MCGL: A NEW METHOD FOR MOELLING THE CHOICE BEHAVIOR OF THE DECISION MAKER

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ABSTRACT

This research proposes the development and implementation of a novel method for multi-criteria decision analysis (MCDA). The proposed method (MCGL: Multi Criteria Gain Loss) is based on tenets of prospect theory and the complexity theory. The simple preference elicitation in MCGL improves the performance of the decision maker (DM). The usage of descriptive theories of decision making to process these preferences increases the accuracy of MCGL in modeling the choice behavior of the DM. Through a pilot study, the effectiveness of MCGL is tested by collecting responses from 23 university students using the MCGL and the Analytic Hierarch Process (AHP) on a hypothetical mobile selection problem. When the ranking of mobile phones obtained from the AHP and from the MCGL are analyzed, the rankings obtained from the MCGL are more similar to the direct rankings given by the respondents. The results indicate that the MCGL captures the human decision making process more accurately than the AHP. Applicability measures like the number of decisions, time and the cognitive burden strongly favors the MCGL.

Keywords: Multi-Criteria Decision Analysis, Prospect Theory, Complexity Theory, AHP

INTRODUCTION

Last couple of decades has witnessed the evolution of plethora of methods for discrete alternative Multi-Criteria Decision Analysis (MCDA). These methods deal with many criteria and/or attributes to assess various alternatives. The extant methods are based on idiosyncratic axioms and differ by the complexities involved in eliciting preferences from the decision maker (DM) and thereafter, processing them. Many of the MCDM methods (like AHP, ANP, MAUT, PROMETHEE) have normative foundations, assuming that the DM is rational, has unlimited cognitive capacity and will always tend to optimize his/her decision. But there are instances in reality where the actual decision differs from the optimum one. Descriptive theories from the domains like psychology can better explain such irrational behavior of the DM. During last couple of decades, results of the experimental psychology strengthened the assumptions of novel theories of behavioral decision making. These theories have widely been used to explain the
human choices in single criterion decision making under uncertainty, mostly in the areas of economics and finance. According to Larichev (1999), the complexities of the extant MCDM methods create a gap between the requirements of the MCDM methods (normative) and the capabilities of human information processing system (descriptive). The objective of this paper is to explore and verify how some of the latest developments in the behavioral and psychological research can be used to enrich discrete MCDA under certainty. Further, we also intend to decrease the complexity of MCDM method in order to improve the performance of the DM in the process of choice.

This research proposes the development of a method for multi-criteria decision making under certainty. Here, certainty is about the objective value of the alternatives in different criterion. The proposed method (Multi Criteria Gain Loss: MCGL) is based on the tenets of prospect theory (which is currently the main descriptive theory of decision making) (Kahneman & Tversky, 1979) and the complexity theory (Kauffman, 1993). Rest of the paper is organized as follows. Following section covers the MCGL in detail. A pilot study is conducted to test the effectiveness of the MCGL. Details of the pilot study and its results are discussed in the subsequent sections.

PROPOSED METHOD- MULTI CRITERIA GAIN LOSS (MCGL)

Any MCDM method begins with the identification of the criteria, attributes and the alternatives followed by arranging them in a structure to represent real decision setting. The preference elicitation and processing largely depends upon how the problem is structured. The MCGL is novel in its way of preference elicitation and processing. The identification of criteria, attributes and the alternatives is in accordance with the principles described in Keeney & Raifa (1976). For the purpose of preference elicitation from the DM, the structure of the problem is assumed to be hierarchical having top to bottom influence only. This means that each criterion determines the importance of the alternatives but not vice versa. The preference elicitation in MCGL is divided in to two steps as described below. The elicited preferences from the DM in the first two steps are then processed in the third step.

**Step 1- Assign importance weight to each criterion:** The DM gives importance weight to each criterion by dividing a total score of 100 among the criteria. The score given by the DM to a criterion reflect its relative importance in evaluation of the given alternatives. The number of decision required to be given by the DM in this step is equal to the number of criteria/sub-criteria used for the evaluation of the alternatives.

