LOGISTICS FACILITY SITE SELECTION USING FUZZY ANALYTIC HIERARCHY PROCESS

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ABSTRACT

Logistics facility location decisions are difficult to make as all parameters pertaining to the problem may not be deterministic. In addition, to realize significant cost savings and serve major markets the decision makers must understand the critical factors that drive logistics facility location selection. As a result, choosing the right logistics facility location site has become increasingly strategic decision for logistics companies competing in highly competitive business environments. In this study, we demonstrate how Fuzzy Analytic Hierarchy Process can be employed to identify the optimal facility location site for a logistics company. Potential feasible locations have been analyzed based on such criteria as proximity to the market and suppliers, workforce, investment cost, and growth potential, and the optimal location site has been identified based on a real business case.

Key Words: Fuzzy Analytic Hierarchy Process, Multi-criteria decision making, logistics, facility site selection.

INTRODUCTION

Logistics facility site selection has become strategically important in today’s highly competitive business environment as companies try to reduce costs of operations while increasing customer satisfaction and the quality of customer service. Hence, companies looking for the best logistics facility location need to evaluate factors such as proximity to markets and suppliers, labor costs, and transportation infrastructure. As companies become more focused on optimizing operational efficiency and reducing costs, locational determinants are becoming even more important. Primary locational determinants and the impact that they may have on prospective locational alternatives should be carefully studied using a facility location analysis.

Choosing the most suitable site with maximum efficiency and minimum operating costs for a logistics company involves a multi-criteria decision making process with multiple factors. When
identifying the optimal location, the situation becomes even more complicated as decision makers have to deal with multiple geographically dispersed logistics facility location sites.

Logistics facility location decisions are difficult to make as all parameters pertaining to the problem may not be deterministic. Further, ranking of the alternative sites and deciding on the most suitable and optimum location for a logistics company is a multi-criteria decision. As a result, choosing the right facility location site has become increasingly strategic decision for logistics companies competing in highly competitive business environments.

The main objective of this study is to determine the location of a logistics facility needed to serve a growing customer base. Although a number of models and approaches have been proposed for solving facility location problems, in this study, we demonstrate how Fuzzy Analytic Hierarchy Process can be employed to identify the optimal facility location site for a logistics company as it is one of the best models used to solve problems with multiple criteria, subjective factors, and linguistic variables. The criteria have been analyzed and the optimal location site has been identified based on a real business case using input from the firm’s management.

**BACKGROUND**

Selecting the most appropriate site for a logistics company is an important consideration. As suggested by (Ertugrul and Karakasoglu, 2007; Javid and Azad, 2010) decisions involving selecting a plant location are critical decisions for firms as they are costly and difficult to reverse, and they entail long term commitment. Correira et al., (2010) point out that a logistics facility location decision may have a significant impact on the company’s competitive position in terms of operating cost, customer service levels, and delivery speed performance. Hence, logistics facility site selection considerations have been extensively studied. For instance, Pirkul and Jayaraman (1998) proposed an integer programming model called the PLANWAR model to provide the optimal solution for a supply chain management problem with a multi-commodity and multi-plant facility location so that total operating costs are minimized. Similarly, Revelle and Eiselt (2005) focused on the mathematical science of facility selection and proposed various algorithms and formulations that can be employed in diverse settings in the private sector and the public sector.

Because facility site selection problems involve multiple criteria and objectives, various models such as the Analytic Hierarchy Process (AHP) have been offered to study such problems. Saaty (1980) has proposed AHP as a systematic method for comparing a list of objectives or alternatives. AHP has been used widely because it allows decision makers to solve problems having multiple criteria as it allows the user to systematically and carefully evaluate the importance of each criterion in relation to the others in a hierarchical manner (Levary and Wan, 1999). In the AHP, decision-makers make pairwise comparisons between different criteria to obtain values of their relative importance (Kordi and Brandt, 2012). In addition, the AHP model is a useful tool to tackle complex problems because of its ability to handle both qualitative and quantitative variables (Wang et al., 2012).

The AHP model has been verified in various environments. For example, Che et al., (2007) discuss and develop a manufacturing quality yield model for forecasting 12 in. silicon wafer
slicing based on the AHP framework. Their study demonstrates and verifies the feasibility and effectiveness of the proposed AHP-based algorithm.

Although the traditional AHP method has the ability to handle the input from multiple users, it is problematic in that it uses an exact value to express the decision maker’s opinion in a comparison of alternatives (Ertugrul and Karakasoglu, 2007). However, as pointed out by (Wang and Chen, 2007), human thoughts are full of uncertainty, so the decision-makers cannot make exact pairwise comparisons. Since the AHP model initially only dealt with exact values in the pairwise comparisons, later it has been modified and adapted to also consider fuzzy values (Kordi and Brandt, 2012). Moreover, because costs, demands, travel times, and other inputs to classical facility location models may be highly uncertain (Snyder, 2006), and AHP does not consider the uncertainty and the ambiguity associated with the judgment of the decision makers (Hauser and Tadikamalla, 1996), fuzzy set principle was integrated into AHP to control the uncertainty and to determine the best solution to a multi-criteria problem (Chen, 1996).

