robustness of this approach with downstream SC disruptions.

MODELING INFRASTRUCTURE

This research deals with a three-echelon SC for bulk products that are distributed through warehouses in several different regions. The SC under investigation is predefined and has *s* suppliers, *f* production facilities, *p* products, and *w* warehouses. Customers' demands are aggregated and allocated to the warehouses. The locations of suppliers, production facilities, and warehouses are hypothetically given and the transportation of raw materials and finished goods is assumed to be done by third-party logistics (3PL) providers (thus avoiding the issues of consolidating shipments and related delay due the shipment consolidation process). Research data were adapted from the literature (Tsiakis et al., 2001) with amendments made to accommodate the purposes of this research. Figure 1 illustrates the supply chain structure.

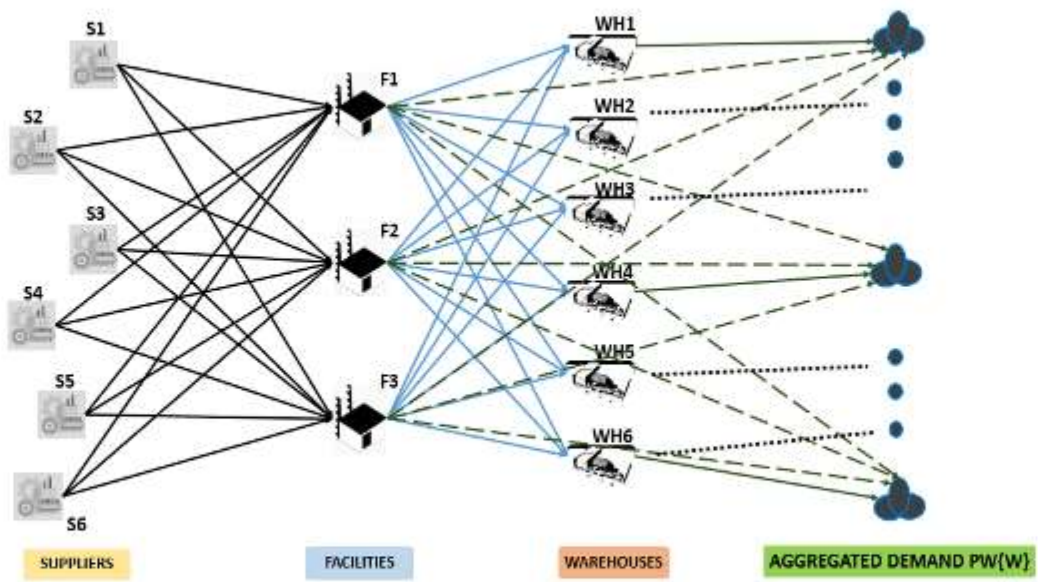

Figure 1 Research Supply Chain Structure

To capture the stochastic elements in the SC, we employ a simulation model with an embedded optimizer and perform iterative updates to the plan with a rolling horizon. This enables us to develop strategies that minimize expected costs or maximize expected contributions to profit while maintaining a designated level of service with explicit recognition of uncertainty. The hybrid model is constructed on the Statistical Analysis System (SAS) 9.4 platform.

A schedule-revision period is chosen. SC solutions are revised with the optimizing model after simulating business activity in the revision period and accounting for stochastic demands and transit times for flows in the SC network during the interval. Updates are applied to initial inventories at plants and warehouses and to goods in transit with their expected arrival dates. The optimizing model reschedules SC activity (material inflows, production and product deliveries) accordingly. This iterative process continues until the last day in the experimental period. Multiple iterations of the simulation are performed to assess service levels and produce estimates of financial performance using standard cash-flow accounting methods. Figure 1-1 illustrates the interactive process of the hybrid model.

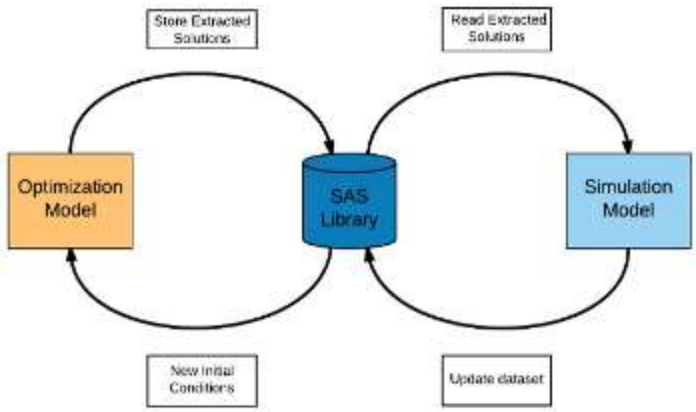

Figure 2 Interaction Between Simulation and Optimization

With a goal of maximizing net profit contribution, the optimization model includes procurement, production, and distribution plans. Buffer inventories (raw materials, finished goods at plants and products at warehouses) are built into the SC as part of the risk-mitigation strategy. Supply chain risk-event management is represented by allowing finished goods to be shipped directly from plants to customers when shortages occur at customer service centers (warehouses) or by adding additional shifts when production must be intensified (possibly due to disruptions that have occurred in the supply chain).

