SUPPLY CHAIN PLANNING AT A CHEMICAL PROCESS INDUSTRY

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ABSTRACT

The purpose of this paper is to develop a mathematical optimization model that can be used as a
decision support tool for the supply chain planning at Perstorp Oxo AB, a global company in the
process industry. At their site in Stenungsund, Perstorp Oxo AB produce chemicals to customers
in a variety of branches and for further refinement at other Perstorp sites in Gent, Castellanza and
Perstorp. The customers are mainly in branches such as food and feed, leather and textile, plastic
and safety glass production. Since Perstorp Oxo sells products to customers worldwide, two
large inventory facilities are located in Antwerp (Belgium) and Tees (United Kingdom) for five
product types each and two smaller facilities in Philadelphia (USA) and Aveiro (Portugal) for
one type respectively. The developed model is a mixed-integer linear program, where the
objective function maximizes the profit margin, that is, the difference between the selling price
and the cost of production, transportation, inventory carrying and outsourcing. A solution to the
model shows the quantities to be transported between the different sites, production rates,
inventory levels, setups and purchases from external suppliers, each with its respective cost. The
results of a baseline scenario show that there is a potential to increase profit margin by using a
decision support tool based on an optimization model.

Keywords: Process Industry, Supply Chain, Mixed Integer Programming, Optimization

INTRODUCTION

At Perstorp Oxo AB in Stenungsund, production of chemical intermediates is carried out. These
products are delivered to industrial customers, where they are added to other products used in
daily life, for example protective glass and windscreens in cars. It is not an easy task to plan the
production, inventories and transportation of these products with regard to variations in demand,
selling price and the availability of the production plants. Today, Perstorp Oxo AB does not use
any advanced planning tools or optimization algorithms when planning the supply chain at the
site in Stenungsund. It is believed that by having an optimization tool for the planning of the
supply chain, the supply chain at the site could be improved and be more profitable.
The need to involve profit margins in the planning process is crucial to make the supply chain more efficient. According to Shah (2005), the supply chain benchmarks for the process industries do not measure up to other sectors. The author gives examples of such: high inventory levels throughout the whole supply chain, high supply chain cycle times and low material efficiencies, meaning that in some branches, only a small proportion of material entering the supply chain ends up as end products. The companies within the process industry are therefore striving to improve their efficiency and responsiveness in order to be more competitive. The need for optimization and more efficient use of available resources through a supply chain perspective is therefore necessary.

There are many examples of supply chain optimization in the literature. For a broad overview see Grossman (2004), and a more recent review of the area in Barbosa-Póvoa (2012). Examples of industrial cases can be found in Stadtler and Kilger (2005). In his doctoral thesis (Persson, 2002), Persson deals with the production scheduling and shipment planning of oil refineries, in cooperation with the Nynas refinery located in Nynäshamn, Sweden. The scheduling problem of an ethylene plant is described in Tjoa et al. (1997), and examples of forest supply chain optimization are found in Bredström et al. (2004) and Gunnarsson et al. (2007).

The purpose of this paper is to develop a mathematical optimization model that can be used as a decision support tool for the supply chain planning at Perstorp Oxo AB. In Section 2 we describe the problem at hand, and in Section 3 a mathematical model is derived. In Section 4 results are presented, and finally conclusions and future work are found in Section 5.

**PROBLEM DESCRIPTION**

The Perstorp group has 13 production sites located around the world while the majority of the employees are located in the EU, see Figure 1. Perstorp Oxo AB, formerly named Neste Oxo before the integration in the Perstorp group, produces oxo products such as aldehydes, alcohols, esters and acids mainly for use in the water-based paint and lacquer industries.

![Figure 1: Perstorp sites world wide.](image)
The site at Stenungsund, which is the focus of this paper, is divided into 8 areas and produces 11 different products. Each area has the capability of producing one or more products. Stenungsund is the petrochemical centre of Sweden, which means that the site is close to many of its suppliers. The first step is the production of products P1 and P2 which is performed at Area 1, and product P3 at Area 2. This is done using a synthesis gas R1 which is mixed with two other reactants, R2 and R3, respectively. These are the basis for further processing of which some is carried out at other sites in the Perstorp group. Figure 2 shows the production schemes for the products produced at each area in Stenungsund respectively and the raw materials used.

![Production schemes for site Stenungsund](image)

Figure 2: The production schemes for each area. One or two reactants are combined to produce a new product. For many areas, one of the reactants are one of the earlier products.

**Supply Chain at Perstorp**

The supply chain at Perstorp Oxo AB in Stenungsund is described in Figure 3, which shows the key relationships of produced products and their distribution. According to the general classification by Acar et al. (2009), this is a deterministic supply chain model since no consideration to uncertainty is taken. Further, process manufacturing is defined in Cox and Blackstone (2002) as production which adds value by mixing, separating, forming, and/or chemical reactions, performed in either batch or continuous mode. This is certainly true for the production at the site in Stenungsund, and it is performed in both batch and continuous mode.