**Step 2- Assign preference ratings to various alternatives in each criterion/attribute separately:** Here, the DM needs to define his/her reference point for each of the criterion. This reference point could be the DM’s aspiration level required for that criterion or his/her current status. To model loss aversion and reference dependence effects on brand choice, Hardie et al, (1993) used most recently purchased brand as the reference point of the consumers. In MCGL, the reference point can also represent a level above which the DM thinks to be satisfied and below which he/she does not. Once the reference point is fixed, the DM can edit the alternatives in each criterion by removing duplicates. Then, the DM needs to arrange the alternatives in the
Table 1
Bipolar scale to rate the alternatives

<table>
<thead>
<tr>
<th>Score</th>
<th>Preference Description</th>
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<tbody>
<tr>
<td>+100</td>
<td>Very Strong Positive Preference from Reference Point</td>
</tr>
<tr>
<td>+75</td>
<td>Strong Positive Preference from Reference Point</td>
</tr>
<tr>
<td>+50</td>
<td>Definite Positive Preference from Reference Point</td>
</tr>
<tr>
<td>+25</td>
<td>Weak Positive Preference from Reference Point</td>
</tr>
<tr>
<td>0</td>
<td>Indifferent (required minimum level or reference point or gain-loss boundary or current holdings)</td>
</tr>
<tr>
<td>-25</td>
<td>Weak Negative Preference from Reference Point</td>
</tr>
<tr>
<td>-50</td>
<td>Definite Negative Preference from Reference Point</td>
</tr>
<tr>
<td>-75</td>
<td>Strong Negative Preference from Reference Point</td>
</tr>
<tr>
<td>-100</td>
<td>Very strong Negative Preference from Reference Point</td>
</tr>
</tbody>
</table>

USE INTERMEDIATE VALUES WHEN COMPROMISE IS NEEDED

order of preference from most preferred to least preferred, separately in each criterion. Many behavioral scientists concluded that the reaction to evaluate an alternative either as good or bad occurs very quickly and such reactions includes emotional feelings, distinctive somatic and physiological events (Bechara et al, 2000; Loewenstein, 1996; Zajonc, 1980). After arranging the alternatives in the order of preference, the DM is required to give preference rating to each of the alternative on a bipolar scale (Table 1) with a 0 at the reference point. The preference rating given by the DM is either gain or loss in terms of the extent of satisfaction from his/her reference point in a criterion. The number of decisions required to be given by the DM in this step is m*n where m is the number of criteria/sub-criteria and n is the number of alternatives.

Step3- Processing of preferences given by the DM:
Once the preference ratings are given by the DM, they are to be analyzed and processed in order to generate an aggregate score for each alternative which can then be used to rank the alternatives. The preferences given by the DM are coded in the form of gains and losses. The importance score given by the DM in step 1 to different criterion is the general indicator of the DM’s values and is independent of the scatter of alternatives in any criterion. Goldstein, (1990) term these judgments as global interpretations of relative importance of criteria, a fixed attitude of the DM and a stable characteristic that does not depend on the stimuli involved. Even though the preference ratings given by the DM to various alternatives on each criterion (in step 2) are on the same scale (-100 to +100), the unit of measurement is different. It is the criterion weight which provides the scaling factor to relate a unit of measurement on one criterion to a unit of measurement on another criterion. This meaning of criterion importance or weight is as per Multi Attribute Value Theory (MAVT) (Keeney & Raiffa, 1976) where it is well defined and unambiguous. The weights define the tradeoff among the criteria. The multiplication of preference ratings given to the alternatives in a criterion by respective criteria weight can bring them on a common unit of measurement which is nothing but the part worth value of an alternative in a criterion. In a simple linear aggregation, the sum of such part worth provides overall value of an alternative (equation 1).
Modeling Choice Behavior of the Decision Maker

\[ V_i = \sum_{j=1}^{m} w_j X_{ij} \quad \text{for each } i=1,\ldots,n \]  

where \( V_i \) is the aggregate value of the alternative \( i \), 
\( w_j \) is the weight given by the DM to the \( j \)-th criterion, and 
\( X_{ij} \) is the preference rating of alternative \( i \) for its objective value on the \( j \)-th criterion.