In addition to the AHP model, various other models for determining the optimal facility location have been proposed including stochastic models such as mean outcome models, mean variance models, and probabilistic models and robust models such as minimax costs models and minimax regret models (Snyder, 2006). However, these models are not suitable when it comes to dealing with vague nature of “linguistic variables”. Zadeh (1975) defines a linguistic variable as a variable whose values are words or sentences in a natural or artificial language. Thus, the fuzzy AHP model has been used to solve facility location problems having linguistic variables.

Fuzzy AHP model is a multi-criteria decision making model which has been successfully applied to solve numerous facility location selection problems. For instance, Kahraman et al., (2004) used fuzzy AHP to find the best location for a company providing catering services. Vahidnia et al. (2009) employed the same model to determine the optimal site for a new hospital. Similarly, Ka (2011) used fuzzy AHP to select the best dry port location in China using such criteria as transportation, economic level, infrastructure facilities, trade level, political environment, and cost. The fuzzy AHP model was also utilized by Turgut et al., (2011) to design a decision support system for determining the locations of disaster logistics centers. Choudhary and Shankar (2012) demonstrated how fuzzy AHP can be used for evaluation and selection of the most suitable and efficient location for a thermal power plant.

Recently, fuzzy AHP has been employed to demonstrate how it can be a useful tool when making decisions dealing with linguistic variables and uncertainties. For instance, Pang (2007) employed fuzzy AHP to measure the performance of a supplier by considering some linguistic variables and several factors and criteria. A study by Wang et al. (2012) blends fuzzy logic with AHP to incorporate uncertain parameters into the decision-making process and to form a decision-making model for different green initiatives in the fashion industry by analyzing the associated risk of different alternatives, subject to different factors that may be deterministic or not. Similarly, Shaw et al. (2012) used Fuzzy AHP and fuzzy multi-objective linear programming for selecting the appropriate supplier in the supply chain by analyzing the weights of the multiple factors such as cost, quality rejection percentage, late delivery percentage, greenhouse gas emission and demand. Finally, Ho et al (2012) used fuzzy AHP to evaluate and select the optimal third party logistics service providers.
These studies suggest the fuzzy AHP can be successfully used to deal with problems having linguistic variables, multiple subjective factors, and uncertainties.

**Triangular Fuzzy Numbers**

The major difficulty in most multi-criteria decision making models is that they fail to address the uncertainty and vagueness decision makers experience when solving a problem having ambiguity and fuzziness. To overcome this problem, Zadeh (1965) introduced the fuzzy sets theory, which can be used in a wide range of domains in which information is imprecise.

Triangular fuzzy numbers used in environments where information is incomplete are an important property of the Fuzzy-AHP. A triangular fuzzy number is the special class of a fuzzy number whose membership function is defined by three real numbers, expressed as \((l, m, u)\) (Figure 1) (Thalia et al., 2011).

Triangular fuzzy numbers may be expressed as

\[
\mu_\tilde{A}(x) = \begin{cases} 
0, & x < l \\
\frac{x-l}{m-l}, & l \leq x \leq m \\
\frac{u-x}{u-m}, & m \leq x \leq u \\
0, & x > u
\end{cases}
\]  

where \(l, m\), and, \(u\) are the lowest, best, and highest expected values, respectively.

In Fuzzy-AHP method pairwise comparisons are made using linguistic preference scale ranging from 1 to 9 (Thalia et al., 2011). For simplicity, the reciprocal fuzzy numbers are replaced by individual triangular fuzzy numbers in the pairwise comparison matrix. Triangular fuzzy numbers are compared according to their membership functions.

<table>
<thead>
<tr>
<th>Fuzzy Numbers</th>
<th>Membership functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1, 1, 2)</td>
</tr>
<tr>
<td>2</td>
<td>(1, 2, 3)</td>
</tr>
<tr>
<td>3</td>
<td>(2, 3, 4)</td>
</tr>
<tr>
<td>4</td>
<td>(3, 4, 5)</td>
</tr>
<tr>
<td>5</td>
<td>(4, 5, 6)</td>
</tr>
<tr>
<td>6</td>
<td>(5, 6, 7)</td>
</tr>
<tr>
<td>7</td>
<td>(6, 7, 8)</td>
</tr>
<tr>
<td>8</td>
<td>(7, 8, 9)</td>
</tr>
<tr>
<td>9</td>
<td>(8, 9, 9)</td>
</tr>
</tbody>
</table>
Figure 1. Triangular Fuzzy Number

The Steps of Fuzzy Analytic Hierarchy Process (FAHP)

The steps of FAHP originally developed by Chang (1996) can be summarized as in the following:

\[ M_{g_i}^{1}, M_{g_i}^{2}, \ldots, M_{g_i}^{m} \quad i = 1, 2, \ldots, n \]  

(2)

Where \( g_i \) is a goal set for the decision hierarchy and \( M_{g_i}^{j} \) (\( j = 1, 2, \ldots, m \)) are triangular fuzzy numbers of the decision matrix.