Products P1, P2 and P3 constitute the basis for other products produced in Stenungsund and at other sites in the Perstorp group. Products P2 and P3 are also sold to external customers. The production of P1, P2 and P3 are closely connected by the earlier steps in the production, that is the production of steam and synthesis gas, which adds requirements on the internal relationships of P1, P2 and P3 production.

The flow disperses from the production of the aldehydes to other plants. Product P1 is used at Area 6 to produce P8 and in the production of another product (not considered here) at the site in Perstorp. Product P9 is also produced at Area 6, where the key reactant is product P2, and it is
delivered to the regional inventory facilities in Tees and Antwerp. These facilities only stock products which are to be delivered to customers on the respective market. Product P2 is also used for the production of another product (not considered here) at the site in Perstorp, and in Areas 3 and 4 where products P4 and P5 are produced respectively.

Figure 3: Relations within Perstorp. The product flow, both within areas and transportations between Perstorp locations and customers. All products produced in Stenungsund, except P1, can be sold directly to External Customers on the spot market. Products P7 and P8 are only sold this way. All other products are sold to contract customers from different locations around the world.

The chemicals produced in Area 3 and Area 4 are further processed within the Perstorp group; product P5 is used at Area 5 to produce product P6, and product P4 is used both at Area 7 in Stenungsund where product P10 is produced, and at the site in Gent in the production of another product (not considered here). At Area 5, where product P6 is produced, the production resource must be shared with the production of product P7, which uses reactants from external suppliers. Product P7 is only delivered from Stenungsund directly to external customers, while products P6 and P10 are delivered by boat to the facilities in Tees and Antwerp. Product P3 is the third aldehyde produced at site Stenungsund, used as a reactant to produce product P11 at Area 8. Products P3 and P11 are both sold to external customers and also transported to the production
site in Castellanza (Italy) where they are used in the production of other products. Product P11 is also transported by boat to the regional inventories in Tees and Antwerp. Further, from the inventory in Antwerp, product P6 is shipped by boat to Aveiro (Portugal) and product P10 is shipped by boat to Philadelphia (USA). All products, except product P1, are also sold directly to external customers from the site in Stenungsund, although to a varying extent.

**Transports**

The planning must also be carried out with respect to transportation costs, especially the boat transports to Tees and Antwerp which constraints the planning. These boats are managed by a third party and are normally booked by Perstorp with a 14 days planning horizon, based on actual sales orders and on the forecasts of production and demand. Different boats are used for different paths, all with different capacities ranging from 2,500 to 3,500 tons. To simplify the model, we set boat capacity to 2,700 tons. Boats on these routes arrive within one or two days to their destination.

To Aveiro and Philadelphia, the available capacities are 400-800 and 800-1,200 tons respectively. Again, to simplify the model, we only consider boats with the most frequently used capacity of 800 tons for these paths. These longer journeys take 1 and 2 weeks, respectively, before the boats arrive at their destination. Table 1 shows the boat capacities, fixed costs, and transportation time for each destination.

<table>
<thead>
<tr>
<th>Destination</th>
<th>Capacity</th>
<th>Cost</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tees</td>
<td>2,700</td>
<td>45,000</td>
<td>-</td>
</tr>
<tr>
<td>Antwerp</td>
<td>2,700</td>
<td>45,000</td>
<td>-</td>
</tr>
<tr>
<td>Aveiro</td>
<td>800</td>
<td>33,000</td>
<td>1</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>800</td>
<td>54,000</td>
<td>2</td>
</tr>
</tbody>
</table>

It is obviously desirable to use a boat’s full capacity, but one must also regard the cost of tied up capital in inventories on board the boats and how production of other products is affected.

The planning of land transports does not have an equally great impact on the planning of the production, although also carried out by a third party. The available truck capacities to Castellanza, Gent and Perstorp, respectively, are 26, 26 and 40 tons. The cost of a truck is dependent of the distance, and the transportation costs of products to Castellanza and Gent are approximately 2.1 and 1.7 times greater than the cost of shipping products to Perstorp, respectively.

**Fluctuating Demand**

Products with seasonal fluctuations and production campaigns affect the planning. The production at the site in Castellanza is carried out in campaigns, and each campaign requires product P11 from Area 8, which in turn requires product P3 as input. Due to shared utilities not considered here, such as high pressure systems and water cooling systems, this limits the
production of products P1 and P2 in Area 1, and hence needs to be taken into consideration in the planning.

To handle seasonal fluctuations, building up inventories is sometimes possible. However, small tank sizes limit the possibility of carrying large inventories, which also means high costs of tied up capital in inventories.

