ABSTRACT

A mathematical program is developed and analytically solved in this paper for the problem of optimally allocating and pricing service capacity in a monopolistic market by a service provider under uncertain demand following a uniform probability distribution. This paper also examines the impacts of changes in the price charged and a key model parameter in each segment upon the firm’s expected total profit and the optimal scheme of capacity allocation.

KEYWORDS: Monopolistic market, Capacity allocation, Pricing, Yield management

INTRODUCTION

Yield management is practiced in various service industries including airline, lodging, rental, transportation, healthcare, and satellite transmission (Desiraju & Shugan, 1999). Despite differences in its definition in the literature, there is a consensus among researchers that the primary goal of yield management is to maximize revenues by setting the right amount of capacity at the right price to be sold to the right customer (Feng & Xiao, 2000; Ng et al., 2008). The papers of McGill and van Ryzin (1999) and Kimes (2003) provide lucid reviews of the yield management literature.

Yield management is most effective when demand of consumers can be segmented and price sensitivity varies across heterogeneous market segments (Pinder, 2005). For example, airlines rely largely on varied price sensitivity to segment demand and implement price discrimination, as a business traveler may be less price-sensitive than his leisure counterpart. Using the techniques of yield management, American Airlines increased its revenue by 5% in 1992, which translated to $1.4 billion over a three-year period (Chen et al., 2003). Service firms that implement price discrimination (a key feature of yield management) set the price per unit of certain capacity at various levels, because the perceived value that each consumer attaches to a unit of that capacity could be rather different. For example, airlines often sell identical flight seats at multiple fares. Market segmentation combined with a sound pricing strategy is necessary for effective yield management (Vinod, 2010).
Our study aims to tackle a problem in the context of yield management in a monopolistic market, where a sole firm sets the price and then allocates a certain amount of service capacity to each segment for sale to maximize its expected total profit from the entire market. Three notable studies are relevant to this paper. Lee and Ng (2001) addressed the issue of optimal capacity allocation and analytically derived the optimal scheme of capacity allocation in a monopolistic market comprised of two segments where the demand is deterministic. Deng et al. (2008) studied capacity allocation in a multi-segment monopolistic market where the demand in each segment is assumed to independently follow a Poisson distribution. They presented a marginal-revenue-based capacity management model used to manage stochastic demand in a three-segment market and proposed the policies for the firm to allocate capacity to segments of higher revenue. Zhang and Mesak (2010), based on a demand function of a deterministic nature, developed a nonlinear programming model to find the optimal scheme of capacity allocation and prices over a multi-period planning horizon for a service provider in a monopolistic market under the threat of uncertain competitive entry.

This paper, focusing on a profit-maximizing service provider under uncertain demand at the aggregate level, is significantly different from the three studies cited above in several aspects. First, a monopolistic market comprised of \( n \) heterogeneous segments is taken into account in the modeling framework. Second, the capacity allocated to a segment by the service provider is treated as a decision variable. Third, uncertain demand of each segment for the service provider is modeled as a continuous random variable. Fourth, the optimal scheme of capacity allocation is analytically determined for the demand in each segment following a uniform probability distribution. Fifth, this study analytically and numerically examines the impacts of changes in the price charged and a key model parameter in each segment on the service provider’s profitability and capacity allocation.

It is assumed in this paper that the market segments are well sealed from one another. Consumers in one segment are not allowed to make purchases in the other segments. Such a market exists in reality. For example, moviegoers who live in New York City normally do not travel to San Francisco for the sole purpose of seeing a movie even if the ticket price may be lower at the latter location. The three main research questions that we attempt to address in this paper can be specifically stated as follows: (i) What is the optimal policy for the service provider to allocate a service capacity of \( K \) identical units in a monopolistic market comprised of \( n \) heterogeneous segments under uncertain demand during a single selling season so that its expected total profit is maximized? (ii) What are the impacts of changes in the price charged in a segment on the firm’s profitability and scheme of capacity allocation? (iii) What are the impacts of changes in one key model parameter on the firm’s profitability and scheme of capacity allocation? We make the following main assumptions while addressing the strategic issues stated above:

(i) The price charged by the service provider in each segment is the only independent variable that affects the aggregate demand in that segment.
(ii) The consumers of each segment are well-informed of the price charged in that segment as well as other segments, set at the beginning of the selling season.
(iii) The demand of each segment is heterogeneous and independently follows a continuous probability distribution conditioned by the price charged in that segment.

