A SIMPLIFIED TEACHING APPROACH FOR INVENTORY ORDER SIZING:
ADJUSTING FOR HIGH ORDER COSTS AND LEAD TIMES

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ABSTRACT

Most textbooks fail to address the limitations of using the economic order quantity (EOQ) model in advanced inventory control systems when long lead times and high order costs are present. This paper examines multiple approaches to adjusting order policies, and recommends a low cost, parsimonious, and easy to teach technique.

INTRODUCTION

Ordering policy decisions represent a major challenge for supply chain managers (Dhumal, Sundararaghavan, & Nandkeolyar, 2008). To overcome this challenge, researchers have introduced several techniques, including the economic order quantity (EOQ) (Harris, 1913). EOQ has been widely adopted and is prevalent in supply chain and operations management texts. The deterministic model offers a quick, easy way to calculate order quantities at the lowest cost. As such EOQ covers the most basic components of an inventory policy. And, it is based on assumptions that frequently only hold in a few situations. The reality is that variation occurs regularly in different components of the replenishment cycle increasing the complexity of decisions regarding when and how much to order. For this reason, a more advanced inventory control system is needed.

The purpose of this paper is to review textbook explanations of order quantities and reorder points when variation in demand and lead time is present. Following that assessment, an example problem used in the classroom to teach advanced inventory control is presented. Finally, we recommend an easy to teach a technique that offers lower costs than the classic deterministic EOQ model.
ECONOMIC ORDER QUANTITY

Economic order quantity is a fixed-order quantity model (Jacobs & Chase, 2011) in which an order is placed at a predetermined inventory level, called the reorder point (R). The classic EOQ model is aimed at achieving the lowest total cost finding the minimum point on the total cost curve. The EOQ formula is developed from the total cost equation; it is expressed as

\[ EOQ = \sqrt{\frac{2DS}{IC}} \]  

(1)

where:
- \( EOQ \) = Economic Order Quantity
- \( D \) = demand (annual)
- \( S \) = cost of placing an order
- \( I \) = carrying cost as an annual percent of item value/year
- \( C \) = purchase price, delivered

The results of the EOQ model are an order quantity that balances procurement and inventory carrying costs. Once the order quantity is determined, the next step is to decide when to place an order (Axsäter, 2000).

Order placement occurs when on-hand inventory falls to the reorder point. To determine the reorder point, the first step is to calculate the amount of demand that will occur over lead time and add a safety stock component. This ensures that the inventory level for cycle stock will reach zero just before the next batch of inventory arrives. The reorder point is

\[ R = \bar{d}LT + z\sigma \]  

(2)

where:
- \( R \) = reorder point
- \( \bar{d} \) = average demand, in time units
- \( LT \) = lead time, in time units
- \( z\sigma \) = safety stock

The classic EOQ model assumes that the order quantity is larger than demand during lead time (Axsäter, 2000). As such, in the classroom and most textbook examples EOQ is set to exceed the R quantity. However, as Silver, Pyke, and Peterson (1998) point out, there are instances in which R may exceed EOQ due to long lead times or high order costs. As a result, the classic EOQ model must be adjusted when demand during lead time is greater than EOQ. Though many textbooks discuss EOQ with variability, few cover situations in which demand during lead time exceeds order quantity (see Table 1). The remainder of our discussion focuses on how to adjust order quantities when demand during lead time is larger than EOQ.
TABLE 1