For other products, there is a world-wide demand that exceeds the global supply. There are several reasons for why the supply does not meet the demand. For some products, there are only a handful of producers and Perstorp and its competitors produce less than demanded in order to keep the prices high and steady. The products are sold on a spot market where the prices generally are higher than the prices to the contract customers. Selling products on the spot market is associated with risks. The contract customers are committed to fixed prices and quantities and are also preferred by suppliers such as Perstorp AB since this type of customers decrease the risk and ease forecasting and planning. The important aspect of the spot market is therefore that margin profits are higher for every sold unit compared to sales to contract customers. Spot market sales are only possible though when the committed quantities to contract customers have been fulfilled, and only up to a certain limit.

Cost Structure and Profit Description

The sales prices of the end products are determined by the global market prices. The ICIS organization (ICIS, 2013) conducts market research in order to define the current market price. This price, provided by the ICIS organization, is the base for the price setting towards Perstorp AB's customers. In order to calculate the margin profit of one sold ton of a product, the production costs and the costs of raw materials must be known. Also, to calculate the inventory carrying costs, the production costs must be known beforehand.

At Perstorp, the direct costs are aggregated to the direct variable cost (DVC), which includes the production costs and the cost of raw material. The DVC is also dependent of the production rate, since there are fixed costs included. The DVC is constantly measured and it varies over time. The main reason for large variations depends on the cost of raw material. However, since the sales price also depends on the cost of raw materials, the marginal profit is unchanged. Further costs to include are, as mentioned above, the transport costs within the Perstorp organization.

For Perstorp AB, currency fluctuations also affect the profit. However, in order to simplify the model, we have chosen to not take this into consideration.

MATHEMATICAL MODEL

The mathematical model for the supply chain planning at Perstorp is a linear mixed integer programming model. We start by introducing some notation, defining the necessary sets and variables to be used. In the following subsections, the constraints and the objective function are described. All the parameters used in the model are introduced and explained as they appear in the constraints and objective function.
Notations

Sets

\[ P \] Products \( P_1, \ldots, P_{11} \)
\[ A \] Areas \( A_1, \ldots, A_8 \)
\[ D_T \] Truck destinations Castellanza, Gent, Perstorp
\[ D_B \] Boat destinations Antwerp, Tees, Aveiro, Philadelphia
\[ L \] Locations \( L := D_T \cup D_B \)
\[ T \] Time periods Week 27 to week 52
\[ P_U \] Unlimited spot market P4, P6, P9

\[ P_A[k] \] Set of products \( p \) that are produced at each area \( k \)
\[ P_Q[p] \] Set of products \( q \) that require product \( p \) as input

\( \Gamma \) Transportation flows: triplets \( (p, l, k) \) for possible transports of product \( p \) from location \( l \) to destination \( k \)
\( Q \) Buy from External customer: tuples \( (p, k) \) of products \( p \) bought to location \( k \)
\( Y \) Spot market sales: tuples \( (p, k) \) of valid combinations of product \( p \) and location \( k \)

Variables

Continuous

\( x_{pt} \) Volume of produced product \( p \) in time period \( t \)
\( y_{pkt} \) Volume of sold product \( p \) to spot market customer \( k \) in time period \( t \)
\( I_{pkt} \) Inventory level of product \( p \) at location \( k \) at the end of time period \( t \)
\( T_{plkt} \) Volume of transported product \( p \) from location \( l \) to inventory \( k \) in time period \( t \)
\( Q_{pkt} \) Volume of product \( p \) purchased from external suppliers and delivered to customer \( k \) in time period \( t \)

Integer

\( N_{plkt}^T \) Number of trucks needed to deliver product \( p \) from location \( l \) to destination \( k \) in time period \( t \)

\( N_{kt}^N \) Number of boats needed to deliver products to destination \( k \) in time period \( t \)

Binary

\( \alpha_{pt} \) 1 if production of product \( p \) in time period \( t \)
\( \beta_{kt} \) 1 if production in area \( k \) in time period \( t \)
\( s_{pt} \) 1 if a setup of product \( p \) occurs in time period \( t \)
\( w_{pt} \) 1 if the production of product \( p \) is shut down in time period \( t \)
We will use indices $p$ and $q$ for products, $k$ and $l$ for areas, locations and destinations, and index $t$ for time period. In constraints which couples or extends over more than one time step, index $d$ is used. At one occasion, in constraints (1), index $e$ will denote external customers.