Step 1: Computing the fuzzy synthetic extend value \( (S_i) \) with respect to the \( i \)th object.

Let \( S_i = (S_{li}, S_{mi}, S_{ui}) \) where \( l, m, \) and, \( u \) are the lowest, best and highest expected values, respectively;

\[ S_i = \sum_{j=1}^{m} M_{g_i}^{j} \otimes \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1} \]  

(3)

To obtain the following expression

\[ \sum_{j=1}^{m} M_{g_i}^{j} \text{, which can be expressed as} \]

\[ \sum_{j=1}^{m} M_{g_i}^{j} = (\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j) \]  

(4)
Performing the fuzzy addition operation of $\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j$ for a particular matrix and $M_{gi}^j$ $(j = 1, 2, ..., m)$ values yields:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j = (\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i)$$  \hspace{1cm} (5)

To compute the inverse of the above vector the following formula is used:

$$\left(\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j\right)^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} l_i}, \frac{1}{\sum_{i=1}^{n} m_i}\right)$$  \hspace{1cm} (6)

**Step 2: Calculating the degree of possibility**

$$M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$$  \hspace{1cm} (7)

$$V (M_2 \geq M_1) = hgt (M_1 \cap M_2) = \mu_{M_2} (d)$$  \hspace{1cm} (8)

$$= \begin{cases} 1, & \text{if } m_2 \geq m_1 \\ 0, & \text{if } l_1 \geq u_2 \\ \frac{t_1-u_2}{(m_2-u_2)-(m_1-l_1)}, & \text{otherwise} \end{cases}$$  \hspace{1cm} (9)

where (d) is the ordinate of the maximum intersection point (D) between $\mu_{M_1}$ and $\mu_{M_2}$. To make a comparison between $M_1$ and $M_2$, both $V (M_2 \geq M_1)$ and $V (M_1 \geq M_2)$ values are needed.

**Step 3:**

The degree of possibility for a convex fuzzy number to be greater than K convex fuzzy numbers $M_i (i=1, 2, ..., k)$ may be defined as the following:

$$V (M \geq M_1, M_2, ..., M_k) = V[(M \geq M_1) \ and \ (M \geq M_2) \ and \ ... \ (M \geq M_k)]$$  \hspace{1cm} (10)

$$= \min V (M \geq M_i)$$  \hspace{1cm} \(i = 1, 2, 3, ..., k\)

$$k = 1, 2, 3, ..., n \ ; \neq j \ , \ assume \ that \ d'(A_i) = \min V (S_i \geq S_k) \ then$$

$$W' = \left(d'(A_1), d'(A_2), ..., d'(A_n)\right)^T$$
Step 4: Computing the normalized weight vector using the following formula:

\[ W = \left( d(A_1), d(A_2), \ldots, d(A_n) \right), \text{ where } W \text{ is a non-fuzzy number.} \]  

(11)

In the following section we employ the Fuzzy AHP model to find the most suitable location for a logistics company to better serve a growing customer base.

**RESEARCH METHOD**

**Company Profile**

This study was carried out at Ekol Logistics (http://www.ekol.com/en), headquartered in Istanbul, Turkey. With a population of almost 14 million, Istanbul is among the largest cities in the world. Ekol Logistics with a total of 300,000 m² closed area distribution centers, a fleet of 1100 vehicles, and more than 2,000 employees, is one of the leading providers of integrated logistics services in Turkey and Europe. Recently, the company has experienced a huge demand for its logistics services, which motivated this study.

**Method**

In this study, a new logistics facility site selection problem was solved. Criteria and alternatives determined by the firm’s management were evaluated using Saaty’s 1-9 scale along with their corresponding linguistic variables (Saaty, 1980).

The company decided to establish a new distribution center in a new city (region). The following four cities in Turkey were identified by the firm’s management a result of the initial assessment.

- Sultanbeyli (A1)
- Gebze (A2)
- Umraniye (A3)
- Tuzla (A4)

Having identified the cities, the following five evaluation criteria were identified.

- Proximity to the Market (C1)
- Proximity to Suppliers (C2)
- Workforce (C3)
- Investment Cost (C4)
- Growth Potential (C5)

Later, the problem is decomposed into three levels of hierarchy consisting of one goal, five criteria, and four alternatives (Figure 2).
In this study, we used the following linguistic variables and their corresponding fuzzy numbers (Table 2). Triangular fuzzy numbers, $\tilde{1}$ to $\tilde{9}$, are used to represent subjective pairwise comparisons of logistics facility location alternatives.