Now, as the gains are positive numbers and losses are negative numbers, they need to be treated differently. For example, Mittal et al. (1998) investigated the existence and nature of asymmetric and non linear relation (if any) between satisfactions and attribute level performance and found different satisfaction along the positive and negative side of attribute performance. Given a set of alternatives to be assessed on various criteria, the DM chooses an alternative by analyzing two way interactions (across the criteria and across the alternatives). When multiple criteria are involved, the DM evaluates an alternative by analyzing its performance on all the criteria and at the same time the performance of other alternatives on the same set of criteria. An alternative’s inferior performance on most important criterion is enough for the DM to discard that alternative. In any MCDA, elimination of negative performance on any criterion is more important than increasing the performance in positive direction (Mittal et al., 1998). The behavior of the DM to overweight losses is termed as the “loss aversion” phenomenon. Individuals are found to be more sensitive to the criterion in which they are losing relative to their reference point (Kahneman et al., 1991) and this sensitivity further depends upon how much importance is given by the DM to that criterion (Tversky & Kahneman, 1991; Tversky, Sattath, & Slovic, 1988; Viscusi et al., 1987). Presence of asymmetry in satisfaction above and below the reference point may not allow the DM to use same weights to arrive at an aggregate preference for an alternative as it is generally followed in linear compensatory strategy. Many of the existing MCDM methods follow linear compensatory strategy in which the alternatives are judged by weighted average of their performance of different criterion (equation 1). Studies have shown instances of failure of such strategy (e.g. Grabisch & Labreuche, 2008, Marichal, 2000). As an example, suppose two candidates are to be judged on the basis of two criteria and suppose both the candidates have same mean score but one candidate has moderate score on both the criteria while the other has very high and very low score on the same two criteria. If the decision follows linear compensatory strategy, both candidates will receive same aggregate score. But ideally the DM would prefer the candidate having less scatter of scores on the two criteria. If the DM relies on the nonlinear-non compensatory strategy, (e.g Einhorn, 1970) the scatter may affect the output of such MCDA. According to Ganzach, (1995), if the decision follows conjunctive (disjunctive) strategy, the candidate with higher (lower) scatter will receive a lower (higher) evaluation. Ganzach, (1995) also found that when the DM’s commitment is high in the decision making process, their evaluation found to be conjunctive making them to put more emphasis on the negative aspect of input information. Thus in order to incorporate the asymmetry in the negative and the positive aspects of different criterion in a MCD setting, the linear compensatory strategy may not be useful. A different aggregation approach is required which can take in to account the positive and negative asymmetry and overweight negative aspects to avoid them.

Similarly, an alternative’s superior performance on most important criteria is enough for the DM to select that alternative provided it is also performing considerably well on all the other criteria. According to Kauffman’s (1993) complexity theory (NK Landscape), the fitness contribution of
a criterion to the aggregated evaluation of an alternative also depends upon that alternative’s performance on other criteria. In the Kauffman’s NK model, N is the number of criterion under consideration for the evaluation of a set of alternatives while K is the level of interaction of one criterion with every other criterion. If K=0, the landscape is considered to be smooth which means that there is no interaction and the fitness contribution of any criterion to the aggregated evaluation of alternative is independent of that alternative’s performance on any other criterion. However, if K=N-1, the landscape is rugged which means that the fitness contribution of any criterion to the aggregated evaluation of an alternative also depends upon that alternative’s performance on every other criterion. When the DM selects an alternative from a given choice set, he/she generally analyzes how the alternatives are performing within a criterion and across the criterion. Any MCDM method should capture this two way interaction in order to correctly model the choice behavior of the DM. Studies have found that the preference of the DM often remain context dependent i.e. the choice set (Simonson & Tversky, 1992; Tversky & Simonson, 1991).

