A SYMMETRIC MODEL OF ADVERTISING-INVENTORY COMPETITION: SOME THEORETICAL AND EMPIRICAL RESULTS

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ABSTRACT

Following a game theoretic approach that uses a market share attraction model of advertising competition, this article examines analytically the sensitivity of the optimal order quantity and optimal advertising expenditure to changes in the parameters of a competitive inventory-advertising model in a symmetric setting. In the reported empirical application it is demonstrated that if model parameters change, asymmetric rivals in the studied oligopolistic industry for which every market share is less than 50% should adjust their order quantity and advertising spending in a manner consistent with the symmetric duopoly structure for which all market shares are equal.

*Keywords:* Advertising; Economic ordered quantity; Game theory; Market-share attraction models; Sensitivity analysis.

INTRODUCTION

At the individual level of the firm, the business enterprise usually commits significant amount of resources on acquiring inventory and spending on advertising. Because change is the only constant in today’s market environment, it is advantageous for a vigilant competitor to ascertain how to adapt its procurement and advertising strategies in response to changes in the market to maintain or enhance its position (Mesak, 2003).

This paper focuses on the development of a symmetric competitive inventory model with advertising-dependent demand. The present study has two main objectives. The first objective is to investigate analytically how a firm would adapt optimum order quantity and optimum advertising expenditure in response to changes in any of its own parameters, rival parameters, or the parameters that are common to all firms in a symmetric competitive market. The second objective attempts to answer a research question that has not been fully addressed before. In a truly oligopolistic market for which every market share is less than 50%, should rivals adjust their order quantities and advertising expenditures in response to changes in market conditions in a manner consistent or inconsistent with a symmetric duopoly for which all rivals share the market equally?

Whitin (1955) was perhaps the first to jointly consider purchasing and marketing decisions by incorporating the effect of price on demand within the inventory model where the retailer has to decide both the price and order quantity optimally. The price-dependent demand model was followed by another stream of research to integrate the inventory model with advertising-
dependent demand (Freeland, 1980; Subramanyam and Kumaraswamy, 1981; Urban, 1992; Khouja and Robbins, 2003; Sana and Chaudhuri, 2008). While these research works considered only a single firm, there have been a few studies within a static framework on advertising competition in duopoly/oligopoly (Friedman, 1958; Mills, 1961; Gupta and Krishnan, 1967; Mesak and Calloway, 1995, 1999; Mesak, 2003). The above studies, however, do not consider inventory related costs in the modeling effort.

To the best knowledge of the authors, this article is the first work in the literature that attempts to derive and assess the robustness of sensitivity analysis (comparative statics) related to a symmetric competitive inventory model with advertising-dependent demand. It is demonstrated empirically in this article using panel data that if model parameters change, all rivals in the industry should adjust their order quantities and advertising budgets in a manner consistent with the symmetric competitive structure for which all market shares are equal. Gupta et al. (2006, p. 443) emphasize that the combined use of analytical, and empirical research techniques would offer greater insights into the issues being investigated.

The rest of the paper is organized as follows. Second section outlines the theoretical model. Third section derives the sensitivity analysis for a symmetric duopoly. Fourth section estimates a market share attraction model using the carbonated soft drink industry data in the US. Fifth section assesses the robustness of comparative statics related to symmetric competition. Sixth section summarizes and concludes the paper. Derivation of key formulas and mathematical proofs of theoretical findings are available upon request from the authors.

**MATHEMATICAL MODEL**

This research considers simple one-step competing supply chains in which each supplier (vendor) strives to meet an advertising-dependent demand of potential customers. In doing so, the inventory replenishment process of each supplier of a differentiated product is represented by an economic order quantity (EOQ) model for which its classical assumptions are satisfied (mainly, the demand rate is deterministic and constant, shortages are not allowed, the order quantity is the same for each order). It is further assumed that any intermediate levels in each chain coordinate their activities fully with their related supplier.