<table>
<thead>
<tr>
<th>Linguistic Variable</th>
<th>Triangular Fuzzy Numbers</th>
<th>Inverse of Triangular Fuzzy Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{1}$</td>
<td>(1, 1, 2)</td>
<td>$(1/2,1,1)$</td>
</tr>
<tr>
<td>$\tilde{2}$</td>
<td>(1, 2, 3)</td>
<td>$(1/3,1/2,1)$</td>
</tr>
<tr>
<td>$\tilde{3}$</td>
<td>(2, 3, 4)</td>
<td>$(1/4,1/3,1/2)$</td>
</tr>
<tr>
<td>$\tilde{4}$</td>
<td>(3, 4, 5)</td>
<td>$(1/5,1/4,1/3)$</td>
</tr>
<tr>
<td>$\tilde{5}$</td>
<td>(4, 5, 6)</td>
<td>$(1/6,1/5,1/4)$</td>
</tr>
<tr>
<td>$\tilde{6}$</td>
<td>(5, 6, 7)</td>
<td>$(1/7,1/6,1/5)$</td>
</tr>
<tr>
<td>$\tilde{7}$</td>
<td>(6, 7, 8)</td>
<td>$(1/8,1/7,1/6)$</td>
</tr>
<tr>
<td>$\tilde{8}$</td>
<td>(7, 8, 9)</td>
<td>$(1/9,1/8,1/7)$</td>
</tr>
<tr>
<td>$\tilde{9}$</td>
<td>(8, 9, 9)</td>
<td>$(1/9,1/9,1/8)$</td>
</tr>
</tbody>
</table>

**Data Analysis**

First, relative weights of each criterion were determined by making a pair-wise comparison for all the selected criteria at each level, and a matrix of pairwise comparisons was established. Then the weights of alternatives pertaining to each criterion are calculated, as shown in the tables in the following sections. Values were calculated using the matrix of pairwise comparisons of
synthetic benchmarks. In addition, synthetic weights of alternatives were calculated for each criterion.

Table 3: A matrix of comparisons among the criteria

<table>
<thead>
<tr>
<th></th>
<th>Proximity to the Market</th>
<th>Proximity to Suppliers</th>
<th>Workforce</th>
<th>Investment Cost</th>
<th>Growth Potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proximity to the Market</td>
<td>1,00 1,00 1,00</td>
<td>0,33 0,50 1,00</td>
<td>2,00 3,00 4,00</td>
<td>1,00 2,00 3,00</td>
<td>3,00 4,00 5,00</td>
</tr>
<tr>
<td>Proximity to Suppliers</td>
<td>1,00 2,00 3,00</td>
<td>1,00 1,00 1,00</td>
<td>3,00 4,00 5,00</td>
<td>2,00 3,00 4,00</td>
<td>4,00 5,00 6,00</td>
</tr>
<tr>
<td>Workforce</td>
<td>0,25 0,33 0,50</td>
<td>0,20 0,25 0,33</td>
<td>1,00 1,00 1,00</td>
<td>0,33 0,50 1,00</td>
<td>1,00 2,00 3,00</td>
</tr>
<tr>
<td>Investment Cost</td>
<td>0,33 0,50 1,00</td>
<td>0,25 0,33 0,50</td>
<td>1,00 2,00 3,00</td>
<td>1,00 1,00 1,00</td>
<td>2,00 3,00 4,00</td>
</tr>
<tr>
<td>Growth Potential</td>
<td>0,20 0,25 0,33</td>
<td>0,17 0,20 0,25</td>
<td>0,33 0,50 1,00</td>
<td>0,25 0,33 0,50</td>
<td>1,00 1,00 1,00</td>
</tr>
</tbody>
</table>

Evaluation of Criteria

Each city was evaluated in terms of the five criteria. This analysis allowed the firm’s management to see which city offers better opportunities.

**Proximity to the Market**: It is important that the facility is established very close to the regions of potential customers both in terms of enhancing customer satisfaction and reducing costs.

Table 4: Proximity to the Markets

<table>
<thead>
<tr>
<th>Proximity to the Market</th>
<th>Sultanbeyli</th>
<th>Gebze</th>
<th>Umraniye</th>
<th>Tuzla</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sultanbeyli</td>
<td>1,00 1,00 1,00</td>
<td>3,00 4,00 5,00</td>
<td>1,00 2,00 3,00</td>
<td>2,00 3,00 4,00</td>
</tr>
<tr>
<td>Gebze</td>
<td>0,20 0,25 0,33</td>
<td>1,00 1,00 1,00</td>
<td>0,25 0,33 0,50</td>
<td>0,33 0,50 1,00</td>
</tr>
<tr>
<td>Umraniye</td>
<td>0,33 0,50 1,00</td>
<td>2,00 3,00 4,00</td>
<td>1,00 1,00 1,00</td>
<td>1,00 2,00 3,00</td>
</tr>
<tr>
<td>Tuzla</td>
<td>0,25 0,33 0,50</td>
<td>1,00 2,00 3,00</td>
<td>0,33 0,50 1,00</td>
<td>1,00 1,00 1,00</td>
</tr>
</tbody>
</table>

Having analyzed the alternatives in terms of "Proximity to the market" criterion, Sultanbeyli emerges as the best alternative. Umraniye is slightly less important than Sultanbeyli. These alternatives are followed by Gebze and Tuzla, respectively.