### Constraints

The inventory balance at each production area in Stenungsund is controlled by constraints (1). There is an inventory for each product $p$ that each area $k$ produces, and the inflow to an inventory is the production $x_{pt}$ and the purchased amount $Q_{pkl}$ from external suppliers. The outflow is the amount of product $p$ to be transported to locations $l$ outside Stenungsund, the given demands of contract customers $D^C_{pkt}$ and the amount sold from Stenungsund to External Customers, $y_{pkt}$ and finally, the amount of product $p$ needed to supply other plants in the supply chain for further processing, described by parameter $a_{pq}$. Here we assume that the demand from contract customers must be strictly fulfilled.

\[
I_{pk(t-1)} + x_{pt} + \sum_{(p,k)\in\Gamma} Q_{pkl} = I_{pkr} + \sum_{(p,k,l)\in\Gamma} T_{pkl} + D^C_{pkl} + y_{pkt} + \sum_{q\in\Gamma} a_{pq} x_{qt} \quad k \in A, p \in P_{A[k]}, t \in T \quad (1)
\]

The inventory balance constraints for each location $k \in L$ outside Stenungsund are found in (2). The inflow of products is either by truck or boat, described by variable $T_{plkt}$ or bought from external suppliers, $Q_{pkl}$. Parameter $\tau_k$ is the transportation time to destination $k$. Boats with product volumes $T_{plkt}$ arrive to inventory $k$ at time period $t$ when being sent from location $l$ at time period $t-\tau_k$. The outflow is the amount of product $p$ to be transported to other inventories $l$, the demands of contract customers $D^C_{pkl}$, and the amount sold to spot market customers, $y_{pkt}$.

\[
I_{pk(t-1)} + \sum_{(p,l,k)\in\Gamma} T_{plkt} + \sum_{(p,k)\in\Gamma} Q_{pkl} = I_{pkr} + \sum_{(p,k,l)\in\Gamma} T_{pkl} + D^C_{pkl} + y_{pkt} \quad k \in L, (p,k) \in Y, t \in T \quad (2)
\]

### Inventory Limitations

Constraints (3) assure that the inventory level of product $p$ at location $k$ respects the inventory capacity limits $I_{pk}^{min}$ and $I_{pk}^{max}$.

\[
I_{pk}^{min} \leq I_{pkr} \leq I_{pk}^{max} \quad p \in P, k \in A \cup L, t \in T \quad (3)
\]

The lower bound is a specified safety stock, while the upper bound is related to physical limitations.

### Demand and Sales

The demand from spot market customers, defined in parameter $D^S_{pkl}$, is given in constraints (4) and (5). Some products have an “unlimited” spot market, which means that if the demand from
the spot market is not met within one time period, it can instead be fulfilled in the next time period. This is achieved by limiting the accumulated sales by the accumulated demands up to the current time step.

\[
\sum_{d=1}^{t} y_{pkd} \leq \sum_{d=1}^{t} D_{pdk}^S \quad p \in P_U, (p, k) \in Y, t \in T
\]

(4)

The rest of the products have a “limited” spot market. If the demand of products on the limited spot market is not met in period \( t \), it is not possible to compensate in later time periods.

\[
y_{pkt} \leq D_{pkt}^S \quad p \in P \setminus P_U, (p, k) \in Y, t \in T
\]

(5)

**Boats and Trucks**

Constraints (6) and (7) are used to calculate how many trucks, \( N_{pklt}^T \), and boats, \( N_{kt}^B \), are needed to transport all products \( p \) to regional and offshore inventories. The truck and boat capacities, \( C_{pk}^T \) and \( C_{k}^B \) are given parameters. A truck can only transport one type of product at a time, which is regulated in constraints (6). However, a boat can carry different types of products and the only limitation is the weight of the cargo, which is regulated in constraint (7).

\[
T_{pklt} \leq C_{pk}^T \cdot N_{pklt}^T \quad k \in D_T, (p, l, k) \in \Gamma, t \in T
\]

(6)

\[
\sum_{(p, l, k) \in \Gamma} T_{pklt} \leq C_{k}^B \cdot N_{kt}^B \quad k \in D_B, t \in T
\]

(7)

**Production limitations**

Constraints (8) state that the production \( x_{pt} \) does not exceed the maximum capacity \( X_p^{\text{max}} \), nor violates the minimum capacity \( X_p^{\text{min}} \). If the production level is too low, the production must shut down. The binary variable \( \alpha_{pt} \) is equal to 1 if production of product \( p \) occurs at time period \( t \), otherwise zero.

\[
\alpha_{pt} \cdot X_p^{\text{min}} \leq x_{pt} \leq X_p^{\text{max}} \cdot \alpha_{pt} \quad p \in P, t \in T
\]

(8)

In a similar way, constraints (9) enforce the total production at area \( k \) to either stay within its given capacities, \( A_k^{\text{min}} \) and \( A_k^{\text{max}} \), or close down. Set \( P_at \) contains all products produced at a certain area \( k \). The binary variable \( \beta_{kt} \) is equal to 1 if area \( k \) is active in time period \( t \), otherwise zero.

\[
\beta_{kt} \cdot A_k^{\text{min}} \leq \sum_{p \in P_at} x_{pt} \leq A_k^{\text{max}} \cdot \beta_{kt} \quad k \in A, t \in T
\]

(9)
Obviously, for areas where only one product is produced, these constraints are equivalent to (8). To strengthen the LP-relaxation of the model, we also add the following constraints that couple the $\alpha_{pt}$ and $\beta_{kt}$ variables.

\begin{align}
\alpha_{pt} & \leq \beta_{kt} \quad k \in A, p \in P_{d[k]} \quad t \in T \\
\beta_{kt} & \leq \sum_{p \in P_{d[k]}} \alpha_{pt} \quad k \in A, t \in T
\end{align}

(10)

Constraints (10) states that if an area is closed, no production can take place there. And in constraints (11) the opposite needs also to be true, if none of the products in an area are being produced, the area is closed.