COMPARISON OF INVENTORY MANAGEMENT CONTROL TECHNIQUES

<table>
<thead>
<tr>
<th>Text</th>
<th>Pages</th>
<th>Model Name</th>
<th>Variable</th>
<th>Variability</th>
<th>ROP&gt;Q*</th>
<th>Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballou (2004)</td>
<td>344-356</td>
<td>Economic Order Quantity</td>
<td>Q*</td>
<td>Yes</td>
<td>Presented</td>
<td>Multiple Order During Lead Time</td>
</tr>
<tr>
<td>Benton (2007)</td>
<td>85-88, 96-101</td>
<td>Simple Classical Economic Lot-Size Model</td>
<td>Q*</td>
<td>Yes</td>
<td>None</td>
<td>None</td>
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<tr>
<td>Bowersox, Closs, and Cooper (2010)</td>
<td>164-168</td>
<td>Economic Order Quantity</td>
<td>EOQ</td>
<td>Yes</td>
<td>None</td>
<td>None</td>
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<tr>
<td>Chopra and Meinl (2010)</td>
<td>251-255</td>
<td>Economic Order Quantity</td>
<td>Q*</td>
<td>No</td>
<td>None</td>
<td>None</td>
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<tr>
<td>Chrisopher (2011)</td>
<td>104-108</td>
<td>Reorder Point Method</td>
<td>EOQ</td>
<td>No</td>
<td>None</td>
<td>None</td>
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<tr>
<td>Coyle, Langley, Gibson, Novack, and Bardi (2008)</td>
<td>389-400</td>
<td>EOQ Approach</td>
<td>Q</td>
<td>Yes</td>
<td>None</td>
<td>None</td>
</tr>
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<td>Gaither and Frazier (2002)</td>
<td>358-377</td>
<td>Fixed Order Quantity Systems, Two-bin System, EOQ System</td>
<td>EOQ</td>
<td>No</td>
<td>None</td>
<td>None</td>
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<tr>
<td>Heizer and Render (2004)</td>
<td>457-469</td>
<td>EOQ Model</td>
<td>Q*</td>
<td>Yes</td>
<td>None</td>
<td>None</td>
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<tr>
<td>Jacobs and Chase (2011)</td>
<td>565-572</td>
<td>Fixed-order quantity model (Economic Order Quantity, Q-model)</td>
<td>Q_{opt}</td>
<td>Yes</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Jacoby (2009)</td>
<td>26</td>
<td>None</td>
<td>None</td>
<td>No</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Krajewski, Ritzman, and Malhotra (2010)</td>
<td>417-422</td>
<td>EOQ Model</td>
<td>EOQ</td>
<td>Yes</td>
<td>None</td>
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<td>Lambert (2008)</td>
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<td>None</td>
<td>None</td>
<td>No</td>
<td>None</td>
<td>None</td>
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<tr>
<td>Monczka, Handfield, Giunipero, and Patterson (2011)</td>
<td>None</td>
<td>None</td>
<td>None</td>
<td>No</td>
<td>None</td>
<td>None</td>
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<tr>
<td>Monczka, Trent, and Handfield (1998)</td>
<td>28</td>
<td>EOQ</td>
<td>None</td>
<td>No</td>
<td>None</td>
<td>None</td>
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<tr>
<td>Schroeder (2000)</td>
<td>309-312</td>
<td>Wilson EOQ</td>
<td>Q</td>
<td>Yes</td>
<td>Presented</td>
<td>Multiple Orders During Lead Time</td>
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<tr>
<td>Silver et al. (1998)</td>
<td>149-179, 326-331</td>
<td>Economic Order Quantity</td>
<td>Q_{opt} or EOQ</td>
<td>Yes</td>
<td>Presented</td>
<td>Order multiples of Q</td>
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<td>Stock and Lambert (1987)</td>
<td>236-243</td>
<td>Economic Order Quantity</td>
<td>EOQ</td>
<td>Yes</td>
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<td>Wisner, Tan, and Leong (2009)</td>
<td>177-181</td>
<td>Economic Order Quantity</td>
<td>EOQ</td>
<td>None</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

Alternative Order Quantity Calculations

When demand during lead time exceeds EOQ, if the classic EOQ model is used, shortages will occur between placing an order and the order arriving. To overcome this issue, the literature offers three separate approaches.

The first approach (Hadley and Whitin, 1963; Schroeder, 2000; Ballou, 2004) recommends tracking effective inventory, defined as inventory on-hand plus inventory quantity on order. Using effective inventory the on-hand inventory never exceeds demand over lead time, and so
inventory carrying costs are kept lower. Unlike the classic EOQ, which maintains at most one order outstanding at any time, this method requires placing multiple orders during lead time (Hadley & Whitin, 1963; Schroeder, 2000; Ballou, 2004). Allowing multiple EOQ orders during lead time can still minimize procurement and inventory carrying costs. However, as seen in Figure 1, this can lead to a more complicated process for tracking inventory levels when multiple orders are outstanding. Additionally, this method may be more difficult to teach in the classroom. Students can often find it difficult to calculate the current inventory position with multiple orders outstanding or to understand that inventory orders may need to be placed several cycles before the actual arrival of a shipment. We were unable to find any texts that graphically portray tracking and managing the simultaneous orders created by the method. Instead of allowing multiple orders over lead time, another method may enable the system to maintain the assumption of one order outstanding at any time and allow for adjustments to the service level and safety stock to achieve desired system performance.

**FIGURE 1**

**USING EFFECTIVE INVENTORY TO ORDER MULTIPLE EOQ QUANTITIES**

(SCHROEDER, 2000; BALLOU, 2004)

Two alternate, but similar methods have been proposed for calculating an adjusted order quantity when EOQ is less than demand during lead time. First, instead of ordering multiple times, Axsäter (1996) recommends calculating EOQ and multiplying it by a number large enough to make the on-hand inventory greater than the reorder point when the order arrives, as shown in Silver et al. (1998).