**Proximity to Suppliers**: Goods to be provided to the logistics company by suppliers holds great importance. Therefore, proximity to suppliers is an important criterion.
Table 5: Proximity to Suppliers

<table>
<thead>
<tr>
<th>Proximity to Suppliers</th>
<th>Sultanbeyli</th>
<th>Gebze</th>
<th>Umraniye</th>
<th>Tuzla</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sultanbeyli</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td></td>
</tr>
<tr>
<td>Gebze</td>
<td>0,25</td>
<td>0,33</td>
<td>0,33</td>
<td></td>
</tr>
<tr>
<td>Umraniye</td>
<td>0,33</td>
<td>0,50</td>
<td>1,00</td>
<td></td>
</tr>
<tr>
<td>Tuzla</td>
<td>0,20</td>
<td>0,25</td>
<td>0,33</td>
<td></td>
</tr>
</tbody>
</table>

Proximity to suppliers criterion is similar to the criteria of proximity to the market. Sultanbeyli is the best alternative, followed by Umraniye, Gebze and Tuzla, respectively.

**Investment Cost:** Costs for the land where the facility is to be established and geotechnical and environmental conditions are important criteria for a company.

Table 6: Investment Cost

<table>
<thead>
<tr>
<th>Investment Cost</th>
<th>Sultanbeyli</th>
<th>Gebze</th>
<th>Umraniye</th>
<th>Tuzla</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sultanbeyli</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td></td>
</tr>
<tr>
<td>Gebze</td>
<td>2,00</td>
<td>3,00</td>
<td>4,00</td>
<td></td>
</tr>
<tr>
<td>Umraniye</td>
<td>0,33</td>
<td>0,50</td>
<td>1,00</td>
<td></td>
</tr>
<tr>
<td>Tuzla</td>
<td>1,00</td>
<td>2,00</td>
<td>3,00</td>
<td></td>
</tr>
</tbody>
</table>

Due to low land prices, Gebze appears to be the best alternative, followed by Tuzla, Sultanbeyli, and Umraniye in descending order of importance.

**Workforce:** It is important for businesses to easily find talented staff to work. In addition, the workers' wage expectations may vary according to the region's economic conditions.

Table 7: Workforce

<table>
<thead>
<tr>
<th>Workforce</th>
<th>Sultanbeyli</th>
<th>Gebze</th>
<th>Umraniye</th>
<th>Tuzla</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sultanbeyli</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td></td>
</tr>
<tr>
<td>Gebze</td>
<td>4,00</td>
<td>5,00</td>
<td>6,00</td>
<td></td>
</tr>
<tr>
<td>Umraniye</td>
<td>1,00</td>
<td>2,00</td>
<td>3,00</td>
<td></td>
</tr>
<tr>
<td>Tuzla</td>
<td>3,00</td>
<td>4,00</td>
<td>5,00</td>
<td></td>
</tr>
</tbody>
</table>
In terms of workforce, Gebze appears to be the best alternative. This option is followed by less important Tuzla. In decreasing order of importance are then Umranıye and Sultanbeyli.

**Growth Potential:** The long term growth opportunity should be carefully evaluated as the company will plan on expanding its operations.

<table>
<thead>
<tr>
<th>Growth Potential</th>
<th>Sultanbeyli</th>
<th>Gebze</th>
<th>Umranıye</th>
<th>Tuzla</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sultanbeyli</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Gebze</td>
<td>0.25</td>
<td>0.33</td>
<td>0.50</td>
<td>1.00</td>
</tr>
<tr>
<td>Umranıye</td>
<td>0.33</td>
<td>0.50</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Tuzla</td>
<td>1.00</td>
<td>2.00</td>
<td>3.00</td>
<td>4.00</td>
</tr>
</tbody>
</table>

Based on the criteria of growth potential, Tuzla appears to be the most favorable option, followed by Sultanbeyli, Umranıye and Gebze, respectively.

**The Values of Fuzzy Synthetic Extend of all Criteria**

Because fuzzy AHP relies on the computation of fuzzy synthetic extent values, after the initial assessments as spelled out above, fuzzy synthetic extent values of all criteria were computed.

\[ S_1 = (7.33, 10.50, 14.00) \otimes (27.64, 38.69, 51.41)^{-1} \]

\[ S_1 = (7.33/51.41, 10.50/38.69, 14.00/27.64) = (0.143, 0.271, 0.507) \]

\[ S_2 = (11.00, 15.00, 19.00) \otimes (27.64, 38.69, 51.41)^{-1} \]

\[ S_2 = (11.00/51.41, 15.00/38.69, 19.00/27.64) = (0.214, 0.388, 0.687) \]

\[ S_3 = (2.78, 4.08, 5.83) \otimes (27.64, 38.69, 51.41)^{-1} \]

\[ S_3 = (2.78/51.41, 4.08/38.69, 5.83/27.64) = (0.054, 0.105, 0.211) \]

\[ S_4 = (4.58, 6.83, 9.50) \otimes (27.64, 38.69, 51.41)^{-1} \]

\[ S_4 = (4.58/51.41, 6.83/38.69, 9.50/27.64) = (0.089, 0.176, 0.344) \]

\[ S_5 = (1.95, 2.28, 3.08) \otimes (27.64, 38.69, 51.41)^{-1} \]

\[ S_5 = (1.95/51.41, 2.28/38.69, 3.08/27.64) \]
\[ S_5 = (0.038, 0.059, 0.111) \]

Then, using the fuzzy synthetic extend values, the importance of criteria weights are calculated.