However, many of the extant MCDM methods are based on the principle of “independence of irrelevant alternatives” which means that the DM has complete preference order for each of the alternatives and it does not gets affected by the addition or deletion of alternatives (Luce & Raiffa, 1957 pp 288). The experimental evidences discovered violation of this principle (for example, Huber et al, 1982; Simonson & Tversky, 1992). Tversky & Simonson (1991) presented a context dependent model of choice incorporating the tradeoff contrast and the extremeness aversion effect. Their model suggested that the preference for an alternative often also depends upon the choice set and the scatter of the alternatives on different criterion. Such models of MCDA rely on “nonlinear-noncompensatory” strategies (also see Einhorn, 1970). Such findings on behavioral decision making can be used to model the choice behavior of the DM correctly. The MCGL is an attempt to harness the findings of research on behavioral decision making to correctly model the choice behavior of the DM.

The multi-criteria choice systems linking the objective measure of the alternative on different criterion to the observed choices assume that the process of choice can be described by three fundamental relations (Anderson, 1981; Louviere, 1988; Lynch, 1985; Meyer & Johnson, 1995) (1) the valuation rules (mapping objective measures of alternatives on different criterion to their perceived attractiveness) (2) the integration rules (mapping the perceptions of the attractiveness of alternatives on different criterion to their overall impressions) and (3) the choice or behavioral rules (mapping the overall impressions to overt behaviors, most commonly choices). This research is an attempt to model the aforementioned component relations in a multi-criteria decision setting under certainty. Here, certainty is about the objective value of the alternatives on different criterion.

The linear compensatory model presented in the equation (1) is modified to equation (2) with an additional variable $\beta_{ij}$ to overweight losses and underweight gains at different intensity depending upon context in place. The aggregate value of an alternative (equation 2) can vary between $(-2.71*100)$ to $(100)$ where -100 to +100 is the rating scale and the 2.71 is for the exponential of 1.
$V_i = \sum_{j=1}^{m} w_j X_{ij} \beta_{ij}$ \hfill (2)

where $\beta_{ij} = \begin{cases} S_{ij} & \text{if } w_j X_{ij} \geq 0 \\ \exp(I_{ij}) & \text{if } w_j X_{ij} < 0 \end{cases}$ \hfill (3)

$SW_i$ is a row vector of $m$ elements (number of criterion) measuring the superiority of $i^{th}$ alternative in $m$ criteria (equation 5). This superiority is context independent and is local to the alternative. The condition of an alternative being superior in a criterion is that it is a gain ($\geq 0$). The condition of an alternative being superior in a criterion with respect to any other criterion is that its weighted preference on first criterion is a gain ($\geq 0$) and is greater than or equal to this alternative’s weighted preference on any other criterion. $SW_i$ is a row vector of $1^m$ and is normalized after taking sum of the row elements of the superiority matrix $SM_i$. $A_{jk}$ (j and k are index to the criterion) are the elements of the superiority matrix $SM_i$ and is the superiority of ith alternative on jth criterion with respect to kth criterion (if $j \neq k$). It is also the superiority with respect to reference point if $j=k$. It is calculated as (equation 4)

$$A_{jk} = \begin{cases} w_j X_{ij} - w_k X_{ik} & \text{if } (j \neq k \text{ and } w_j X_{ij} \geq 0 \text{ and } w_j X_{ij} \geq w_k X_{ik}) \\ w_j X_{ij} & \text{if } (j = k \text{ and } w_j X_{ij} \geq 0) \end{cases}$$ \hfill (4)

