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Effects of Transaction Costs and Bidders' Valuations on Discrete Dutch Auction Design

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ABSTRACT

This research focuses on the optimal design of discrete Dutch auction to balance the tradeoff between achieving the higher selling price and committing more resources. The objective is to maximize the seller's expected net revenue by determining the number of bid levels and the asking price at each level. For various bidders' valuation distributions, the results show that the optimal number of bid levels is a concave function of the number of bidders and a non-increasing function of transaction cost per bid level. In addition, complete knowledge about the bidding population size results in a higher seller's expected net revenue.

KEYWORDS: Dutch auction; Discrete bid level; Net revenue maximization; Transaction cost; Bidders' valuation

INTRODUCTION

The auctioneer in a Dutch auction starts with an extremely high asking price and continues to lower it according to a predetermined schedule until a bidder indicates the willingness to pay by pushing a button at the desk in the auction room or calling out "mine". One of the common results in the current discrete Dutch auction literature is that the expected selling price generally increases with the number of bid levels set. However, the use of more bid levels results in longer duration and higher transaction costs could occur to seller in the longer auctioning process, such as, office administration cost, personnel expenses, storage and maintenance cost.

As a result, the net revenue generated from the entire auction could be lower. Obviously, there is a tradeoff between achieving the higher selling price and committing more resources.

Another purpose of this work is to investigate the effect of uncertainty about the number of bidders. While the number of bidders in an auction was often assumed to be given in publications prior to the 1980s, it has been treated as a random variable in more recent research (McAfee and McMillan, 1987; Levin and Ozdenoren, 2004; Isaac et al., 2012).

AUCTION MODEL

We consider discrete Dutch auctions whose objective is to maximize the seller's expected net revenue by determining the number of bid levels to be used and the asking price at each level. Under the IPV setting with symmetric information, given $n \geq 0$ bidders, it is assumed that the private valuation of bidder *j*, v_j, is an independent and identically distributed continuous random variable with probability density function (PDF) *f* and cumulative distribution function (CDF) *F*, *j* = 1, 2, …, n.

Suppose that $m \geq 1$ bid levels $l_1, l_2, \ldots, l_m$ are to be set with $l_1 \leq l_2 \leq \ldots \leq l_m$. The auctioneer begins with an arbitrarily high asking price l_{m+1} and lowers it to each of $l_m, l_{m-1}, \ldots, l_2, l_1$ sequentially. Let *c* be the transaction costs at each bid level. The selling price is l_i if and only if $q \geq 1$ bidders' valuations are in the range $[l_i, l_{i+1})$, no one is willing to pay the higher price l_{i+1} announced previously, and the remaining $n-q$ bidders' valuations are below l_i, $i \in \{1, 2, \ldots, \mathrm{m}\}$. In case two or more bidders' valuations fall in the aforementioned range (i.e., *q* ≥ 2), the first bidder to call out "mine" or push the button is the winner.

Let $P(l_i)$ denote the probability that the object is sold at the bid level l_i, *i* = 1, 2, …, m. We have

$$\begin{aligned} P(l_i) &= \sum_{q=1}^{n} \binom{n}{q} F(l_i)^{n-q} \left[F(l_{i+1}) - F(l_i)\right]^q \\ &= F(l_{i+1})^n - F(l_i)^n \end{aligned} \tag{1}$$

Consistent with David et al. (2007), Gallien and Gupta (2007), and Li and Kuo (2013), the bidder arrivals are assumed to be Poisson distributed with mean $\lambda \geq 0$. In light of (1), the probability of selling the object at the bid level l_i, *i* = 1, 2, …, m, is

$$\begin{aligned} P(l_i) &= \sum_{n=1}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} \left[F(l_{i+1})^n - F(l_i)^n\right] \\ &= e^{-\lambda}\left[e^{\lambda F(l_{i+1})} - e^{\lambda F(l_i)}\right] \end{aligned} \tag{2}$$

Observe that the object won't be auctioned off if no bidders are present (i.e., *n* = 0) and the probability of such an occurrence is

$$\begin{aligned} P(n=0) &= \frac{\lambda^0 e^{-\lambda}}{0!} \\ &= e^{-\lambda} \end{aligned} \tag{3}$$

Based on (1), (2), and (3), the seller's expected net revenue in this model, Z_{I} , can be written as

$$Z_{\mathrm{I}} = \sum_{i=1}^{m}\left[l_i - (m+1-i)c\right]P(l_i) - mc\left[1 - P(n=0) - \sum_{i=1}^{m}P(l_i)\right]$$
$$= e^{-\lambda}\sum_{i=1}^{m} l_i\left[e^{\lambda F(l_{i+1})} - e^{\lambda F(l_i)}\right] - ce^{-\lambda}\sum_{i=2}^{m+1} e^{\lambda F(l_i)} + mce^{-\lambda}$$