Because \( m_i \geq m_j \) for \( V(M_i \geq M_j) = 1 \):

- \( V(S_1 > S_2) = 1 \)
- \( V(S_1 > S_3) = 1 \)
- \( V(S_1 > S_4) = 1 \)
- \( V(S_1 > S_5) = 1 \)
- \( V(S_2 > S_1) = 1 \)
- \( V(S_2 > S_3) = 1 \)
- \( V(S_2 > S_4) = 1 \)
- \( V(S_2 > S_5) = 1 \)
- \( V(S_3 > S_2) = 1 \)
- \( V(S_3 > S_4) = 1 \)
- \( V(S_3 > S_5) = 1 \)
- \( V(S_4 > S_3) = 1 \)
- \( V(S_4 > S_5) = 1 \)

\( u_i < l_j \) for \( V(M_i > M_j) = 0 \):

- \( V(S_3 > S_2) = 0 \)
- \( V(S_5 > S_1) = 0 \)
- \( V(S_5 > S_2) = 0 \)

And, using the following formula

\[
\mu = \frac{l_2 - u_1}{(l_2 - m_2) - (u_1 - m_1)}
\]

- \( V(S_1 > S_2) = (0.214 - 0.507)/(0.214 - 0.338) - (0.507 - 0.271) = 0.814 \)
- \( V(S_3 > S_1) = (0.143 - 0.211)/(0.143 - 0.271) - (0.211 - 0.105) = 0.290 \)
- \( V(S_3 > S_4) = (0.089 - 0.211)/(0.089 - 0.176) - (0.211 - 0.105) = 0.632 \)
- \( V(S_4 > S_1) = (0.143 - 0.344)/(0.143 - 0.271) - (0.344 - 0.176) = 0.679 \)
- \( V(S_4 > S_2) = (0.214 - 0.344)/(0.214 - 0.388) - (0.344 - 0.176) = 0.380 \)
- \( V(S_5 > S_3) = (0.054 - 0.111)/(0.054 - 0.105) - (0.111 - 0.059) = 0.553 \)
- \( V(S_5 > S_4) = (0.089 - 0.111)/(0.089 - 0.176) - (0.111 - 0.059) = 0.158 \)

Thus, weights indicating the degree of preference are computed as follows.

- \( V(S_1 > S_2, S_3, S_4, S_5) = \min (0.814, 1, 1, 1) = 0.814 \)
- \( V(S_1 > S_2, S_3, S_4, S_5) = \min (1, 1, 1, 1) = 1 \)
- \( V(S_3 > S_1, S_2, S_4, S_5) = \min (0.290, 0, 0.632, 1) = 0 \)
- \( V(S_4 > S_1, S_2, S_3, S_5) = \min (0.679, 0.380, 1, 1) = 0.380 \)
- \( V(S_5 > S_1, S_2, S_3, S_4) = \min (0, 0, 0.553, 0.158) = 0.158 \)
\( W' = (0.814, 1, 0, 0.380, 0.158) \)

If we normalize the vector of weights, we find
\[ W = (0.346, 0.425, 0, 0.162, 0.067) \]

The same procedure is followed to compute the other matrices.

**Proximity to the Market: fuzzy synthetic extent values**

\[
S_1 = (7.00, 10.00, 13.00) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_1 = (7.00/30.33, 10.00/22.41, 13.00/15.69) \\
S_1 = (0.231, 0.446, 0.829) \\
S_2 = (1.78, 2.08, 2.83) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_2 = (1.78/30.33, 2.08/22.41, 2.83/15.69) \\
S_2 = (0.059, 0.093, 0.180) \\
S_3 = (4.33, 6.50, 9.00) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_3 = (4.33/30.33, 6.50/22.41, 9.00/15.69) \\
S_3 = (0.143, 0.290, 0.574) \\
S_4 = (2.58, 3.83, 5.50) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_4 = (2.58/30.33, 3.83/22.41, 5.50/15.69) \\
S_4 = (0.085, 0.171, 0.351)
\]

\( m_i \geq m_j \) for \( V(M_i \geq M_j) = 1; \)
- \( V(S_1 > S_2) = 1 \)
- \( V(S_1 > S_3) = 1 \)
- \( V(S_1 > S_4) = 1 \)
- \( V(S_3 > S_2) = 1 \)
- \( V(S_3 > S_4) = 1 \)
- \( V(S_4 > S_2) = 1 \)

\( u_i < l_j \) for \( V(M_i > M_j) = 0; \)
- \( V(S_2 > S_1) = 0 \)