**Production rate changes**

Large changes in the production rate is not desirable and would wear down the equipment at a fast pace. Constraints (12) and (13) prevent this unwanted effect by setting a limit $\delta^{UB}_p$ (in percent) on the allowed production rate increase from one period to the next. Likewise, $\delta^{LB}_p$ is a lower bound on how many percent the production rate can decrease from one period to another, assuming that the production is not shut down.

\begin{align}
x_{pt} & \leq \delta^{UB}_p \cdot x_{p,t-1} + X_{p}^{\text{max}} \cdot (1 - \alpha_{p,t-1}) \quad p \in P, t \in T \\
x_{pt} & \geq \delta^{LB}_p \cdot x_{p,t-1} - X_{p}^{\text{max}} \cdot (1 - \alpha_{pt}) \quad p \in P, t \in T
\end{align}

(12)

(13)

If production shutdowns are not taken into account, constraints (12) and (13) would cause infeasibility. Therefore, in order to encourage a continuous production for consecutive weeks, we introduce auxiliary binary variables $w_{pt}$ which are used in constraints (14), forced to become 1 if production shuts down, and in the objective function (28) to penalize shutdowns.

\begin{align}
w_{pt} & \geq \alpha_{p,t-1} - \alpha_{pt} \quad p \in P, t \in T
\end{align}

(14)

**Special limitations Area 1**

In Area 1, where products P1 and P2 are produced, there are limitations of how big the proportion of P1 can constitute of the areas total production. Constraints (15) limit the production of P1 between $\gamma^{\text{min}}_{p1}$ and $\gamma^{\text{max}}_{p1}$, the minimum and maximum allowed proportions.

\begin{align}
\gamma_{p1}^{\text{min}} \cdot (x_{1t} + x_{2t}) \leq x_{1t} \leq \gamma_{p1}^{\text{max}} \cdot (x_{1t} + x_{2t}) \quad t \in T
\end{align}

(15)

**Special limitations Area 5**

The production of P6 and P7, which takes place at Area 5, requires special constraints. They are produced in batches, one product at a time, which is regulated in constraints (16). If the
production of \( p \) was zero in the previous time step \( t-1 \), but is active in time \( t \), constraints (17) force the binary variable \( s_{pt} \) to become one.

\[
\begin{align*}
\alpha_{et} + \alpha_{7t} & \leq 1 \quad t \in T \\
\alpha_{pt} + \alpha_{p,t-1} & \leq s_{pt} \quad p \in P_A[A5], t \in T
\end{align*}
\] (16) (17)

The minimum production length is two weeks, which is ensured by constraints (18). Within this time, the other product cannot be produced, and this is achieved by constraints (16). Also, if a product was produced the previous week, and still is, there cannot be any startup this week. This is captured by constraints (19)

\[
\begin{align*}
s_{pt} & \leq \alpha_{p,t+d} \quad d = 0,1, p \in P_A[A5], t \in T \\
s_{pt} & \leq 2 - \alpha_{pt} - \alpha_{p,t-1} \quad p \in P_A[A5], t \in T
\end{align*}
\] (18) (19)

Constraints (18) will also ensure that if product \( p \) is not produced in time period \( t \), that is \( \alpha_{pt} = 0 \), then obviously the production cannot start either, that is \( s_{pt} = 0 \).

**Variables**

Constraints (20) define all continuous variables and restrict them to be greater or equal to zero. Similarly, constraint (21) defines all binary and integer variables.

\[
\begin{align*}
y_{pkt}, x_{pt}, I_{pkt}, T_{plkt}, Q_{pkt} & \geq 0 \\
\alpha_{pt}, \beta_{pt}, \gamma_{jt}, s_{jt}, s_{gt} & \in \{0,1\}, \quad N_{plkt}^T, N_{kt}^B \in N^+
\end{align*}
\] (20) (21)

**Objective function**

The objective function (22) maximizes the difference between the earnings from sold products and the costs for production, carrying inventory, shipment inventory, transports, shutdown penalties, and costs for purchasing products from external suppliers.

\[
\begin{align*}
\max \; z = Z_{profit} - (Z_{prod} + Z_{inv} + Z_{ship} + Z_{tran} + Z_{shut} + Z_{pur})
\end{align*}
\] (22)