The analytical model is developed with the following assumptions:

1) The managerial goal is to maximize net contribution to profit.
2) The profit contribution net of shipping costs is realized when customer demand is satisfied from the warehouses or directly from the plant.
3) Inventory replenishment is recognized at the end of each business day.
4) Aggregate customer demands for products are registered at the beginning of each day at the warehouses.
5) Suppliers who provide the same raw material are geographically separated (and therefore subject to different disruption risks).
6) Each production facility can produce all products, ship products to all warehouses, and perform alternative delivery of finished goods via expedited shipping to satisfy customer demand as alternatives to deliveries from warehouses.
7) Customer demand of product is aggregated and assigned to the designated warehouse every day. Alternative deliveries from other warehouses are not considered in this research.

Daily production at plants, shipments to warehouses, and deliveries to customers are planned with consideration of production capacities across plants, lower and upper inventory limits at plants and in warehouses, transit times to warehouses, and the possibility of expedited shipping from production facilities directly to the customer (at higher cost) or accepting lost sales in the event of stockouts at the warehouses. A mixed-integer programming (MIP) model is employed to determine “optimal” allocations of production capacity each day and shipments to warehouses from which customer demands are satisfied. It allows us to experiment with different planning horizons, intervals for updating plans, start-up conditions, future demands, shipping times, etc. The mathematical model is provided as an appendix.

We compare simulated financial performance using standard cash-flow accounting of costs and revenues with a penalty for lost sales. The corresponding MIP objective function is composed as follows:

Standard Objective function:

> Maximize [Net Profit Contribution = (Profit contribution from warehouse deliveries + Profit contribution from alternative deliveries – Costs of lost sales – Product inventory holding costs at plants and warehouses – Raw material inventory holding costs at plants – Product inventory shortage costs at plants and warehouses – Raw material inventory shortage costs at plants – Product inventory overstocking costs at plants and warehouses – Raw material inventory overstocking costs at plants – Product shipping costs – Product in transit costs – Raw material shipping costs – Raw material in transit costs – Plant setup costs – Plant idle costs)]

With such an objective, there is no incentive in the optimizing model to ship goods to a warehouse if they do not reach the destination in time to realize revenue from sales at the warehouse (except to meet constraints imposed on ending inventories). To deal with this problem, we propose a value-added objective function. Optimizing with a value-added approach, we would recognize revenues from all deliveries to customers when they occur and also recognize the benefit derived from shipping goods to warehouses from plants which will not be delivered to customers within the current planning horizon.

As an approximation to such a value-added objective, we include potential revenues for shipments that occur from production facilities to warehouses if they occur within the minimum downstream lead time plus one day of the end of the planning horizon.

Value-Added Objective Function:

> Maximize [Net Profit Contribution = (Profit contribution from warehouse deliveries to customers + Profit contribution from alternative deliveries + Expected profit contribution from shipments to warehouses within the minimum downstream lead time plus one day of the end of the planning horizon – Costs of lost sales – Product inventory holding costs at plants and warehouses – Raw material inventory holding costs at plants – Product inventory shortage costs at plants and warehouses – Raw material inventory shortage costs at plants – Product inventory overstocking costs at plants and warehouses – Raw material inventory overstocking costs at plants – Product shipping costs – Product in transit costs – Raw material shipping costs – Raw material in transit costs – Plant setup costs – Plant idle costs)].

To assess the impact of optimizing with one model versus another, we conduct 25 replications for each planning scenario and perform statistical analysis to determine the extent to which differences in performance metrics are attributable to systematic versus random variation. Each replication constitutes 90 days of simulated activity (an entire season) under planning with a rolling optimization horizon. In this instance, we revise schedules at the end of each simulated day to offer maximum responsiveness to immediate demands. Solutions from the optimizing model for the first day are extracted and saved in a SAS dataset used by the simulation model to induce production, flows of finished goods, orders from warehouses and orders of raw material in the optimization model. The solutions contain the following information:

1) Raw material inventory level at each plant.
2) Outstanding orders of raw materials at each plant.
3) Raw materials in transit to each plant.
4) Amount of each product produced at each plant.
5) Finished product inventories at each plant.
6) Finished product inventories at each warehouse.
7) Outstanding orders of products at each warehouse.
8) Finished products in transit at each warehouse.

The first period's product demands are presumed to be known with certainty. For successive days, the simulation model generates product demands and delivery dates randomly according to specified distributions, implements the MIP solution, and updates datasets that represent the new states of the system including finished goods in transit and raw materials in transit. Randomly generated delivery dates for raw materials and finished goods are set when orders are placed and goods are shipped. They are not altered as successive iterations occur on the rolling horizon.