The second method from the literature is illustrated in an inventory case study in the 2nd edition of Stock and Lambert (1987). This method recommends an adjustment to the order quantity itself instead of placing multiple orders for the same quantity. After calculating EOQ, Method 2 requires the calculation of a second possible order quantity, \( Q^* \), equal to the demand during lead time (Eq. 3), as seen in Figure 2.

\[
Q^* = \bar{d}LT
\]  

(3)

When EOQ<\( Q^* \), the order quantity is adjusted to be \( Q^* \) instead of EOQ. This change is seen in Figure 2. This method is much less complicated than tracking 2-3 orders at one time and, as shown below, reduces total relevant cost.

**FIGURE 2**

ORDERING THE LARGER OF EOQ OR ROP (STOCK & LAMBERT, 1987)

To illustrate the two approaches, making multiple orders (Schroeder, 2000; Ballou, 2004) and making one large order (Stock & Lambert, 1987), we offer an example below that we have used in the classroom. This example allows students to calculate EOQ and R, to determine an adjusted order quantity when necessary, and to analyze cost and service level tradeoffs for the solutions.

**EXAMPLE PROBLEM**

Choosing which method to use involves calculating EOQ and R, then deciding whether to hold the safety stock or service level constant. The following example shows the difference in total costs and service levels between the two methods recommended: placing multiple orders, Method 1 (Schroeder, 2000; Ballou, 2004) or ordering the larger of EOQ or R, Method 2 (Stock & Lambert, 1987).

In the example the following inputs are used:

- Forecasted demand (\(\bar{d}\)) = 325 cases per week
- Annual Demand (D) = 16,900 cases per year
- Forecast error, standard deviation (\(s_d\)) = 95 cases per week
- Lead-time (LT) = 1.5 weeks (10.5 days)
- Lead time, standard deviation (\(s_{LT}\)) = 0.5 weeks
- Annual carrying cost, inventory (I) = 20%
- Purchase price, delivered (C) = $50 per case
- Purchase order cost (S) = $10 per order
- Stock out cost (k) = $35 per case
- Probability of being in stock during lead time, (P) = 88%

To evaluate the effectiveness of each method, total relevant costs are calculated. As stated previously, not all costs associated with inventory policy should be included in the calculation, but only those that change with different methods. Therefore, the relevant total cost equation (Eq. 5) includes components for procurement cost, carrying cost of cycle stock, carrying cost of safety stock, and stock-out costs (For a more detailed explanation of each cost, see Ballou, 2004, p. 352).

\[
TAC = \left( \frac{D}{Q} \right) * S + \left( I * C * \frac{Q}{2} \right) + \left( I * C * Z * \sigma \right) + \left( \frac{D}{Q} * k * \sigma E(d) \right)
\] (5)

The first step in either method is to calculate the economic order quantity (Eq. 3) and the reorder point (Eq. 4). Using Equations 1 and 2 the EOQ is 183.5 or 184 units, safety stock is 168 units and R is 655 units. EOQ is much lower than the reorder point. In fact, demand during lead time is more than twice as large as EOQ, meaning that one EOQ order will not bring inventory levels up to a point that will avoid shortages. Students are offered two methods to overcome this issue.

Using Method 1 there are 2-3 EOQ amounts on order at all times (Fig. 1). The reorder point remains the same, but tracking both on-hand and effective inventory becomes far more complicated. On-hand inventory levels remain relatively low compared to the other approaches, but inventory levels reach the zero point twice between placing and receiving order 3. The lower inventory levels put the firm at a higher risk of stockout, increasing projected out-of-stock costs. At the same time, between ordering and receiving order 3, two more orders are placed. Keeping
track of three or more orders at a time and increasing out-of-stock risks makes this system far more complicated than the other approach proposed. In addition, having this many orders in process at the same time reduces the firm’s short term flexibility. This is of particular concern when operating conditions are dynamic and/or lead times are long, e.g. weeks versus days.

Method 2 recommends an adjustment to the order quantity itself instead of placing multiple orders for the same quantity. After calculating EOQ, Method 2 requires the calculation of a second possible order quantity, $Q^*$, equal to the demand during lead time (Eq. 6).

$$Q^* = dLT$$  \hspace{1cm} (6)

When $EOQ = Q^*$, the order quantity is adjusted to be $Q^*$ instead of EOQ. This change is seen in Figure 2. This method is much less complicated than tracking 2-3 orders at one time and, as shown below, reduces total relevant cost.