The earnings from sold products are found in (23). The sales prices \( R_{pk}^C \) and \( R_{pk}^S \) vary per product \( p \) and customer \( k \). All demand \( D_{pkt}^C \) must be fulfilled in each time period, and the variable \( y_{pkt} \) is the amount of product \( p \) sold to customer \( k \) at the spot market in time period \( t \).

\[
Z_{profit} = \sum_{t \in T} \sum_{(p,k) \in Y} R_{pk}^C \cdot D_{pkt}^C + \sum_{t \in T} \sum_{(p,k) \in Y} R_{pk}^S \cdot y_{pkt}
\] (23)
Cost of production is defined in (24). The parameter $c_p$ is the cost to produce 1 kg of product $p$, and variable $x_{pt}$ is the production plan which states the amount of product $p$ produced in time period $t$.

$$Z_{prod} = \sum_{t \in T} \sum_{p \in P} c_p \cdot x_{pt}$$  \hspace{1cm} (24)$$

The cost of carrying inventory, found in equation (25), is defined as the product of the annual inventory carrying interest $\rho$ and the inventory values. The average inventory value at location $k$ is calculated as the average inventory level multiplied by $c_{pk}$, the accumulative product value of product $p$ at location $k$.

$$Z_{inv} = \frac{\rho}{52} \sum_{t \in T} \sum_{p \in P} \sum_{k \in D} I_{pk(t-1)} + I_{pkl} \cdot \frac{c_{pk}}{2}$$  \hspace{1cm} (25)$$

There is also a cost to carry inventory on the boats with journey time of 1-2 weeks, that is, the transports to Aveiro and Philadelphia. As stated in equation (26), this cost is defined as the product of the transportation time $\tau_k$, the transported amount of product $p$ to inventory $k$, described by variable $T_{pklk}$, and the accumulative product value $c_{pk}$.

$$Z_{inv} = \frac{\rho}{52} \sum_{t \in T} \tau_k \sum_{l \in L, j \in J, (p,l,k) \in \Gamma} T_{pklk} \cdot c_{pk}$$  \hspace{1cm} (26)$$

In equation (27), the transportation costs are calculated as the sum of the number of trucks $N_{pkl}$ and boats $N_{kl}$ used to destination $k$ with costs $\tilde{c}_{pk}$ and $\tilde{c}_k$ respectively.

$$Z_{tran} = \sum_{t \in T} \sum_{(p,l,k) \in \Gamma: \tau_{l,j} > 0} \tilde{c}_{pk} \cdot N_{pkl}^T + \sum_{t \in T} \sum_{k \in D} \tilde{c}_k \cdot N_{kl}^B$$  \hspace{1cm} (27)$$

To prevent production shutdowns, the auxiliary variables $w_{pt}$ keep track of production shutdowns, which is penalized by parameter $S_p$ in equation (28).

$$Z_{shut} = \sum_{t \in T} \sum_{p \in P} S_p \cdot w_{pt}$$  \hspace{1cm} (28)$$

It is also possible to purchase products from external suppliers in case Perstorp Oxo AB cannot supply its customers. The cost is defined in (29), where $P_{pk}$ is the sales price from external suppliers of product $p$ to regional inventory $k$. Variable $Q_{pkl}$ is the amount of purchased products from external suppliers in each time period.

$$Z_{pur} = \sum_{t \in T} \sum_{(p,k) \in Q} P_{pk} \cdot Q_{pkl}$$  \hspace{1cm} (29)$$
That concludes the optimization model, which is characterized as a MIP-model.

**Data**

The data used in this paper, such as demands of contract customers, $D_{pkC}$, and spot market demands, $D_{pkS}$, has been collected from Perstorp Oxo AB's ERP system by extracting all orders during the second half of 2007. This data was cleaned from faulty entries and items outside the scope. We have excluded sales orders with extreme prices and low quantities. These orders are crediting orders, which are used when product swaps with other companies are carried out.

Further, internal transportation orders were originally included in the sales data and thus had to be removed in order to avoid quantities to be counted twice. The customers have then been classified manually according to the type of customer, and at the same time also been associated with an inventory location. This was performed by the planning manager at Perstorp Oxo AB.

Furthermore, sales data originally contained orders where the product was purchased from an external supplier and delivered directly to the customer. These orders could be part of swaps or crediting orders, thus they had to be distinguished. Product P1 had in some cases been purchased to the site in Gent due to capacity constraints, which made it possible to transport the produced amount of P1 in Area 1 to Perstorp instead. Historically, product P8 has also been purchased from external suppliers and delivered directly to external customers at times when the need of product P1 at Perstorp or Gent was great, and hence transported there instead of being used at Area 6 (where product P8 is produced).