The model described in this section and the one to follow considers first, as in Little (1979), Monahan (1987) and Mesak (2003), a market share attraction model in a duopoly market, with one advertising spending variable \( x_j \) measured in real dollars per unit time for each firm \( j \), \( j = 1, 2 \). Sales of firm \( l \), \( D_l(x_1, x_2) \), in a market of two competitors is expressed as follows:

\[
D_1(x_1, x_2) = \frac{mf(x_1)}{f(x_1) + f(x_2)} = \frac{m \beta_1 x_1^{\delta_1}}{\beta_1 x_1^{\delta_1} + \beta_2 x_2^{\delta_2}}, \quad \text{and} \quad D_2(x_1, x_2) = m - D_1(x_1, x_2),
\]

where \( m \) is the market potential (considered constant for a mature market) and \( f(x_1) \) and \( f(x_2) \) are the attraction functions of firms 1 and 2, respectively. The theoretical justification of the advertising-driven market share function associated with \( D_l(x_1, x_2) \) given by (1) is discussed in Bell et al. (1975). While some other elaborate forms of \( f(x_j) \) can be used, their implications appear to be empirically consistent (see e.g., Mesak and Means, 1998). All rivals are assumed to have concave advertising attraction functions, such that \( f'(x_j) > 0 \) and \( f''(x_j) < 0 \) for \( x_j > 0 \). For
all the attraction functions, the \( \beta \)'s and \( \delta \)'s are strictly positive parameters, and for a concave
attraction function \( 0 < \delta < 1 \). Designating the unit price, the marginal cost, the advertising cost
function, the ordered quantity, the ordering cost per order, the inventory holding cost per unit
held per unit time, and the fixed costs by \( P_j, MC_j, C(x_j), Q_j, C_{o1}, C_{o2}, \) and \( F_j \) respectively, and
introducing \( \gamma_j \) as the profit margin per unit sold \( (P_j - MC_j) \), the profit function, \( \pi_j \) of firm \( j \) per unit
time takes the form
\[
\pi_j = \gamma_j D_j(x_1, x_2) - C(x_j) - \frac{Q_j}{2} C_{o1} - \frac{D_j(x_1, x_2)}{Q_j} C_{o2} - F_j, \quad j = 1, 2. \tag{2}
\]

It is also assumed that the advertising cost function is convex and of the form \( C(x_j) = d_j x_j^2 \), where \( d_j \) and \( \varepsilon_j \) are positive constants such that \( d_j > 0 \), and \( \varepsilon_j \geq 1 \) (Piconni and Olson, 1978) with underlying properties, \( C'(x_j) > 0 \) and \( C'(x_j) \geq 0 \), for \( x_j > 0 \). Defining \( \Pi_j = \pi_j / \gamma_j m \), for
convenience of analysis, and substituting for \( D_j(x_1, x_2) \), \( j = 1, 2 \), expression (2) takes the following form:
\[
\Pi_1 = \frac{f(x_1)}{f(x_1) + f(x_2)} \left[ 1 - \frac{C_{o1}}{\gamma_1 Q_1} \right] - \frac{C(x_1)}{\gamma_1 m} - \frac{C_{o1}}{2 \gamma_1 m} Q_1 - \frac{F_1}{\gamma_1 m}. \tag{3}
\]
\[
\Pi_2 = \frac{f(x_2)}{f(x_1) + f(x_2)} \left[ 1 - \frac{C_{o2}}{\gamma_2 Q_2} \right] - \frac{C(x_2)}{\gamma_2 m} - \frac{C_{o2}}{2 \gamma_2 m} Q_2 - \frac{F_2}{\gamma_2 m}. \tag{4}
\]
The analysis of this situation is based on the Nash equilibrium solution concept of game theory.
Researchers have widely used the Nash equilibrium solution concept in an oligopoly (e.g., Mills,
1961; Mesak, 2003). The Nash equilibrium solution requires that no single rival unilaterally
changes its advertising budget and order quantity given the other rivals’ optimal advertising
budgets and order quantities.

The first-order optimality conditions for an interior solution \( (x_j^* > 0 \) and \( Q_j^* > 0 \) \) implies that
\[
\frac{\partial \Pi_j}{\partial x_j} = 0 \quad \text{and} \quad \frac{\partial \Pi_j}{\partial Q_j} = 0, \quad j = 1, 2. \tag{5}
\]
Also to ensure that expressions (5) indicate profit maximization, both the second-order
derivatives \( \frac{\partial^2 \Pi_j}{\partial x_j^2} \) and \( \frac{\partial^2 \Pi_j}{\partial Q_j^2} \) must be negative and \( \frac{\partial^2 \Pi_j}{\partial x_j^2} \) \( \frac{\partial^2 \Pi_j}{\partial Q_j^2} \) \( \frac{\partial^2 \Pi_j}{\partial x_j \partial Q_j} \)
must be positive for all \( j \). Sensitivity analysis is discussed next.