$$SW_i = \left[ \frac{\sum_{j=1}^{m} A_{1j}}{\sum_{j=1}^{m} \sum_{k=1}^{m} A_{jk}} \quad \frac{\sum_{j=1}^{m} A_{2j}}{\sum_{j=1}^{m} \sum_{k=1}^{m} A_{jk}} \quad \cdots \quad \frac{\sum_{j=1}^{m} A_{mj}}{\sum_{j=1}^{m} \sum_{k=1}^{m} A_{jk}} \right]$$ \hfill (5)

$$SM_i = \begin{bmatrix} A_{11} & \cdots & A_{1m} \\ \vdots & \ddots & \vdots \\ A_{m1} & \cdots & A_{mm} \end{bmatrix}$$ \hfill (6)

Similarly the $IW_i$ (the inferiority of ith alternative on m criteria) and the $IM_i$ (Inferiority matrix for $i^{th}$ alternative) are calculated. This inferiority is context independent and is local to the alternative. The condition of an alternative being inferior in a criterion is that it is a loss ($< 0$). The condition of an alternative being inferior in a criterion with respect to any other criterion is that its weighted preference on first criterion is a loss ($< 0$) and is less than or equal to this alternative’s weighted preference on any other criterion. $IW_i$ is a row vector of $1^m$ and is normalized after taking sum of the row elements of the inferiority matrix $IM_i$. $A_{jk}$ (j and k are index to the criterion) are the elements of the inferiority matrix $IM_i$ and is the inferiority of ith alternative in jth criterion with respect to kth criterion (if $j \neq k$). It is also the inferiority with respect to reference point if $j=k$. It is calculated as (equation 7)
\[ A_{jk} = \begin{cases} 
Abs(w_jX_{ij} - w_kX_{ik}) & \text{if (j ≠ k and } w_jX_{ij} < 0 \text{ and } w_jX_{ij} ≤ w_kX_{ik}) \\
Abs(w_jX_{ij}) & \text{if (j = k and } w_jX_{ij} < 0) 
\end{cases} \quad (7) \]

The superiority of an alternative in a criterion is normalized with respect to superiority of all the alternatives in the same criterion (equation 8). This is to incorporate context dependent contrast effect by which attractiveness (or unattractiveness) of an alternative in a criterion is calculated with respect to attractiveness (or unattractiveness) of all the other alternatives in the same criterion. For example, a circle appears to be large when surrounded by small circles and vice versa. Similarly, it is done for inferiority of an alternative (equation 9).

\[ S_{ij} = \frac{SW_{ij}}{\sum_{i=1}^{n} SW_{ij}} \]
for each j=1 to m, i=1 to n \quad (8)

\[ I_{ij} = \frac{IW_{ij}}{\sum_{i=1}^{n} IW_{ij}} \]
for each j=1 to m, i=1 to n \quad (9)

**PILOT STUDY**

According to Smith & Winterfeldt (2004), the mathematical models developed using descriptive studies of human psychology and behavior can be judged by the extent to which the predictions done by such models correspond to the actual choices made by the DM. Hence, the effectiveness of the MCGL is tested by collecting response from 23 university students on a hypothetical mobile selection problem. We designed a hypothetical mobile selection problem with four alternatives and four criteria viz Camera size, Battery Talk time, Brand and Color of handset. The hierarchy is shown in Figure 1 and alternatives are shown in Table 2. We have chosen mobile selection problem because almost everyone carries a mobile phone and respondents are aware about the important features of mobile phones. The mode of response was paper and pencil. Response also collected from the students using AHP because AHP is one of the well established methods for MCDM.