As with Method 1, the reorder point with Method 2 remains the same, but only one order is needed to bring on-hand inventory back to the reorder point. Even with demand and lead time uncertainty, the increased order quantity ($Q^* = 655$ units) and identical safety stock ($168$ units) allows for an overall increase in service level between Methods 1 and 2, from $88\%$ to $95\%$, according to Equation 2. This increase in service level has no effect on the procurement and inventory carrying costs, but the overall change in order quantity does create a situation in which the two components of EOQ do not need to be balanced.

Ordering larger quantities in fewer orders decreases procurement costs, but disproportionately increases carrying costs as well. Along with the unchanged cost of carrying safety stock, the cost of Method 2 is $30\%$ higher than Method 1. When out-of-stock costs are added, however, the results show a significant improvement from Method 1 to Method 2, as shown in Table 2. The out-of-stock cost for Method 1 is over three times that of Method 2. When stockout costs can be estimated, Method 2 offers a much lower cost solution than Method 1. On the other hand, as stockout costs are difficult to estimate, service levels are often considered instead of stockout costs (Axsäter, 2000). In this situation, Models 1 and 2 should be compared based on service level and safety stock tradeoffs.
### TABLE 2

A COMPARISON OF TOTAL COST BY INVENTORY METHOD

<table>
<thead>
<tr>
<th></th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 2 (Adjusted SL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order Cost ($)</td>
<td>918.48</td>
<td>346.31</td>
<td>346.31</td>
</tr>
<tr>
<td>Carrying Cost (CS) ($)</td>
<td>920.00</td>
<td>2440.00</td>
<td>2440.00</td>
</tr>
<tr>
<td>Carrying Cost (SS) ($)</td>
<td>1680.00</td>
<td>1680.00</td>
<td>471.61</td>
</tr>
<tr>
<td>Total Cost (Excluding Stockout Cost)</td>
<td>3518.48</td>
<td>4464.17</td>
<td>355.77</td>
</tr>
<tr>
<td>Stockout Cost ($)</td>
<td>71958.29</td>
<td>27131.81</td>
<td>70980.00</td>
</tr>
<tr>
<td>Service Level</td>
<td>0.88</td>
<td>0.95</td>
<td>0.88</td>
</tr>
<tr>
<td>Total Cost</td>
<td>75476.77</td>
<td>31598.13</td>
<td>74237.92</td>
</tr>
</tbody>
</table>

As stated earlier, inventory policies can hold either the safety stock or service level constant. When constant safety stock is maintained, the service level increases which in turn reduces the out-of-stock costs. On the other hand, when the service level is maintained, safety stock levels decrease, lowering the cost of carrying safety stock, while increasing the stockout cost at the same time. When stockout costs cannot be estimated, Method 1 and Method 2 should be compared through total relevant costs excluding the cost of a stockout. Instead, procurement cost, inventory carrying cost, and cost of carrying safety stock should be used.

Although the example includes the cost of a stockout, in certain situations these costs cannot be determined. However, to achieve the same service level as Method 1 (88%) while implementing Method 2, the safety stock could be drastically reduced (to 47 units) as seen in Figure 3. This results in Method 2 giving a lower total cost, while also offering a less complicated system to implement for managers and to teach conceptually in class.
CONCLUSION

For any situation in which demand during lead time is less than the order quantity, the EOQ approach is a simple and effective inventory policy. Unfortunately, variation in demand and long lead times may create situations in which EOQ is less than the reorder point. Most textbooks surveyed offer no solution for this issue. Two techniques for adjusting the EOQ method that involve changing either order quantities or number of orders to avoid inventory shortages are examined in this paper. Our classroom experience shows that the method recommended by Stock and Lambert (1987), which is not presented in any current editions of the textbooks surveyed, is a simple and straightforward way to address this issue. By setting order quantities to the amount of demand during lead time, safety stock can be reduced, total relevant costs decreased, and the model becomes much less complicated.

As noted by Occam’s razor, simpler explanations are generally better than more complex ones with all other things being equal (Domingos, 1999). Student mastery of inventory management and control techniques, as demonstrated through the example problem offered, indicates that the straightforward approach of Method 2 is a highly effective way to illustrate the classic tradeoffs between service level, safety stock and total costs. Moreover, the adjustment to the order quantity proposed in this paper has been shown to be an efficient and effective way to manage...
and control inventory. Based on our analysis, it is recommended that texts and educators use the method and example offered here as a parsimonious and competitive way for dealing with situations where the demand during lead time exceeds the EOQ.
REFERENCES