**SENSITIVITY ANALYSIS**

Monahan (1987) and Mesak (2003) assert that parameter changes occur mostly sequentially in an
oligopoly. This section provides a theoretical derivation of the impact of changes in each of the
shift parameters namely \( \gamma_j, \beta_j, \delta_j, m, d_j, \varepsilon_j, C_{o1}, C_{o2} \) for a given rival \( j \) on its optimal advertising
expenditures \( x_j^* \) and optimal order quantity \( Q_j^* \) (self-sensitivity analysis) as well as on similar
quantities related to the competitors (cross - sensitivity analysis). In a symmetric duopoly, firms
are as similar as possible in all economic respects that compete against each other for the same
potential buyers in an industry producing a storable, homogeneous product. Symmetric
competition also stipulates that rivals will have the same production and inventory costs, acquire real promotion on the same terms, charge the same fixed price and face symmetric demand functions (Schmalensee, 1976). Therefore, all comparable shift parameters are assumed to be equal and firms are assumed to have equal market shares.

Although equations (5) for a symmetric competitive structure could be solved explicitly for \( x_j^* \) and \( Q_j^* \) in terms of model parameters, several of the self-comparative statics obtained afterwards would have been missing and all of the cross-comparative statics would have not been disclosed. Therefore, the implicit function theory IFT (see Bertsekas, 1999) is used instead to arrive at expressions for self and cross comparative statics. Because of the curse of dimensionality when the number of decision variables per rival is more than one and when the number of competitors \( N \geq 3 \), the theoretical analysis is confined to a duopoly. The generality of the obtained results to an asymmetric oligopoly is assessed empirically in Section 5. To study the sensitivity of each \( x_j^* \) and \( Q_j^* \), \( j = 1, 2 \); to a change in one of the model parameters \( \theta \), equations (5) are partially differentiated with respect to \( \theta \) to arrive, after minor manipulation, at the following equations expressed in a matrix format:

\[
\begin{bmatrix}
\frac{\partial^2 \Pi_1}{\partial x_1^2} & \frac{\partial^2 \Pi_1}{\partial x_1 \partial Q_1} & \frac{\partial^2 \Pi_1}{\partial x_1 \partial x_2} & \frac{\partial^2 \Pi_1}{\partial x_1 \partial Q_2} \\
\frac{\partial^2 \Pi_1}{\partial Q_1 \partial x_1} & \frac{\partial^2 \Pi_1}{\partial Q_1^2} & \frac{\partial^2 \Pi_1}{\partial Q_1 \partial x_2} & \frac{\partial^2 \Pi_1}{\partial Q_1 \partial Q_2} \\
\frac{\partial^2 \Pi_1}{\partial x_2 \partial x_1} & \frac{\partial^2 \Pi_1}{\partial x_2 \partial Q_1} & \frac{\partial^2 \Pi_1}{\partial x_2^2} & \frac{\partial^2 \Pi_1}{\partial x_2 \partial Q_2} \\
\frac{\partial^2 \Pi_1}{\partial Q_2 \partial x_1} & \frac{\partial^2 \Pi_1}{\partial Q_2 \partial Q_1} & \frac{\partial^2 \Pi_1}{\partial Q_2 \partial x_2} & \frac{\partial^2 \Pi_1}{\partial Q_2^2}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial x_1}{\partial \theta} \\
\frac{\partial x_2}{\partial \theta} \\
\frac{\partial \theta}{\partial Q_1} \\
\frac{\partial \theta}{\partial Q_2}
\end{bmatrix}
= -
\begin{bmatrix}
\frac{\partial^2 \Pi_1}{\partial x_1 \partial \theta} \\
\frac{\partial^2 \Pi_1}{\partial Q_1 \partial \theta} \\
\frac{\partial^2 \Pi_1}{\partial x_2 \partial \theta} \\
\frac{\partial^2 \Pi_1}{\partial Q_2 \partial \theta}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial x_1}{\partial \theta} \\
\frac{\partial x_2}{\partial \theta} \\
\frac{\partial \theta}{\partial Q_1} \\
\frac{\partial \theta}{\partial Q_2}
\end{bmatrix}.
\tag{6}
\]