The MIP model reads information from the updated datasets at the end of the simulated day as its new initial conditions and re-solves the problem for the fixed number of days in the planning horizon (e.g., Day2 to Day 16 in the second iteration of a 15-day planning horizon). This iterative process continues until it reaches the end of the planning horizon (where the solution is developed for Day 90 to Day 104 and just implemented for Day 90). The optimization model and simulation models are thus used in concert to develop SC plans that attempt to maximize the net profit contribution while controlling for risk.

EXPERIMENTAL RESULTS

Considering delivery lead times, the SC under study has a theoretical planning horizon of at least 18 days. To demonstrate the effect of using the two versions of the MIP model with different planning horizons, we perform the simulation with 10-day and 20-day planning horizons and generate quarterly reports closely approximating accounting income statements. Table 1 summarizes financial performance from simulations using the MIP with standard objective function and 10-day planning horizon (STDOBJ_H10), value-added objective function with 10-day planning horizon (VAOBJ_H10), standard objective function with 20-day planning horizon (STDOBJ_H20), and value-added objective function with 20-day planning horizon (VAOBJ_H20). The value-added approach not only effectively reduced the length of the planning horizon, but also outperformed the standard approach with longer planning horizon.

Shortening the planning horizon with the standard objective from 20 days to 10 days resulted in a 12% decrease in average daily profit (from $18,299 to $16,114). With the value-added objective, the higher level of performance was achieved with either planning horizon.

Table 1 – Average Financial Performance with Different MIP Objectives and Planning Horizons

Daily Net Profit Contribution Statement for 90-day period								
	STDOBJ_H10		VAOBJ_H10		STDOBJ_H20		VAOBJ_H20	
	$	$	$	$	$	$	$	$
GROSS PROFIT CONTRIBUTION		17,154.10		19,629.55		19,269.27		19,748.32
Product sold at warehouses	17,154.10		19,629.55		19,269.27		19,748.32	
PLANT EXPENSES		1,040.18		1,045.27		969.89		987.97
Finished Product Inventory Costs	9.34		10.32		10.26		10.47	
Finished Product in Transit Costs	17.70		22.93		22.40		24.37	
Raw Material Inventory Costs	33.11		38.04		39.37		39.25	
Raw Material in Transit Costs	22.44		26.00		31.64		30.82	
Raw Material Shipping Costs	449.50		497.98		416.22		433.06	
Idle Costs	58.49		0.00		0.00		0.00	
Setup Costs	449.60		450.00		450.00		450.00	
NET PROFIT CONTRIBUTION		16,113.92		18,584.28		18,299.38		18,760.35

To test the robustness of this finding, we imposed disruptions downstream by randomly setting 20% of finished-product inventories at warehouses to zero (as perhaps could occur with damage in processing or shipment or following surges in demand due to interruptions in supply chains of competitors). All other initial conditions were the same as they were in the previous simulations. The random changes in inventory and outages were imposed at the beginning of the planning period in each replication. Outages could occur in any product-warehouse combination and the amounts of other inventories held in the system could be any value between min and max at beginning of each replication. The results in Table 2 show expected deterioration in average performance under each of the four scenarios and the benefits of the value-added approach persist.

Table 2 – Comparable Average SC Financial Performance with Disruptions

	Daily Net Profit Contribution Statement for 90-day period							
Scenario Two	STDOBJ and H10		VAOBJ and H10		STDOBJ and H20		VAOBJ and H20	
	$	$	$	$	$	$	$	$
GROSS PROFIT CONTRIBUTION		15,189.62		18,611.58		18,127.16		18,620.75
Products sold at warehouses	15,189.62		18,611.58		18,127.16		18,620.75	
PLANT EXPENSES		1,045.40		1,045.36		971.10		987.78
Finished Product Inventory Costs	9.19		10.33		10.26		10.49	
Finished Product in Transit Costs	17.03		23.15		22.41		24.53	
Raw Material Inventory Costs	33.94		38.00		39.36		39.27	
Raw Material in Transit Costs	22.46		26.02		31.60		30.79	
Raw Material Shipping Costs	444.50		497.86		417.47		432.70	
Idle Costs	68.48		0.00		0.00		0.00	
Setup Costs	449.80		450.00		450.00		450.00	
NET PROFIT CONTRIBUTION		14,144.22		17,566.22		17,156.06		17,632.97

We use average daily gross profit contribution per product to assess SC financial performance at the overall level. The left panel in Table 3 compares the averages resulting from the simulations with no disruptions; the right panel compares the averages for simulations with downstream disruptions. Note in the left panel of Table 3 that the average daily gross profit contribution per product when optimizing with VAOBJ_H20 is $3291.38. The corresponding VAOBJ_H20 entry in Table 1 is six times that amount ($19,748.32). Statistical analysis of the findings using ANOVA and pairwise comparisons with Duncan's multiple range tests (illustrated in Table 3) affirm that our findings demonstrate systematic rather than random benefits in employing the value-added objective in the MIP formulation.