The rest of the paper is organized as follows. In the next section, the profit functions for the service provider are developed, and a mathematical programming model is formulated to find the optimal scheme of capacity allocation. The third section presents ten propositions that analytically highlight the optimal allocation scheme and the impacts of changes in the price
charged and a key model parameter in each segment. Numerical illustrations of the optimal capacity allocation and pricing are given in the fourth section. Finally, our study concludes in its fifth section with a summary of its contributions, limitations, and directions for future research. Proofs of all propositions are available from the first author upon request.

MODEL FRAMEWORK

Let us consider a sole service provider that has a service capacity of \( K \) identical units to be priced and allocated during a single selling season in a monopolistic market comprised of \( n \) heterogeneous segments, denoted as segment \( i \) (\( i = 1, 2, \ldots, n \)). Several terms are defined below to formulate the firm’s profit functions:

- \( y_i \): the capacity to be allocated by the service provider to segment \( i \) (a decision variable);
- \( C \): the cost per unit of capacity incurred by the service provider (\( C > 0 \));
- \( P_i \): the price per unit of capacity charged by the service provider in segment \( i \) (\( P_i > C \));
- \( d_i \): the aggregate demand of consumers in segment \( i \);
- \( \omega_i \): the demand parameter in segment \( i \), which is a function of \( P_i \) (\( \omega_i > 0 \));
- \( f(x|\omega_i) \): the probability density function (p.d.f.) of \( d_i \) being a continuous random variable;
- \( E(d_i) \): the expected value of \( d_i \);
- \( \pi_i \): the service provider’s profit yielded from segment \( i \);
- \( E(\pi_i) \): the expected value of \( \pi_i \);
- \( \pi \): the service provider’s total profit yielded from the entire \( n \)-segment market; and
- \( E(\pi) \): the expected value of \( \pi \).

Price is one of the most effective variables that managers can manipulate to encourage or discourage demand in the short run. The aggregate demand of consumers in a market segment is usually uncertain (e.g., Shah & Jha, 1991; Deng et al., 2008). Since buyers are assumed to be well-informed of the prices charged by the service provider in each segment, they would take the prices into consideration while making their purchases. Hence, the demand in segment \( i \), \( d_i \) (\( i = 1, 2, \ldots, n \)), can be modeled as a random variable following a probability distribution conditioned by the price charged in that segment, \( P_i \). The p.d.f. of \( d_i \) takes the form of \( f(x|\omega_i) \) if \( d_i \) is a continuous random variable.

In segment \( i \) (\( i = 1, 2, \ldots, n \)), if the demand \( (d_i) \) exceeds the capacity allocated by the service provider \( (y_i) \), its profit \( (\pi_i) \) will equal the profit per unit of capacity multiplied by the number of units sold. On the other hand, if \( d_i \) is smaller than \( y_i \), a portion of the allocated capacity, \( y_i - d_i \), will be unsold and its cost stands for a loss to the service provider. Therefore, the provider’s profit yielded from segment \( i \) (\( i = 1, 2, \ldots, n \)) is expressed as

\[
\pi_i = \begin{cases} 
P_i d_i - C y_i, & d_i \leq y_i, \\
(P_i - C) y_i, & d_i > y_i.
\end{cases}
\]  

(1)

If \( d_i \) is a continuous random variable, the expected profit yielded from segment \( i \) is derived from (1) as follows:

\[
E(\pi_i) = P_i \int_0^{y_i} f(x|\omega_i)dx - C y_i \int_0^{y_i} f(x|\omega_i)dx + (P_i - C) y_i \int_{y_i}^{\infty} f_i(x|\omega_i)dx.
\]  