When AHP is used to generate rank order of mobile phones, a total of 30 decisions were taken from each individual to form five pair-wise comparison matrices. When the MCGL is used to generate rank order of mobile phones, a total of 20 decisions collected from each individual. Students were also asked to give their preferred rank order (1-most preferred, 4-least preferred) of mobile phone considering all four criteria simultaneously. The rank order generated by the MCGL and the AHP are compared with direct rank order given by respondent. We used three validity measures (two for predictive validity and one for convergent validity) to test the effectiveness of AHP and MCGL for the same problem. 1) HIT1- is the percentage of respondents for whom the topmost rank generated by MCGL or AHP resemble the topmost rank given directly by them. 2) HIT2- is the percentage of respondents for whom the second rank
generated by MCGL or AHP resemble the second rank given directly by them. 3) CV - is the percentage of respondents for whom the ranks of all four mobile phones generated by MCGL or AHP resemble the rank order given directly by them. HIT1 and HIT2 are the measures of predictive validity while CV is the measure of convergent validity.

RESULTS

The main strength of AHP lies in pair-wise comparison based on the assumption that the human beings are good at comparing two things at a time. But when these comparisons require to be connected, it may give rise to problems of in-transitivity and inconsistency. In AHP, if the inconsistency measure in a pair wise comparison matrix exceeds 10%, the DM is advised to review his/her preferences in that matrix. In our study, we have not asked any of the students to review his/her preferences, even though average inconsistency found to be very high. Table 3 shows the average inconsistency among the 23 respondents for all 5 decision matrices. In MCGL, there is no problem of intransitivity as because the DM arranges the alternatives for a criterion in the order of preference and then rates them. Table 4 shows the validity measures. The resemblance between direct ranks and ranks obtained from MCGL found to be higher than resemblance between direct ranks and AHP ranks for all the three validity measures viz HIT1, HIT2 and CV. This implies that the MCGL is able to capture the human decision making process more accurately. Applicability measures like number of decisions required, time and cognitive burden strongly favors MCGL.

Figure 1
Decision hierarchy for mobile selection problem
CONCLUSION

A novel method for MCDA is proposed in this paper using some of the latest descriptive theories of decision making. The results of the empirical study shows that the prospect theory is able to model the decision making process of individuals under certainty also. The new non linear non compensatory aggregation model using the prospect theory and the complexity theory is able to resemble the decision making process of individuals. The new approach of aggregation is able to model how individuals analyze the two way interaction (across the criteria and across the alternatives) in the process of choice. The simple preference elicitation in MCGL reduces the gap between normative and descriptive requirements of MCDA.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Four alternatives with four criteria</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>Camera (mp) 2</td>
</tr>
<tr>
<td>B</td>
<td>Camera (mp) 10</td>
</tr>
<tr>
<td>C</td>
<td>Camera (mp) 4</td>
</tr>
<tr>
<td>D</td>
<td>Camera (mp) 6</td>
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<table>
<thead>
<tr>
<th>Table 3</th>
<th>Inconsistency (in %) in five decision matrices when using AHP</th>
</tr>
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<tbody>
<tr>
<td>Criteria Matrix</td>
<td>21.19%</td>
</tr>
<tr>
<td>Alternatives Matrix with respect to each criteria</td>
<td>40.28%</td>
</tr>
<tr>
<td>Alternatives Matrix with respect to each criteria</td>
<td>24.07%</td>
</tr>
<tr>
<td>Alternatives Matrix with respect to each criteria</td>
<td>17.61%</td>
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<tr>
<td>Alternatives Matrix with respect to each criteria</td>
<td>21.7%</td>
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<table>
<thead>
<tr>
<th>Table 4</th>
<th>Validity Measures</th>
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<tr>
<td>MCGL Rank Vs Direct Rank</td>
<td>Hit1 86.96%</td>
</tr>
<tr>
<td>AHP Rank vs Direct Rank</td>
<td>Hit1 69.57%</td>
</tr>
</tbody>
</table>

Hit1: Predictive Validity-Hit Rate for topmost rank
Hit2: Predictive Validity-Hit Rate for second rank
CV: Convergent Validity- Ranks of all four alternative
REFERENCES


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