Table 3 - Duncan Test Results for Average Gross Revenue per Product

Without Downstream Disruptions

Alpha	0.05
Error Degrees of Freedom	96
Error Mean Square	761.4838

Number of Means	2	3	4
Critical Range	15.49	16.30	16.84

Means with the same letter are not significantly different.			
Duncan Grouping	Mean	N	treatment
A	3291.387	25	VAOBJ_H20
B	3271.592	25	VAOBJ_H10
C	3211.545	25	STDOBJ_H20
D	2859.017	25	STDOBJ_H10

With Downstream Disruptions

Alpha	0.05
Error Degrees of Freedom	96
Error Mean Square	11225.88

Number of Means	2	3	4
Critical Range	59.49	62.60	64.67

Means with the same letter are not significantly different.			
Duncan Grouping	Mean	N	treatment
A	3103.46	25	VAOBJ_H20
A			
A	3101.53	25	VAOBJ_H10
B	3021.19	25	STDOBJ_H20
C	2531.60	25	STDOBJ_H10

CONCLUSION AND FUTURE RESEARCH

Using discrete-event simulation in concert with an MIP optimizing model, we have demonstrated the sensitivity of SC performance to the choice of planning horizon when standard cash-flow accounting of costs and revenues (with penalty for lost sales) is employed in a SC optimizing model. We produced an alternative MIP formulation that uses a value-added objective which proved to be much more robust. The advantages of the alternative formulation persist when downstream disruptions are imposed randomly.

Of course, disruptions can occur anywhere in the SC, and in the future research, the SC under investigation can be further stressed by incorporating disruptions upstream or at plants. With our analytical approach, researchers can test the effectiveness of combining routine SC risk reduction strategies with strategies for managing adverse events. The platform created in this research can facilitate the investigation of possible changes in demand patterns, interrelationships among stochastic elements, and possibilities of disruptive events.

APPENDIX

Set notation employed

Set	***Description***
R{r}	Set of raw materials
S{s}	Set of suppliers
F{f}	Set of production facilities
P{p}	Set of products
W{w}	Set of warehouses
D{d}	Set of days in planning horizon
SR{r}	Set of suppliers for raw material r
RP{p}	Set of raw materials used in producing product p
PF{f}	Set of products produced in production facility f
PR{r}	Set of products require raw material r for production
RF{f}	Set of raw materials used in producing products at production facility f
PW{w}	Set of products distributed through warehouse w
WP{p}	Set of warehouses to which product p is delivered
DRMS {r, s, f}	Set of days on which raw material r from supplier s is scheduled to arrive at production facility f
DFGS {p, f, w}	Set of days on which product p from production facility f is scheduled to arrive at warehouse w

Optimization Model Parameters

Parameter	***Description***
mrR_rP_p	Units of raw material r required to produce one unit of product p
$mininvP_pF_f$	Minimum inventory of product p desired at production facility f
$maxinvP_pF_f$	Maximum inventory of product p desired at production facility f
$mininvR_rF_f$	Minimum inventory of raw material r desired at production facility f
$maxinvR_rF_f$	Maximum inventory of raw material r desired at production facility f
$ShtPenaltyR_rF_f$	Daily penalty ($ per unit) for shortage of raw material r inventory at production facility f
$ShtPenaltyP_pF_f$	Daily penalty ($ per unit) for shortage of product p inventory at production facility f
$ShtPenaltyP_pW_w$	Daily penalty ($ per unit) for shortage of product p inventory at warehouse w
$OvrPenaltyR_rF_f$	Daily penalty ($ per unit) for excess raw material r inventory at production facility f
$OvrPenaltyP_pF_f$	Daily penalty ($ per unit) for excess of product p inventory at production facility f
$OvrPenaltyP_pW_w$	Daily penalty ($ per unit) for excess of product p inventory at warehouse w
$mininvP_pW_w$	Minimum inventory of product p desired at warehouse w (including outstanding orders)
$maxinvP_pW_w$	Maximum inventory of product p desired at warehouse w (including outstanding orders)

$dem_p whse_w$	Assigned aggregated average daily demand for product p at warehouse w
$shiptimeF_f W_w$ δ (f, w)	Shipping time (days) from production facility f to warehouse w
$shiptimeS_s F_f$ θ (s, f)	Shipping time (days) from supplier s to production facility f
$spcF_f$	Production setup costs at production facility f incurred each day that production occurs (including idle cost associated with set up time)
$pcP_p W_w$	Unit profit contribution of product p delivered from warehouse w
$scP_p F_f W_w$	Supply cost per unit of product p from production facility f to warehouse w (including variable production cost at production facility f and shipping cost from production facility f to warehouse w, but excluding raw material and goods in transit costs)
$scR_r S_s F_f$	Supply cost per unit of raw material r from supplier s to production facility f (including ordering and shipping costs, but excluding raw material in transit costs)
$scP_p W_w$	Shipping cost per unit of product p from warehouse w to customer
$icP_p F_f$	Inventory carrying cost for finished product p at production facility f
$icR_r F_f$	Inventory carrying cost of raw material r at production facility f
$itcP_p F_f W_w$	Unit cost of carrying product p in transit from production facility f to warehouse w