Designating the 4 x 4 square matrix of the second-order partial derivatives in (6) by matrix \( H \), equations (6) can be rewritten as

\[
\begin{bmatrix}
\frac{\partial x_1}{\partial \theta} \\
\frac{\partial x_2}{\partial \theta} \\
\frac{\partial \theta}{\partial Q_1} \\
\frac{\partial \theta}{\partial Q_2}
\end{bmatrix}
= -H^{-1}
\begin{bmatrix}
\frac{\partial^2 \Pi_1}{\partial x_1 \partial \theta} \\
\frac{\partial^2 \Pi_1}{\partial Q_1 \partial \theta} \\
\frac{\partial^2 \Pi_1}{\partial x_2 \partial \theta} \\
\frac{\partial^2 \Pi_1}{\partial Q_2 \partial \theta}
\end{bmatrix}.
\tag{7}
\]

It can be shown that the Heissian matrix of second partial derivatives \( H \) is nonsingular and negative semi-definite, so that its inverse does exist and the Nash equilibrium solution of the system of equations (5) is unique (Gruca et al., 1992).

Based on the solution of equations (7) for a symmetric competitive EOQ inventory model with advertising-dependent demand, the following self-explanatory proposition is introduced with related terms defined as shown below:
\[ a = \frac{f'}{4f} \frac{C_o}{\gamma Q^2} \text{ (positive term),} \tag{8} \]
\[ b = \frac{(f^* f - f'^2)}{4f^2} \left[ 1 - \frac{C_o}{\gamma Q} \right] - \frac{C^*}{\gamma m} \text{ (negative term), and} \tag{9} \]
\[ \Delta_i = -\frac{b C_o}{\gamma Q^3} - a^2 \text{ (positive term).} \tag{10} \]

**Proposition:** For a symmetric duopoly

(i) An increase in the parameters \( \gamma, \beta, \) or \( \delta \) of a rival should be responded to by an increase in the equilibrium advertising and order quantity of its own but a decrease in the equilibrium advertising and order quantity of its competitor.

(ii) An increase in either of the parameters \( d \) or \( \varepsilon \) of a rival should be responded to by a decrease in the equilibrium advertising and order quantity of its own but an increase in the equilibrium advertising and order quantity of its competitor.

(iii) An increase in parameter \( C_h \) of a rival should be responded to by a decrease in the equilibrium advertising and order quantity of its own but an increase in the equilibrium advertising and order quantity of its competitor.

(iv) An increase in parameter \( C_o \) of a rival should be responded to by a decrease in the equilibrium advertising and an increase (decrease) in the equilibrium order quantity if \( \Delta_i - 2a^2 > (<) 0 \) of its own but an increase in the equilibrium advertising and order quantity of its competitor.

(v) An increase in parameter \( m \) should be responded to by an increase in the equilibrium advertising and order quantity of both rivals.

(vi) For each model parameter cross comparative statics do not exceed, in magnitude, the respective self-comparative statics.

It is conjectured here that the results depicted in the proposition are generalized to an oligopoly of \( N \) competitors. It is worthy to mention here that the signs of the comparative statics relating the changes in the marketing parameters (\( \gamma, \beta, \delta, d, \varepsilon, \) and \( m \)) to changes in optimal advertising are similar to those reported in Mesak (2003) though such earlier study does not consider inventory related costs. The remaining comparative statics are unique to the present study.

The theoretical findings reported in the above proposition are derived under ideal symmetric conditions that almost never materialize in practice. This article attempts to assess the robustness of the theoretical sensitivity results to deviations from these ideal conditions through examining the applicability of these results to a practical oligopoly setting. Section 4 estimates the market share attraction model in an oligopoly using actual data related to the United States carbonated soft drink industry. The estimated parameters are then used in Section 5 to determine the elements of expressions (7), upon obtaining the equilibrium advertising and order quantity levels using equations (5). Robustness is assessed by comparing the sensitivity analysis results obtained empirically upon applying expressions (7) with their theoretical counterparts highlighted in the proposition.
ESTIMATION OF MARKET SHARE ATTRACTION MODEL

The market share of firm \( j \), \( MSHR_j \), in a market of \( N \) competitors is expressed as follows:

\[
MSHR_j = \frac{f_j(x_j)}{\sum_{i=1}^{N} f_i(x_i)} = \frac{\beta_j x_j^\delta}{\sum_{i=1}^{N} \beta_i x_i^\delta}
\]