(2)
As in the study of Azoury (1985), the aggregate demand of consumers in each of the \( n \) segments is modeled in this paper as a random variable following a uniform probability distribution. The p.d.f. of the demand in segment \( i, d_i \), is assumed to take the following continuous form:

\[
f(x|\omega_i) = \begin{cases} 
1/\omega_i & \text{if } 0 < x \leq \omega_i, \\
0 & \text{if } x > \omega_i.
\end{cases}
\]  

(3)

The expected demand for the service provider in segment \( i \), based on (3), is given by

\[
E(d_i) = \int_0^{\omega_i} \frac{x}{\omega_i} \, dx = \frac{\omega_i}{2}.
\]  

(4)

Expression (4) shows that the demand parameter in segment \( i, \omega_i \), equals twice the expected demand, \( E(d_i) \), and hence serves as an indicator of the aggregate demand in segment \( i \).

Substituting (3) into (2) and carrying out the integrations yield

\[
E(\pi_i) = (P - C)y_i - \frac{P}{2\omega_i}y_i^2, \quad \text{if } y_i < \omega_i;
\]  

(5)

\[
E(\pi_i) = \frac{P}{2}\omega_i - Cy_i, \quad \text{if } y_i \geq \omega_i.
\]  

(6)

The service provider’s expected total profit from the \( n \)-segment market can be expressed as

\[
E(\pi) = \sum_{i=1}^{n} E(\pi_i).
\]  

(7)

Given a service capacity \( K \), which is to be allocated during a single selling season in an \( n \)-segment monopolistic market for sale by the service provider, we aim at finding the optimal capacity to be allocated to segment \( i \) \((i = 1, 2, \ldots, n)\), \( y_i \), to maximize the firm’s expected total profit. Thus, the problem is formulated as follows:

\[
\text{Max} \sum_{i=1}^{n} E(\pi_i)
\]

s.t. \( \sum_{i=1}^{n} y_i \leq K \),

\( y_i \geq 0 \) for \( i = 1, 2, \ldots, n \).  

(8)

The iso-elastic model is one of the price-dependent demand models in a monopolistic setting, which are “commonly used to characterize the demand for a product that only depends on its price of the product itself (see Huang, et al., 2013).” It is assumed in this paper that the demand parameter of the firm, \( \omega_i \), takes the form of the iso-elastic model:

\[
\omega_i = \beta_i P^{-\alpha_i},
\]  

(9)

where \( \alpha_i > 1 \) and \( \beta_i > 0 \) \((i = 1, 2, \ldots, n)\). In expression (9), the parameter, \( \alpha_i \), defines how the aggregate demand in segment \( i \) responds to the price charged in that segment. The scaling factor \( \beta_i \) reflects the overall size of segment \( i \). A larger value of \( \beta_i \) indicates stronger demand in segment \( i \). In model (9), the price \( P \) is set relative to some reference price \( P_0 \) (considered reasonable to pay for a unit of service capacity). Without loss of generality, \( P_0 \) is taken equal to 1 (see Moon et al., 2006 for details). Model (8) is analytically solved next.
OPTIMAL CAPACITY ALLOCATION

Following the approach of Lee and Ng (2001), this study only focuses on an optimal interior solution for model (8) such that \( \sum_{i=1}^{n} y_i^* < K \). In each of the following two cases, we first provide a solution to model (8) and then analytically examine the impacts of changes in the price charged and the model parameter \( \alpha_i \) in each segment upon the firm’s optimal expected total profit and optimal scheme of capacity allocation.

Case 1. \( y_i < \omega_i \) (\( i = 1, 2, \ldots, n \))

In this case, the capacity allocated to each segment of the market by the service provider is less than \( \omega_i \). Five propositions are introduced below, for which the proofs are available from the first author upon request.