$itcR_rS_sF_f$	Unit cost of raw material r in transit from supplier s to production facility f
$acP_pF_fW_w$	Unit cost of alternative supply from production facility f for product p at warehouse w
icP_pW_w	Inventory carrying cost for product p at warehouse w
$opcostP_pW_w$	Unit opportunity cost of lost sales for product p at warehouse w
$DemP_pW_wD_d$	Demand for product p (units) at warehouse w on day d
$sutimeP_pF_f$	Product p production setup time at production facility f
$idlePenF_f$	Idle penalty cost per hour at production facility f
$MXprodF_f$	Maximum daily throughput (units) at production facility f
$minsysinvP_p$	Desired minimum system inventory of product p (across all production facilities)
$maxsysinvP_p$	Desired maximum system inventory of product p (across all production facilities)

The algebraic formulation of the problem is presented below:

$$
\begin{aligned}
&Max \sum_{d \in D\{d\}} \left\{ \sum_{w \in W\{w\}} \sum_{p \in PW\{w\}} \left[\left(pcP_pW_w - scP_pW_w \right) * DelP_pW_wD_d \right.\right. \\
&+ \sum_{f \in F\{f\}} \left(pcP_pW_w - acP_pF_fW_w \right) * AltP_pF_fW_wD_d - opcostP_pW_w * LSP_pW_wD_d \\
&- icP_pW_w * Inv\,P_pW_wD_d - ShtPenaltyP_pW_w * USP_pW_wD_d - OvrPenaltyP_pW_w \\
&\left. * OSP_pW_wD_d \right] \\
&- \sum_{f \in F\{f\}} \left[\sum_{p \in P\{p\}} \left(icP_pF_f * InvP_pF_fD_d + ShtPenaltyP_pF_f * USP_pF_fD_d \right.\right. \\
&+ OvrPenaltyP_pF_f * OSP_pF_fD_d \\
&\left. + \sum_{w \in W\{w\}} \left(scP_pF_fW_w * ShpP_pF_fW_wD_d + itcP_pF_fW_w * TrP_pF_fW_wD_d \right) \right) + spcF_f \\
&* SUF_fD_d + idlePenF_f * idleF_fD_d \\
&+ \sum_{r \in RP\{p\}} \left(icR_rF_f * InvR_rF_fD_d + ShtPenaltyR_rF_f * USR_rF_fD_d \right. \\
&+ OvrPenaltyR_rF_f * OSR_rF_fD_d \\
&\left.\left.\left. + \sum_{s \in SR\{r\}} \left(scR_rS_sF_f * ShpR_rS_sF_fD_d + itcR_rS_sF_f * TrR_rS_sF_fD_d \right) \right) \right] \right\}
\end{aligned}
$$

Subject to the following constraints:

1. Product p can't be produced at production facility f on day d unless the necessary set up is completed (constraint $STP_pF_fD_d$). For each production facility and day for each $p \in PF\{f\}$,

$$ProdP_pF_fD_d \;\; \leq \;\; MxprodP_pF_f \; * \; SUP_pF_fD_d.$$

2. Consumption of raw materials r at production facility f on day d cannot exceed the quantities available at beginning of day d (constraint $UBR_rF_fD_d$). For each production facility and day for each $p \in PR\{r\}$ and each $r \in RF\{f\}$,

$$\sum_{p \in PR\{r\}} mrR_rP_p * ProdP_pF_fD_d \leq InvR_rF_fD_d.$$

Notice that if units of raw material r required to produce each unit of product p are significantly different across production facilities because of labor, technology or machinery etc., then raw material conversion rates could be defined as $mrR_rP_pF_f$. For this research, we assume that there is no significant difference or bill of materials for product p produced at each production facility.

This constraint also assumes that raw materials received during the day will not be available for production until the next day.

3. Sum of production times used on day d at production facility f cannot exceed total available operating time (constraint $TPRODF_fD_d$). For each production facility and day,

$$\sum_{p \in PF\{f\}} \left(\left(\frac{1}{unitperhr} \right) * ProdP_pF_fD_d + sutimeF_f * SUF_fD_d + IdleF_fD_d \right)$$
$$= 8 * maxshifts$$

SUF_fD_d = [0,1]. If setup times are negligible, these binary constraints may be relaxed.

4. Production of product p at production facility f on day d cannot occur unless the production facility is activated for production on that day (constraint $FSUF_fD_d$). For each production facility and day for each $p \in PF\{f\}$,

$$\sum_{p \in PF\{f\}} SUP_pF_fD_d \le SUF_fD_d.$$

$SUP_pF_fD_d$ values attribute set up time to the production of individual product. If separate set up were required for each product, these equations would be replaced with sets of equations for set up of individual product. For this research, we assume that there is a single setup required if a production facility is to be activated for production during the day. $SUP_pF_fD_d$ in this formulation allocates production capacity to individual products. We, therefore, add a constraint that creates a single binary variable for each production facility during the day that accounts for setup to activate and shut down the production at production facility.