And,
\[
\mu = \frac{l_2 - u_1}{(l_2 - m_2) - (u_1 - m_1)}
\]

- \( V(S_2 > S_3) = 0.158 \)
- \( V(S_2 > S_4) = 0.549 \)
- \( V(S_3 > S_1) = 0.687 \)
- \( V(S_4 > S_1) = 0.304 \)
- \( V(S_4 > S_3) = 0.636 \)
\[ W' = (1, 0, 0.687, 0.304); \]

\[ W_1 = (0.502, 0, 0.345, 0.153). \]

**Proximity to Suppliers: fuzzy synthetic extent values**

\[
S_1 = (7.00, 10.00, 13.00) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_1 = (7.00/30.33, 10.00/22.41, 13.00/15.69) \\
S_1 = (0.231, 0.446, 0.829) \\
S_2 = (2.58, 3.83, 5.50) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_2 = (2.58/30.33, 3.83/22.41, 5.50/15.69) \\
S_2 = (0.085, 0.171, 0.351) \\
S_3 = (4.33, 6.50, 9.00) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_3 = (4.33/30.33, 6.50/22.41, 9.00/15.69) \\
S_3 = (0.143, 0.290, 0.574) \\
S_4 = (1.78, 2.08, 2.83) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_4 = (1.78/30.33, 2.08/22.41, 2.83/15.69) \\
S_4 = (0.059, 0.093, 0.180)
\]

- \[ V(S_2 > S_1) = 0.304 \]
- \[ V(S_2 > S_3) = 0.636 \]
- \[ V(S_3 > S_1) = 0.687 \]
- \[ V(S_4 > S_3) = 0.158 \]
- \[ V(S_4 > S_2) = 0.549 \]

\[ W' = (1, 0.304, 0.687, 0); \]

\[ W_2 = (0.502, 0.153, 0.345, 0). \]

**Proximity to Workforce: fuzzy synthetic extent values**

\[
S_1 = (1.70, 1.95, 2.58) \otimes (19.48, 26.03, 33.41)^{-1} \\
S_1 = (1.70/33.41, 1.95/26.03, 2.58/19.48) \\
S_1 = (0.051, 0.075, 0.132) \\
S_2 = (9.00, 12.00, 15.00) \otimes (19.48, 26.03, 33.41)^{-1} \\
S_2 = (9.00/33.41, 12.00/26.03, 15.00/19.48) \\
S_2 = (0.269, 0.461, 0.770) \\
S_3 = (2.45, 3.58, 4.83) \otimes (19.48, 26.03, 33.41)^{-1} \\
S_3 = (2.45/33.41, 3.58/26.03, 4.83/19.48) \\
S_3 = (0.073, 0.138, 0.248) \\
S_4 = (6.33, 8.50, 11.00) \otimes (19.48, 26.03, 33.41)^{-1} \\
S_4 = (6.33/33.41, 8.50/26.03, 11.00/19.48) \\
S_4 = (0.189, 0.327, 0.565)
\]

- \[ V(S_1 > S_3) = 0.484 \]
- \[ V(S_3 > S_4) = 0.238 \]
- \[ V(S_4 > S_2) = 0.688 \]
\( W' = (0, 1, 0, 0.688); \)
\( W_3 = (0, 0.592, 0, 0.408). \)

**Proximity to Investment Cost: fuzzy synthetic extent values**

\[
\begin{align*}
S_1 &= (2.58, 3.83, 5.50) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_1 &= (2.58/30.33, 3.83/22.41, 5.50/15.69) \\
S_1 &= (0.085, 0.171, 0.351) \\
S_2 &= (7.00, 10.00, 13.00) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_2 &= (7.00/30.33, 10.00/22.41, 13.00/15.69) \\
S_2 &= (0.231, 0.446, 0.829) \\
S_3 &= (1.78, 2.08, 2.83) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_3 &= (1.78/30.33, 2.08/22.41, 2.83/15.69) \\
S_3 &= (0.059, 0.093, 0.180) \\
S_4 &= (4.33, 6.50, 9.00) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_4 &= (4.33/30.33, 6.50/22.41, 9.00/15.69) \\
S_4 &= (0.143, 0.290, 0.574) \\
\end{align*}
\]

- \( V(S_1 > S_2) = 0.304 \)
- \( V(S_1 > S_3) = 0.636 \)
- \( V(S_2 > S_3) = 0.158 \)
- \( V(S_3 > S_1) = 0.549 \)
- \( V(S_4 > S_2) = 0.687 \)

\( W'' = (0.304, 1, 0, 0.687); \)
\( W_4 = (0.153, 0.502, 0, 0.345). \)