An NLP for Model I is presented below, where $l_1, l_2, \ldots, \text{and}\, l_m$ are the decision variables while *m*, λ and *c* are the parameters:

$$\begin{aligned} &\text{Maximize} \quad Z_{\mathrm{II}} = e^{-\lambda}\sum_{i=1}^{m} l_i\left[e^{\lambda F(l_{i+1})} - e^{\lambda F(l_i)}\right] - ce^{-\lambda}\sum_{i=2}^{m+1} e^{\lambda F(l_i)} + mce^{-\lambda} \\ &\text{subject to:} \quad l_{i+1} \geq l_i,\ i = 1, 2, \ldots, \mathrm{m} \\ &\qquad l_1 \geq 0 \end{aligned} \tag{4}$$

Let the bidders' valuation be drawn from the standard uniform distribution $U(0,1)$. It follows that $F(l_i) = l_i$, *i* = 1, 2, …, m, and (4) reduces to

$$\begin{aligned} &\text{Maximize} \quad Z_{\mathrm{I}} = e^{-\lambda}\sum_{i=1}^{m} l_i\left(e^{\lambda l_{i+1}} - e^{\lambda l_i}\right) - ce^{-\lambda}\sum_{i=2}^{m+1} e^{\lambda l_i} + m\mathit{œ}^{-\lambda} \\ &\text{subject to:} \quad l_{i+1} \geq l_i,\ i = 1, 2, \ldots, \mathrm{m} \\ &\qquad l_1 \geq 0 \\ &\qquad l_m \leq 1 \end{aligned} \tag{5}$$

It can be proven that the objective function in (5) is concave. This along with the fact that the constraint set is convex implies the existence of an optimal solution to the above mathematical program.

Proposition 1 $Z_{\mathrm{I}} = e^{-\lambda}\sum_{i=1}^{m} l_i\left(e^{\lambda l_{i+1}} - e^{\lambda l_i}\right) - ce^{-\lambda}\sum_{i=2}^{m+1} e^{\lambda l_i} + m\mathit{œ}^{-\lambda}$ is concave in $(l_1, l_2, \ldots, l_m)$.

MODEL VARIATION

In this section, we consider a variation of Models I by replacing the standard uniform distribution of bidders' valuations with the exponential distribution. As in David et al. (2007) and Rothkopf and Harstad (1994), the bidders' valuations are assumed to follow an exponential distribution with PDF $f(l_i) = \theta e^{-\theta l_i}$ and CDF $F(l_i) = 1 - e^{-\theta l_i}$, *i* = 1, 2, …, m. In light of (5), Model II as a variation of Model I may be represented by the NLP below, where $l_1, l_2, \ldots, \text{and}\, l_m$ are the decision variables whereas *m*, λ, *c* and θ are the parameters:

$$\text{Maximize} \quad Z_{\mathrm{II}} = e^{-\lambda}\sum_{i=1}^{m} l_i\left[e^{\lambda\left(1-e^{-\theta l_{i+1}}\right)} - e^{\lambda\left(1-e^{-\theta l_i}\right)}\right] - ce^{-\lambda}\sum_{i=2}^{m+1} e^{\lambda\left(1-e^{-\theta l_i}\right)} + mce^{-\lambda}$$
$$\text{subject to:} \quad l_{i+1} \ge l_i,\ i = 1, 2, ..., \mathrm{m}$$
$$l_1 \ge 0 \qquad (6)$$

NUMERICAL EXAMPLES AMD ANALYSIS

In this section, we solve a set of problem instances to investigate the impact of the model parameters on the auction outcome. Based on (5), problem instances involving various combinations of $m \in \{1, 2, \ldots, 30\}$, $\lambda \in \{2, 5, 10, 20, \ldots, 100\}$ and $c \in \{0.01, 0.05, 0.1, 0.15, 0.2\}$ are developed and solved by running LINGO to gain a better understanding of how some model parameters might affect the auction outcome.

A summary of the optimal numbers of bid levels (m^*) and the corresponding seller's maximum expected net revenues ($Z^*_{\mathrm{I},\ m^*}$) for $c \in \{0.01, 0.05, 0.1, 0.15, 0.2\}$ can be found in Table 1. Specifically, $Z^*_{\mathrm{I},\ m^*}$ is an increasing function of λ and a decreasing function of *c*. Besides, it is also seen that m^* is a concave function of λ and a non-increasing function of *c*.