5. Raw materials inventory balance at production facility f (constraint $IBR_rF_fD_d$). For each production facility and day for each $r \in RF\{f\}$ and each $s \in SR\{r\}$,

$$InvR_rF_fD_{d+1} = InvR_rF_fD_d - \sum_{p \in PR\{r\}} mrR_rP_p \; * \; ProdP_pF_fD_d$$
$$+ \sum_{s \in SR\{r\}} (ShpR_rS_sF_fD_{d-\theta(s,f)} + ItsR_rS_sF_fD_d).$$

Note that the $ItsR_rS_sF_fD_d$ variables are defined only for (r, s, f, d) combinations where there are raw materials in transit at beginning of the planning horizon and are scheduled to arrive at production facility f on day d for each $d \in DRMS\{r, s, f\}$.

6. Place order of raw material r at beginning of day d to ensure safety stock at production facility f (constraint $MNOR_rF_fD_d$). For each production facility and day for each $r \in RF\{f\}$ and each $s \in SR\{r\}$,

$$\sum_{s \in SR\{r\}} \left(OR_rS_sF_fD_d + OOR_rS_sF_fD_d \right) + InvR_rF_fD_d$$
$$\ge mininvR_rF_f - USR_rF_fD_{d-1}.$$

Note that under storage of raw materials could occur at the beginning of Day 1.

7. Restrict order of raw material r at beginning of day d to prevent overstock at production facility f (constraint $MXOR_rF_fD_d$). For each production facility and day for each $r \in RF\{f\}$ and each $s \in SR\{r\}$,

$$\sum_{s \in SR\{r\}} \left(OR_rS_sF_fD_d + OOR_rS_sF_fD_d\right) + InvR_rF_fD_d \le maxinvR_rF_f + OSR_rF_fD_{d-1}.$$

Note that over storage of raw materials could occur at the beginning of Day 1.

8 & 9. Update under storage (constraint $AUSR_rF_fD_d$) and overstocking (constraint $AOSR_rF_fD_d$) of raw material r at production facility f at the end of day d. For each production facility and day for each $r \in RF\{f\}$ and each $s \in SR\{r\}$,

$$InvR_rF_fD_d + USR_rF_fD_d - \sum_{p \in PR\{r\}} mrR_rP_p * ProdP_pF_fD_d + \sum_{s \in SR\{r\}} (ShpR_rS_sF_fD_{d-\theta(s,f)} + ItsR_rS_sF_fD_d) \ge minInvR_rF_f.$$

$$InvR_rF_fD_d - OSR_rF_fD_d - \sum_{p \in PR\{r\}} mrR_rP_p * ProdP_pF_fD_d + \sum_{s \in SR\{r\}} (ShpR_rS_sF_fD_{d-\theta(s,f)} + ItsR_rS_sF_fD_d) \le maxInvR_rF_f.$$

Note that the $ItsR_rS_sF_fD_d$ variables are defined only for (r, s, f, d) combinations where there are raw materials in transit at beginning of the planning horizon and are scheduled to arrive at production facility f on day d for each $d \in DRMS\{r,s,f\}$.

10. Total units of raw material r shipped from supplier s at the end of day d to satisfy orders acknowledged from production facility f at beginning of that day (constraint $DLVR_rS_sF_fD_d$). For each day and each $s \in \mathrm{SR}\,\{r\}$ for each production facility,

$$ShpR_rS_sF_fD_d \ge OR_rS_sF_fD_d.$$

11. Update outstanding orders of raw material r at production facility f at beginning of day d (constraint $OOUR_rS_sF_fD_d$). For each production facility and day for each $r \in \mathrm{RF}\,\{f\}$ and each $d \in DRMS\{r,s,f\}$,

$$OOR_rS_sF_fD_{d+1} = OOR_rS_sF_fD_d + OR_rS_sF_fD_d - ShpR_rS_sF_fD_{d-\theta(s,f)} - ItsR_rS_sF_fD_d.$$

Note that the $ItsR_rS_sF_fD_d$ variables are defined only for (r, s, f, d) combinations where there are raw materials in transit at beginning of the planning horizon and are scheduled to arrive at production facility f on day d for each $d \in DRMS\{r,s,f\}$. $OOR_rS_sF_fD_1$ should include sum of the $ItsR_rS_sF_FD_d$ values for each day with scheduled arrivals.