(11)

where all parameters are defined in section 2. Neart and Weverbergh (1981) demonstrate that a model for which all the \( \delta_j \)s are equal provides a better empirical fit with the data than the unrestricted version. The market share attraction model for an oligopoly as given in equation (11) is empirically estimated using actual data related to the United States carbonated soft drink industry. For the carbonated soft drink market, firm specific sales and advertising data for Coke (firm 1), Pepsi (firm 2), Sprite (firm 3), Mountain Dew (firm 4) and Dr. Pepper (firm 5) were obtained from different issues of *Beverage Digest*, *SuperBrands* and *Advertising Age*. The current study employs carbonated soft drink industry data for years 1995 through 2004. Prior to analysis, the advertising data for the carbonated soft drink industry were converted to 1995 dollars. Each of the above five brands are considered as independent profit centers. Defining

\[
S_{jt} = \text{cases in millions sold by firm } j \text{ in year } t,
MSHR_{jt} = \text{market share of firm } j \text{ in year } t \text{ and } x_{jt} = \text{advertising in 1995 million dollars in year } t \text{ for firm } j.
\]

Assuming \( t_0 \) be year 1995 at which consumer price index = 100, an advertising deflator is defined

\[
\text{Advertising deflator in year } t = \frac{\text{US population in year } t}{\text{US population in year } t_0} \times \text{consumer price index in year } t.
\]

Accordingly, advertising expenditure in year \( t \) for firm \( j \), \( x_{jt} \), expressed in \( t_0 \) dollars is given as follows: Advertising in million dollars in year \( t \) for firm \( j \) / Advertising deflator in year \( t \).

Descriptive statistics of the data are shown in Table 1.

The table indicates that all variables \( MSHR_j \) and \( x_j \) assume considerable variability during the studied period.

Defining \( MSHR_t = (MSHR_{j1}, MSHR_{j2}, \ldots, MSHR_{jN})^{1/N} \),

\[
MSHR_{jt}^\prime = \ln (MSHR_{jt}/MSHR_t),
\]

\( x_t = (x_{j1t}, x_{j2t}, \ldots, x_{jNt})^{1/N} \), and

\( x_{jt}^\prime = \ln (x_{jt}/x_t) \)

The market share attraction model (11) is estimated by ordinary least-squares (OLS) regression after log-centering formulation of the data for each period \( t \). Cooper and Nakanishi, 1988, p.128) mention that the OLS procedure appears satisfactory in many market share model applications. The regression equation used to estimate the parameters of the market share attraction model is given by

\[
MSHR_{jt}^\prime = \alpha_1 + \alpha_2 d_2 + \alpha_3 d_3 + \ldots + \alpha_N d_N + \delta x_{jt}^\prime + \epsilon_{jt}^\prime,
\]

(12)

where \( d_j \) is a dummy variable that takes on the value 1 for an observation related to firm \( j \) and 0 otherwise; \( \alpha_j = \alpha_j - \alpha_1 \) for \( j = 2, 3, \ldots, N \); \( \delta = \delta_1 = \delta_2 = \ldots = \delta_N \) are parameters to be estimated; and \( \epsilon_{jt}^\prime \) is an error term related to firm \( j \) in period \( t \). The estimated parameter \( \beta_j \) is given by \( \epsilon_{jt}^\prime \).
Table 1