λ	c = 0.01		c = 0.05		c = 0.1		c = 0.15		c = 0.2	
	m^*	Z^*_{I,m^*}	m^*	Z^*_{I,m^*}	m^*	Z^*_{I,m^*}	m^*	Z^*_{I,m^*}	m^*	Z^*_{I,m^*}
2	15	0.49175	5	0.38363	3	0.29445	2	0.22347	2	0.16100
5	19	0.73295	7	0.62901	5	0.53663	3	0.45720	3	0.38536
10	24	0.84840	11	0.76427	7	0.68545	5	0.61537	4	0.54965
20	24	0.91139	12	0.84269	10	0.77474	8	0.71255	7	0.65316
30	20	0.93379	11	0.87175	9	0.80837	8	0.74943	8	0.69262
40	17	0.94553	11	0.88737	8	0.82658	7	0.76946	7	0.71408
50	17	0.95285	9	0.89727	7	0.83819	7	0.78225	6	0.72778
60	16	0.95788	8	0.90418	7	0.84631	7	0.79120	6	0.73738
70	15	0.96158	8	0.90931	7	0.85235	6	0.79786	5	0.74452
80	14	0.96443	8	0.91329	7	0.85704	6	0.80303	5	0.75006
90	14	0.96669	8	0.91648	7	0.86080	5	0.80718	5	0.75450
100	13	0.96854	7	0.91909	6	0.86389	5	0.81059	5	0.75814

Table 1 Seller's maximum expected net revenue in Model I vs. expected number of bidders and transaction cost per bid level

To perform a comparable numerical analyses, we choose $\theta = 2$ here so that the exponential distribution *Exp*(2) has a mean of $\frac{1}{\theta} = \frac{1}{2} = 0.5$. The optimal numbers of bid levels and the corresponding seller's maximum expected net revenues for Model II is shown in Table 2. It is interesting to note that our findings about Models II resemble those about Model I reported previously.

λ	c = 0.01		c = 0.05		c = 0.1		c = 0.15		c = 0.2	
	m^*	Z^*_{II,m^*}	m^*	Z^*_{II,m^*}	m^*	Z^*_{II,m^*}	m^*	Z^*_{II,m^*}	m^*	Z^*_{II,m^*}
2	16	0.51917	6	0.35580	3	0.24168	2	0.16140	1	0.10024
5	22	0.92648	8	0.72810	5	0.58100	4	0.46790	3	0.37371
10	30	1.27065	13	1.06961	8	0.91913	6	0.80194	5	0.70126
20	34	1.61720	16	1.41614	12	1.26561	9	1.14829	8	1.04736
30	34	1.81993	17	1.61887	12	1.46834	10	1.35102	9	1.25009
40	34	1.96377	17	1.76271	12	1.61218	11	1.49486	9	1.39393
50	34	2.07535	18	1.87428	12	1.72375	11	1.60643	10	1.50550
60	34	2.16651	18	1.96544	13	1.81491	11	1.69759	10	1.59666
70	34	2.24358	18	2.04252	13	1.89199	11	1.77467	10	1.67374
80	34	2.31035	18	2.10928	13	1.95875	10	1.84143	9	1.74050
90	33	2.36924	17	2.16817	12	2.01764	10	1.90032	9	1.79939
100	33	2.42192	17	2.22085	12	2.07032	10	1.95300	9	1.85207

Table 2 Seller's maximum expected net revenue in Model II vs. expected number of bidders and transaction cost per bid level

CONCLUSIONS

This study is motivated by our observation in the discrete Dutch auction that the use of more bid levels can achieve higher expected selling price, however, it also increases the auction duration and results in committing more resources, such as office administration cost, personnel expenses, utility, renting the auction room and holding the products for a longer period of time. To balance this tradeoff, this research focuses on the optimal design of discrete Dutch auction with consideration of the transaction costs incurred by sellers. The main objective is to maximize the seller's expected net revenue by determining the optimal number of bid levels and the asking price at each level.

A few directions for our future research are outlined below. Firstly, we assume that the transaction costs per bid level is a fixed value, as part of our future work, it should be interesting to investigate the situation where the transaction costs per bid level is different and could be considered as a random variable following a certain distribution. Secondly, we assume that the bidders will bid based on their valuations. Bidders may either acquire information to refine their valuations during the dynamic auction process, or take the opportunity cost to wait for a lower price. Thus, bidders may not bid truthfully based on their valuations. We are curious about its impact on the optimal Dutch auction design.

(Proof of propositions is available upon requests from the authors.)

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