12. Update raw materials in transit from supplier s to production facility f (constraint $RITR_rS_sF_fD_d$) at beginning of day d. For each production facility and day for each $r \in \mathrm{RF}\,\{f\}$ and each $d \in DRMS\{r,s,f\}$,

$$TrR_rS_sF_fD_{d+1} = TrR_rS_sF_fD_d + ShpR_rS_sF_fD_d - ShpR_rS_sF_fD_{d-\theta(s,f)} - ItsR_rS_sF_fD_d.$$

Note that the $ItsR_rS_sF_fD_d$ variables are defined only for (r, s, f, d) combinations where there are raw materials in transit at beginning of the planning horizon and are scheduled to arrive at production facility f on day d for each $d \in DRMS\{r,s,f\}$. TrRrSsFFD$_1$ is set to sum of the $ItsR_rS_sF_fD_d$ values for each day with scheduled arrivals of raw materials.

13. Place order for product p at the beginning of day d to ensure desired safety stock at warehouse w (constraint $MNOP_pW_wD_d$). For each warehouse and day for each $p \in PW\{w\}$,

$$\sum_{f\in F\{f\}} (OP_pF_fW_wD_d + OO\,P_pF_fW_wD_d) + InvP_pW_wD_d \geq mininvP_pW_w - USP_pW_wD_{d-1}.$$

Note that under storage of products can occur with associated penalty.

14. Restrict order of product p at the beginning of day d to prevent overstock at warehouse w (constraint $MXOP_pW_wD_d$). For each warehouse and day for each $p \in PW\{w\}$,

$$\sum_{f\in F\{f\}} (OP_pF_fW_wD_d + OO\,P_pF_fW_wD_d) + InvP_pW_wD_d \leq maxinvP_pW_w + OSP_pW_wD_{d-1}.$$

Note that over storage of products can occur with associated penalty.

15. Produce sufficient product p across plants to cover orders and provide production system-wide safety stocks (constraint $MNSYSP_pD_d$). For each day for each product across all plants,

$$\sum_{f\in F\{f\}} (Prod\,P_pF_fD_d + InvP_pF_fD_d) \geq \sum_{f\in F\{f\}} \sum_{w\in PW\{w\}} OP_pF_fW_wD_d + minsysinvP_p.$$

16. Restrict production of product p across plants on day d to prevent overstock in the production system (constraint $MXSYSP_pD_d$). For each day for each product across all plants,

$$\sum_{f\in F\{f\}} (Prod\,P_pF_fD_d + InvP_pF_fD_d) \leq \sum_{f\in F\{f\}} \sum_{w\in PW\{w\}} OP_pF_fW_wD_d + maxsysinvP_p.$$

17. Ship sufficient finished goods from production facility f to cover orders placed at warehouse w on day d (constraint $DLVP_pF_fW_wD_d$). For each production facility and day for each $\mathrm{p} \in \mathrm{PF}\{\mathrm{f}\}$ and each $p \in PW\{w\}$,

$$ShpP_pF_fW_wD_d \;\geq\; OP_pF_fW_wD_d.$$

18 & 19. Update over storage (constraint $AOSP_pF_fD_d$) and under storage (constraint $AUSP_pF_fD_d$) of product p at production facility f at the end of day d. For each production facility and day for each $p \in PF\{f\}$ and each $p \in PW\{w\}$,

$$InvP_pF_fD_d - OSP_pF_fD_d + ProdP_pF_fD_d - \sum_{w \in WP\{p\}} \left(ShpP_pF_fW_wD_d + AltP_pF_fW_wD_d\right) \le maxInvP_pF_f.$$

$$InvP_pF_fD_d + USP_pF_fD_d + ProdP_pF_fD_d - \sum_{w \in WP\{p\}} \left(ShpP_pF_fW_wD_d + AltP_pF_fW_wD_d\right) \ge minInvP_pF_f.$$

20. Limit shipments of product p from production facility f to warehouses on day d to the amount available in production facility inventory (constraint $SLP_pF_FD_d$). For each production facility and day for each $p \in PF\{f\}$ and each $p \in PW\{w\}$,

$$\sum_{w \in WP\{p\}} \left(ShpP_pF_fW_wD_d + AltP_pF_fW_wD_d\right) \le InvP_pF_fD_d.$$

This also implies that production of product p during day d will not be available for delivery until the next day.

21. Account for inventory balance of products at production facility f at the end of day d (constraint $IBP_pF_fD_d$). For each production facility and day for each $p \in PF\{f\}$ and each $p \in PW\{w\}$,

$$InvP_pF_fD_{d+1} = InvP_pF_fD_d + ProdP_pF_fD_d - \sum_{w \in WP\{p\}} \left(ShpP_pF_fW_wD_d + AltP_pF_fW_wD_d\right).$$

22. Deliver goods from warehouse or alternative source (production facility) to satisfy customer demand and acknowledge lost sales if inventory is insufficient (constraint $DLVP_pW_wD_d$). For each warehouse and day for each $p \in PW\{w\}$,

$$DelP_pW_wD_d + LSP_pW_wD_d + \sum_{f \in F\{f\}} AltP_pF_fW_wD_d = DemP_pW_wD_d.$$

23. Account for inventory balance of product p at warehouse w recognizing inbound shipping delays (constraint $IBP_pW_wD_d$) at the end of day d. For each warehouse and day for each $p \in PW\{w\}$,

$$InvP_pW_wD_{d+1} = InvP_pW_wD_d - DelP_pW_wD_d + \sum_{f \in F\{f\}} (ShpP_pF_fW_wD_{d-\delta(f,w)} + ItsP_pF_fW_wD_d).$$

Note that the $ItsP_pF_FW_WD_d$ variables are defined only for (p, f, w, d) combinations where there are finished goods in transit at the beginning of the planning horizon and are scheduled to arrive at warehouse w on day d for each $d \in DFGS\{p, f, w\}$.