<table>
<thead>
<tr>
<th>Firm (brand)</th>
<th>Variable</th>
<th>Minimum</th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coke (classic and diet)</td>
<td>$MSHR_1$</td>
<td>0.4389</td>
<td>0.4463</td>
<td>0.4550</td>
</tr>
<tr>
<td></td>
<td>$x_1^a$</td>
<td>84.3187</td>
<td>143.6455</td>
<td>177.8039</td>
</tr>
<tr>
<td></td>
<td>$Q_1^b$</td>
<td>350.8047</td>
<td>436.5744</td>
<td>498.1475</td>
</tr>
<tr>
<td>Pepsi (classic and diet)</td>
<td>$MSHR_2$</td>
<td>0.2678</td>
<td>0.2808</td>
<td>0.3116</td>
</tr>
<tr>
<td></td>
<td>$x_2^a$</td>
<td>81.0188</td>
<td>102.1788</td>
<td>131.5442</td>
</tr>
<tr>
<td></td>
<td>$Q_2^b$</td>
<td>270.0415</td>
<td>281.5854</td>
<td>291.2763</td>
</tr>
<tr>
<td>Sprite (classic and diet, if any)</td>
<td>$MSHR_3$</td>
<td>0.0732</td>
<td>0.0897</td>
<td>0.0971</td>
</tr>
<tr>
<td></td>
<td>$x_3^a$</td>
<td>14.9018</td>
<td>45.9175</td>
<td>60.3013</td>
</tr>
<tr>
<td></td>
<td>$Q_3^b$</td>
<td>56.7676</td>
<td>88.3876</td>
<td>107.4228</td>
</tr>
<tr>
<td>Mountain Dew (classic and diet)</td>
<td>$MSHR_4$</td>
<td>0.081</td>
<td>0.0964</td>
<td>0.1155</td>
</tr>
<tr>
<td></td>
<td>$x_4^a$</td>
<td>20.5</td>
<td>34.8355</td>
<td>53.1218</td>
</tr>
<tr>
<td></td>
<td>$Q_4^b$</td>
<td>72.0588</td>
<td>97.3578</td>
<td>123.1134</td>
</tr>
<tr>
<td>Dr. Pepper (classic and diet)</td>
<td>$MSHR_5$</td>
<td>0.0819</td>
<td>0.0868</td>
<td>0.0911</td>
</tr>
<tr>
<td></td>
<td>$x_5^a$</td>
<td>36.0914</td>
<td>55.3905</td>
<td>76.2314</td>
</tr>
<tr>
<td></td>
<td>$Q_5^b$</td>
<td>88.4296</td>
<td>111.4862</td>
<td>125.9556</td>
</tr>
<tr>
<td>Industry Sales $^b$</td>
<td>$S$</td>
<td>6289.3</td>
<td>6718.5</td>
<td>6948.1</td>
</tr>
</tbody>
</table>

$^a$ (1995 $^bM$), $^b$ (Cases in Millions-Computed in Appendix B)

Table 2 lists the parameter estimates and their related $p$-values as obtained from linear regression. All parameters are statistically different from zero at better than the 0.05 level.

Table 2
Regression estimates of the parameters related to the market share attraction model of carbonated soft drink industry

| Coefficient estimate of $\delta$ | 0.085692 (0.0288) |
| Coefficient estimate of $\alpha_1$ | 0.982806 (9.49X10$^{-30}$) |
| Coefficient estimate of $\alpha_2'$ | $-0.436788 (6.73X10^{-19})$ |
| Coefficient estimate of $\alpha_3'$ | $-1.505897 (3.59X10^{-30})$ |
| Coefficient estimate of $\alpha_4'$ | $-1.415414 (1.47X10^{-26})$ |
| Coefficient estimate of $\alpha_5'$ | $-1.555932 (1.34X10^{-33})$ |
| Number of observations | 50 |
| $R^2$ | 0.993486 |

Note: $p$-values are in parentheses.
The estimated models have face validity as the value of $\delta$ lies between 0 and 1 suggesting a concave advertising attraction function which is supported by the literature (Simon and Arndt, 1980; Friedman, 1983).

From Table 2, the following estimates were obtained for the carbonated soft drink industry:

- $\beta_1 = e^{\alpha_1} = 2.672462$, $\beta_2 = e^{\alpha_2+\alpha_3} = 1.738461$, $\beta_3 = e^{\alpha_1+\alpha_3} = 0.59274$, $\beta_4 = e^{\alpha_1+\alpha_4} = 0.64921$, and $\beta_5 = e^{\alpha_1+\alpha_5} = 0.563831$.

Upon analyzing companies’ and industry records, descriptive statistics of the obtained order quantities in millions of cases over time are shown in Table 1. The obtained gross profit per case of soft drink in 1995 dollars are given by $\gamma_1 = \gamma_3 = 2.476$, $\gamma_2 = \gamma_4 = 2.6679$ and $\gamma_5 = 2.312$. The obtained holding costs per case per year are given by $C_{h1} = C_{h2} = C_{h3} = C_{h4} = C_{h5} = 0.22$. Finally, the obtained ordering cost per order are given by $C_{o1} = 7.025$, $C_{o2} = 4.666$, $C_{o3} = 1.433$, $C_{o4} = 1.568$, and $C_{o5} = 2.467$ (further details are obtainable from the authors).

**DERIVATION OF EMPIRICAL COMPARATIVE STATICS**

The empirical comparative statics is derived by following the steps described below:

(i) All parameters associated with the market share attraction model (11) are estimated using information from Table 2.