Purchased material that was considered not to be part of crediting orders also describes the demand, and should thus be included in the $D_{pkC}$ and $D_{pkS}$ parameter respectively. Finally, the demand of each customer at each inventory location was aggregated into a weekly demand.

**RESULTS**

In order to verify the mathematical model, and illustrate solution characteristics, we present a baseline scenario where the contract customer demand must be fulfilled exactly, whereas the spot market demand is seen as an upper bound on the sales. The real data from Perstorp cannot be published due to secrecy matters, hence the presented numbers are perturbed and scaled, but still representative.

The baseline scenario is solved for a time horizon of 26 weeks, and gives rise to a problem with 3,477 linear constraints, and contains 2,094 continuous, 780 binary, and 260 integer variables. The model was implemented using AMPL, see Fourer et al. (2003), and the problem was solved on a HP DL160 server with two 6-core Intel Xeon CPUs and 72 GB of RAM memory, running Linux. The MIP solver used was the commercial solver CPLEX/12.5, see ILOG (2012), for 64 bit environment. After AMPL and CPLEX preprocessing, the problem consists of 2,919 rows and 2,828 columns, and contains 613 binary variables and 252 integer variables.
Feasible IP solutions are found within seconds, and the best found integer solution after 50 minutes has a relative MIP-gap of 0.1%, but it must be mentioned that the MIP-gap is of order 0.2% already after 60 seconds.

Production rates

In Figure 4 the production rates, expressed in percentage of the maximum production rate $X_p^\text{max}$, for each product and time step are shown, together with information of the average production rates. Most production rates are stable throughout all weeks, except for product P7 that is not produced at all, and product P8 which is only produced during weeks 43-47 in a small amount.

![Figure 4: Production rates in percentage of the maximum capacity for each product. In the legend, the average production rates are also given. Notice that product P7 is not produced at all during this time horizon, and product P8 is only produced during weeks 43-47.](image)

In total, only one shutdown occurs during all the weeks for all products, and it is product P6 in week 51. Both products P3 and P11 are only produced until week 49, but in contrast to P6 their production is not started again throughout the time horizon. If constraints (14) are not included, that is, shutdowns are not penalized, the corresponding total number of shutdowns becomes 19.

Transportation

The average transportation flows and the purchase of products from external suppliers, both expressed in percentage of the maximum production rates, are displayed in Figure 5. Since products P7 and P8 are only sold to external customers directly from Stenungsund, there are no
transportation flows for these products. Only product P1 is purchased from the external suppliers, mainly to the site in Gent but also some small amounts to Perstorp. From Antwerp, a significant part of product P6 is shipped off to Aveiro and a small part of product P10 to Philadelphia. In total, 875 trucks and 51 boats are used over the time period of 26 weeks, with an average capacity usage of 98% and 91% respectively.

![Figure 5: Average transportation flows of products and purchases from external suppliers, all expressed in percentage of the maximum production rates. Also, the numbers shown in brackets are the total number of trucks (NT) used during the time horizon. The total number of boats (NB) used is shown at each boat destination.]

**Inventories**

With few exceptions, the maximum inventory levels reached during the time horizon are all well below their upper limits. The inventory levels at Stenungsund for products P2 and P5 hit their respective roof at three occasions all in all. As seen in Figure 6, the inventory for product P3 at Castellanza does on the other hand reach its limit in seven occasions over the time horizon.

![Figure 6: Inventory levels for product P3 at Castellanza, in percentage of the maximum inventory level. The maximum limit is reached 7 times during the time horizon.]

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Common for all inventories, the safety stock lower limit was reached during most of the time periods. This is reasonable since inventory levels are part of the objective function as a cost, and should thus be kept at low levels if possible. The average inventory levels for all products at all locations, expressed in percentage of their respective maximum inventory levels, are shown in Figure 7.

![Figure 7: Average inventory levels, in percentage of their respective maximum inventory levels, for all products at all locations.](image)

The average inventory levels are all below 50% of their maximum capacity. It is important though to remember that high fluctuations over the time horizon is common, as seen in Figure 6.

**Sales**

For each product, the average production rates, deliveries to contract customers, and sales to spot market customers (numbers in brackets) are all found in Table 2. Like before, values are given in percentage of the maximum production rate $X_p^{\text{max}}$. Products P1 - P4 are all used to either serve the production of other products or delivered to contract customers, and products P7 and P8 are only produced in small amounts to please customers, whereas the remaining products are sold in larger quantities to both contract customers and to the spot market.

At a first glance, it might seem strange that the sold amounts of products P7 (not produced at all) and P8 exceed their production, but this can be explained by the initial inventory levels being large enough to cover the needs for the time horizon in question. Also, the average production rates for products P2 - P5 are relatively large compared to their sales. This has to do with the fact that these products are used as input to the production of the remaining products P6 and P9 -P11, which are all produced and sold in large quantities.
Table 2: Average production rate and sales of each product, given in percentage of the maximum production rate. For each product, its dependency of another product is stated within brackets. The sales stated are to contract customers and to spot market customers (numbers within brackets) respectively.