**Proximity to Growth Potential: fuzzy synthetic extent values**

\[
\begin{align*}
S_1 &= (4.33, 6.50, 9.00) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_1 &= (4.33/30.33, 6.50/22.41, 9.00/15.69) \\
S_1 &= (0.143, 0.290, 0.574) \\
S_2 &= (1.78, 2.08, 2.83) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_2 &= (1.78/30.33, 2.08/22.41, 2.83/15.69) \\
S_2 &= (0.059, 0.093, 0.180) \\
S_3 &= (2.58, 3.83, 5.50) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_3 &= (2.58/30.33, 3.83/22.41, 5.50/15.69) \\
S_3 &= (0.085, 0.171, 0.351) \\
S_4 &= (7.00, 10.00, 13.00) \otimes (15.69, 22.41, 30.33)^{-1} \\
S_4 &= (7.00/30.33, 10.00/22.41, 13.00/15.69) \\
S_4 &= (0.231, 0.446, 0.829) \\
\end{align*}
\]

- \( V(S_1 > S_4) = 0.687 \)
- \( V(S_2 > S_1) = 0.158 \)
- \( V(S_2 > S_3) = 0.549 \)
- \( V(S_3 > S_1) = 0.636 \)
V(S_3 > S_4) = 0.304

W' = (0.687, 0, 0.304, 1);

W_5 = (0.345, 0, 0.153, 0.502).

The weight vectors of the alternatives based on the criteria are given in table 9.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Weight Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>W_1</td>
<td>(0.502, 0, 0.345, 0.153)</td>
</tr>
<tr>
<td>W_2</td>
<td>(0.502, 0.153, 0.345, 0)</td>
</tr>
<tr>
<td>W_3</td>
<td>(0, 0.592, 0, 0.408)</td>
</tr>
<tr>
<td>W_4</td>
<td>(0.153, 0.502, 0, 0.345)</td>
</tr>
<tr>
<td>W_5</td>
<td>(0.345, 0, 0.153, 0.502)</td>
</tr>
</tbody>
</table>

Combined weights of criteria and alternatives are given below

\[
\begin{bmatrix}
0.502 & 0 & 0.345 & 0.153 \\
0.502 & 0.153 & 0 & 0.345 \\
0 & 0.592 & 0 & 0.408 \\
0.153 & 0.502 & 0 & 0.345 \\
0.345 & 0 & 0.153 & 0.502 \\
\end{bmatrix} = \begin{bmatrix}
0.346 & 0.425 & 0 & 0.162 & 0.067 \\
\end{bmatrix} \cdot \begin{bmatrix}
0.435 & 0.146 & 0.276 & 0.142 \\
\end{bmatrix}
\]

Alternatives and their weights are given in table 10.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sultanbeyli</td>
<td>0.435</td>
</tr>
<tr>
<td>Gebze</td>
<td>0.146</td>
</tr>
<tr>
<td>Umrahiye</td>
<td>0.276</td>
</tr>
<tr>
<td>Tuzla</td>
<td>0.142</td>
</tr>
</tbody>
</table>

Having computed the weights, it appears the city of Sultanbeyli is the best option for establishing the new distribution center, followed by Umrahiye, and Gebze. The executives who supervised this project agree that Sultanbeyli is the most suitable location due to fact that it has easy access to major highways, it offers highly skilled workforce, and low investments cost. Although it is very close to a sea port in the Sea of Marmara, the city of Tuzla, which has the lowest weight, is the least preferred option due to some other unfavorable factors such as high investment costs.

**DISCUSSIONS AND CONCLUSIONS**

Logistics facility location selection is a multi-criteria decision problem involving a number of quantitative and linguistic variables. One would agree that because the selection is costly and
difficult to reverse, it must be based on not only current needs but also on projected needs. In addition, to realize significant cost savings and serve major markets the decision makers must understand the critical factors that drive logistics facility location selection. Thus, location decisions reflect the specific requirements of a company as it focuses on optimizing a number of different objectives.

Although decision makers in the past have used various models to solve complex facility location problems, due to the increased recognition of uncertainties faced by decision makers, and because the traditional models may not be suitable for dealing with vagueness, ambiguity, and imprecision inherent in linguistic variables, later the fuzzy AHP model that reflects the fuzziness between variables has been developed and used to solve logistics facility location selection problems involving linguistic variables and multiple criteria.

In this study, a facility site selection study was carried out at a logistics company employing the fuzzy AHP model. The five criteria used in this study are proximity to the market, proximity to suppliers, workforce, investment cost, and growth potential. Having identified the linguistic variables, triangular fuzzy numbers, $[1/9, 5/9, 9/9]$, are used to represent subjective pairwise comparisons of logistics facility location alternatives. Criteria and weights relative to each other were examined. Then, the weight vectors of the alternatives were examined to decide on the most suitable option. As a result, the city of Sultanbeyli appeared to be the best location. These results are consistent with the view and judgments of the executives supervised this project.

While the literature presents numerous models and methods used to solve problems having multiple criteria, as seen in this study, the fuzzy AHP model is a practical and effective assessment method for dealing with vague nature of linguistic variables and subjective factors used when selecting a facility location.

**REFERENCES**


Pang, B. Multi-criteria Supplier Evaluation Using Fuzzy AHP, Proceedings of the IEEE International Conference on Mechatronics, and Automation, August 5 - 8, 2007, Harbin, China


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