(ii) The parameters $m$, $\gamma_i$, advertising cost function $C(x_i)$, ordering cost $C_{o_j}$, holding cost $C_{h_j}$, and the order quantity $Q_j$ variables are estimated. The above quantities are associated with the profit functions [equations (3) and (4)], extended to $N$ firms (5 for the carbonated soft drink industry).

(iii) Noting that at optimality,

$$\frac{\partial \Pi}{\partial x_j} = 0, \text{ and } \frac{\partial \Pi}{\partial Q_j} = 0.$$  

For all firms $j$, the Nash equilibrium $x_j^*$ and $Q_j^*$ are determined through numerically solving the system of $2N$ simultaneous nonlinear equations.

(iv) The elements of the Heissian matrix $H_{2N \times 2N}$ are computed similar to expression (7), but extended to $N = 5$ firms. It should be noted that elements $H_{jk}$ and $H_{kj}$ need not be equal as these two terms belong to different rivals.

(v) The entries of the post-multiplier column vector of expression (8) extended to $N$ firms is computed.

(vi) Expression (7), extended to $N = 5$ firms, is applied in conjunction with the results obtained from steps (iv) and (v) to assess the signs of the empirical comparative statics.

It is assumed in the empirical analysis for simplicity as in Mesak (2003) that the advertising cost function is linear, i.e., $C(x_j) = x_j$; thus, $C'_j = 1$ and $C''_j = 0$ for all $j$. After applying the above procedure, the obtained empirical statics are reported in Table 3.
Table 3

Empirical comparative statics of soft drink industry data

<table>
<thead>
<tr>
<th>$\partial x^*/\partial \gamma_1$</th>
<th>$\partial x^*/\partial \gamma_2$</th>
<th>$\partial x^*/\partial \gamma_3$</th>
<th>$\partial x^*/\partial \gamma_4$</th>
<th>$\partial x^*/\partial \gamma_5$</th>
<th>$\partial x^*/\partial \beta_1$</th>
<th>$\partial x^*/\partial \beta_2$</th>
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Table 3 shows, without exception, that all of the obtained 260 empirical comparative statics of the carbonated soft drink industry data possess signs that are consistent with their theoretical symmetric counterparts depicted in the proposition. The table shows in particular that the self-comparative statics $\frac{\partial Q_j^*}{\partial C_{oj}}$ are of the same sign (and of larger magnitudes) compared to the cross-comparative statics $\frac{\partial Q_i^*}{\partial C_{oj}}$ for $i \neq j$ (see also parts (iv) and (vi) of the proposition). Such a central finding is alarming provided that, for each year, the raw data summarized in Tables 1 indicate that the market shares of competing firms are dispersed apart from each other.

**CONCLUSIONS**

This study involves theoretical derivation of comparative statics of a symmetric duopoly inventory model with advertising-dependent demand associated with a market share attraction model of advertising competition. The validity and robustness of the theoretical model is empirically examined using the carbonated soft drink industry data in the US for which competition is asymmetric. The results demonstrate remarkable consistency between the theoretical and the empirical comparative statics. The validation procedure requires estimation of the parameters of a market share attraction model (fourth section), which are used afterwards to operationalize expression (7) extended to an oligopoly of more than two competitors. Then the signs of the empirical sensitivity results (fifth section) are compared with their theoretical counterparts involving symmetric competition in a duopoly (Proposition in third section). Of course, analyzing data sets for other industries should help in assessing the generality of the above important findings. The main conclusion drawn from the findings of the present study is of significant strategic appeal for marketing and production/purchasing managers. For an oligopolistic market that is not dominated by a single rival, a firm would adjust its advertising spending and order quantity in response to changes in its own or competitor parameters in a manner consistent with the comparative statics reported in the proposition. Although the theoretical model is developed for a symmetric competition for simplicity, the validation of the results using empirical data from an asymmetric oligopolistic competition suggest that relaxing the assumption of symmetry for theoretical model development will make the analysis more complex without yielding much new insights.

**REFERENCES**


Freeland, J.R. Coordination strategies for production and marketing in a functionally decentralized firm. AIIE Transactions 1980, 12, 126-132.


Subramanyam, E.S., S. Kumaraswamy. EOQ formula under varying marketing policies and conditions. AIIE Transactions 1981, 13, 312-314.