<table>
<thead>
<tr>
<th>PRODUCTION</th>
<th>PRODUCTION</th>
<th>SALES</th>
<th>SALES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Ext.Cust.</td>
<td>Tees</td>
</tr>
<tr>
<td>P1</td>
<td>0.92</td>
<td>—</td>
<td>0.00 (0.00)</td>
</tr>
<tr>
<td>P2</td>
<td>0.92</td>
<td>— (0.01)</td>
<td>0.00 (0.00)</td>
</tr>
<tr>
<td>P3</td>
<td>0.92</td>
<td>— (0.00)</td>
<td>0.00 (0.00)</td>
</tr>
<tr>
<td>P4 (P2)</td>
<td>0.68</td>
<td>0.03 (0.04)</td>
<td>0.16 (0.09)</td>
</tr>
<tr>
<td>P5 (P4)</td>
<td>0.99</td>
<td>0.03 (0.11)</td>
<td>0.05 (0.27)</td>
</tr>
<tr>
<td>P6 (P5)</td>
<td>0.94</td>
<td>0.03 (0.11)</td>
<td>0.05 (0.27)</td>
</tr>
<tr>
<td>P7</td>
<td>—</td>
<td>— (0.00)</td>
<td>0.00 (0.00)</td>
</tr>
<tr>
<td>P8 (P1)</td>
<td>0.01</td>
<td>0.04 (—)</td>
<td>0.00 (0.00)</td>
</tr>
<tr>
<td>P9 (P2)</td>
<td>0.79</td>
<td>0.20 (0.32)</td>
<td>0.11 (0.03)</td>
</tr>
<tr>
<td>P10 (P4)</td>
<td>0.61</td>
<td>0.06 (0.04)</td>
<td>0.29 (0.01)</td>
</tr>
<tr>
<td>P11 (P3)</td>
<td>0.34</td>
<td>0.00 (0.05)</td>
<td>0.05 (0.03)</td>
</tr>
</tbody>
</table>

Relatively large amounts are sold from the regional inventory facilities in Tees and Antwerp, both to contract customers and to the spot market, as one could expect. Moreover, according to the average sales figures in Table 2, the facilities in Aveiro and Philadelphia are also justified.

CONCLUSIONS

In this paper a large-scale model for the supply chain planning problems arising at a chemical company, Perstorp Oxo AB, is developed. The mathematical model includes both integer, binary and continuous variables. The model is detailed and specific for the company in question since it is intended for direct usage at the site in Stenungsund. However, it is general enough to be applicable also for similar large scale supply chain applications within the chemical process industry. As a solver we make use of the commercial solver CPLEX and solution times are within practical time limits.

The results presented here are from a single case study, based on actual sales data from 2007, to motivate the possible gains from an optimization based planning system for the planners at Perstorp. The model allows for easy testing of different scenarios and can probably produce better solutions compared to manual planning. This planning of today is very time consuming and the suggested decision support tool can therefore make the planning more efficient. The solution times of the model are within practical time limits, and the solution quality is very high. Moreover, data needed for the model is available from company databases. Hence, this makes the model and solver usable as a practical decision support tool in the planning process at the company.
Future work

The model provided in this paper is a good basis for further work and implementation at the facility. At the moment, regarding the input data, no consideration is taken to forecasts. Moreover, the demand should be modelled differently in order to better describe what can be sold on the spot market and to contract customers. This would be necessary in an implemented decision support system (DSS), which should also be able to cope with different DVC:s that could vary with the production rate.

As for the mathematical model itself, there are many possible extensions, some which are already ongoing work. At the moment, the demand from contract customers must be fulfilled. This is obviously something that all companies strive for, but is not always possible to achieve due to unplanned shutdowns and issues connected to transportation. Hence we would like to add the possibility of backlog on orders and at the same time introduce a quality of service measure, for example the amount of orders delivered on time, and penalize if this measure gets too low.

When analyzing and discussing the results together with Perstorp, we found that the current aggregation level of weeks is too coarse. As an example, it is not possible to describe inventory levels accurate enough due to the transportation times of days, but less than a week, using boats. Also, when certain areas are shut down, it could take up to three days before production is up and running again, something which is not possible to capture in the model right now. A more realistic time discretization would be single days, or perhaps a 2-day time step. Such an adjustment will certainly increase the number of variables in the model, which will necessitate other solution approaches than just applying AMPL/CPLEX.

One natural solution technique would be a rolling horizon approach, with a 30 day horizon, which better resembles the actual situation for the planners at Perstorp. This approach also enables for scenario analysis, for example, what happens if an area is shut down unexpected or what if a contract customer suddenly wants to increase its orders significantly?

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References