24 & 25. Update over storage (constraint $AOSP_pW_wD_d$) or under storage (constraint $AUSP_pW_wD_d$) of product p at warehouse w at the end of day d. For each warehouse and day for each $p \in PW\{w\}$,

$$InvP_pW_wD_d + USP_pW_wD_d - DelP_pW_wD_d + \sum_{f \in F\{f\}} (ShpP_pF_fW_wD_{d-\delta(f,w)} + ItsP_pF_fW_wD_d) \ge mininvP_pW_w.$$

$$InvP_pW_wD_d - OSP_pW_wD_d - DelP_pW_wD_d + \sum_{f \in F\{f\}} (ShpP_pF_fW_wD_{d-\delta(f,w)} + ItsP_pF_fW_wD_d) \leq maxinvP_pW_w.$$

Note that the $ItsP_pF_FW_WD_d$ variables are defined only for (p, f, w, d) combinations where there are finished goods in transit at the beginning of the planning horizon and are scheduled to arrive at warehouse w on day d for each $d \in DFGS\{p, f, w\}$.

26. Update outstanding orders for product p at warehouse w at the end of day d (constraint $OOUP_pF_FW_wD_d$). For each warehouse and day for each $p \in PW\{w\}$ and each $d \in DFGS\{p, f, w\}$,

$$OOP_pF_fW_wD_{d+1} = OOP_pF_fW_wD_d + OP_pF_fW_wD_d - ShpP_pF_fW_wD_{d-\delta(f,w)} - ItsP_pF_fW_wD_d.$$

Note that the $ItsP_pF_FW_WD_d$ variables are defined only for (p, f, w, d) combinations where there are finished goods in transit at the beginning of the planning horizon and are scheduled to arrive at warehouse w on day d for each $d \in DFGS\{p, f, w\}$. $OOP_pF_FW_wD_1$ should include sum of the $ItsP_pF_FW_WD_d$ values for each day with scheduled arrivals.

27. Update finished goods in transit to reflect shipments and receipts (constraint $GITP_pF_fW_wD_d$) at the end of day d. For each warehouse and day for each $p \in PW\{w\}$ and each $d \in DFGS\{p, f, w\}$,

$$TrP_pF_fW_wD_{d+1} = TrP_pF_fW_wD_d + ShP_pF_fW_wD_d - ShpP_pF_fW_wD_{d-\delta(f,w)} - ItsP_pF_fW_wD_d.$$

Note that the $ItsP_pF_FW_WD_d$ variables are defined only for (p, f, w, d) combinations where there are finished goods in transit at the beginning of the planning horizon and are scheduled to arrive at warehouse w on day d for each $d \in DFGS\{p, f, w\}$. $TrP_pF_fW_wD_1$ is set to sum of the $ItsP_pF_fW_WD_d$ values for each day with scheduled arrivals.

28. As formulated with the warehouse inventory balance constraint (23), products that arrive in a day may be cross-docked and shipped out immediately if there is demand for them on that day rather than putting them into inventory. Such shipments could be delayed until the next day by adding a constraint (constraint $CDP_pW_WD_d$) that delivery of product p at warehouse w in a day can't exceed the beginning inventory of that product in that day. For each warehouse and day for each $p \in PW\{w\}$,

$$DelP_pW_wD_d \leq InvP_pW_wD_d.$$

All variables are nonnegative.

29 & 30. To facilitate extraction of the solution in the report generator, we define variable $ARRP_pF_fW_WD_d$ to be the finished goods shipped from all production facilities that arrive at the warehouse in day d which will be shipped in this planning horizon and establish their equality in constraints that define inbound freight (constraint $IBP_pF_fW_WD_d$). We also define variable $ARRR_rS_sF_fD_d$ to be the amount of raw material r shipped from supplier s to arrive at production facility f on day d. They are set equal to the corresponding outbound shipments as follows (constraint $IBR_rS_sF_fD_d$),

$$ARRP_pF_fW_wD_d = ShpP_pF_fW_wD_{d-\delta(f,w)}.$$

$$ARRR_rS_sF_fD_d = ShpR_rS_sF_fD_{d-\theta(s,f)}.$$

Note that variables of $ItsP_pF_fW_WD_d$ and ItsRrSsFfDd represent goods in transit to arrive as a result of initial conditions, while that of ShpPpFfW$_W$Dd and $ShpR_rS_sF_fD_d$ indicate when goods arrive from shipments in the current planning frame.

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