Proposition 1. Given \( \sum_{i=1}^{n} \frac{(P_i - C_i)}{P_i} \omega_i < K \) and \( P_i > C_i \) (\( i = 1, 2, \ldots, n \)), the service provider’s expected total profit \( E(\pi) \) reaches its maximal level \( E^*(\pi) = \sum_{i=1}^{n} E^*(\pi_i) \) at \( y_i^* = \frac{(P_i - C_i)}{P_i} \omega_i \) for \( i = 1, 2, \ldots, n \), where \( E^*(\pi_i) = \frac{(P_i - C_i)^2}{2P_i} \omega_i \).

It is shown in Proposition 1 that the firm can maximize its expected total profit by allocating the capacity to each segment at the optimal level. The optimal capacity allocated to segment \( i \), \( y_i^* \), is determined by the price charged in the segment, the cost per unit of capacity, and the expected demand for the firm in the segment (see expression (4)). The condition of \( P_i > C_i \) implies \( y_i^* > 0 \) (\( i = 1, 2, \ldots, n \)).

Proposition 2. Given \( P_i > C_i \) (\( i = 1, 2, \ldots, n \)) and all other things being equal:

(i) If \( \frac{\alpha_i + 1}{\alpha_i - 1} > 1 \), then \( E^*(\pi) \) is monotonically decreasing in \( P_i \).
(ii) If \( \frac{\alpha_i + 1}{\alpha_i - 1} < 1 \), then \( E^*(\pi) \) is monotonically increasing in \( P_i \).

Proposition 3. Given \( P_i > C_i \) (\( i = 1, 2, \ldots, n \)) and all other things being equal:

(i) If \( P_i > 1 \), then \( E^*(\pi) \) is monotonically decreasing in \( \alpha_i \).
(ii) If \( P_i < 1 \), then \( E^*(\pi) \) is monotonically increasing in \( \alpha_i \).
(iii) If \( P_i = 1 \), then \( E^*(\pi) \) is constant in \( \alpha_i \).

Proposition 2 provides guidelines for the service provider on when to raise or lower the price charged in each segment to maximize its expected total profit. It is noted that the expected total profit reaches the highest level if the price charged in segment \( i \) is set at \( \frac{\alpha_i + 1}{\alpha_i - 1} C_i \) (\( i = 1, 2, \ldots, n \)).

Proposition 3 states that the service provider may influence parameter \( \alpha_i \) to change to enhance its profitability. It is interesting to note that if the price charged in a segment is equal to one, then any change in \( \alpha_i \) will not affect the firm’s profitability.
Proposition 4. Given $P_i > C$ ($i = 1, 2, \ldots, n$) and all other things being equal, the optimal amount of capacity allocated to segment $i$, $y_i^*$, is monotonically decreasing in $P_i$.

Proposition 5. Given $P_i > C$ ($i = 1, 2, \ldots, n$) and all other things being equal:

(i) If $P_i > 1$, then the optimal amount of capacity allocated to segment $i$, $y_i^*$, is monotonically decreasing in $\alpha_i$.

(ii) If $P_i < 1$, then $y_i^*$ is monotonically increasing in $\alpha_i$.

(iii) If $P_i = 1$, then $y_i^*$ is constant in $\alpha_i$.

Proposition 4 shows that the service provider should reduce the amount of capacity allocated to a segment as the price charged in that segment rises to enhance its profitability. Proposition 5 gives the directions in which the firm adjusts its optimal scheme of capacity allocation in response to changes in parameter $\alpha_i$.

Case 2. $y_i \geq \alpha_i$ ($i = 1, 2, \ldots, n$)

The capacity allocated to each segment of the market by the service provider is more than or equal to $\alpha_i$ in this case. As in Case 1, we also introduce five propositions here, for which the proofs are available from the first author upon request.

Proposition 6. Given $\sum_{i=1}^{n} \omega_i \leq K$, the firm’s expected total profit $E(\pi)$ reaches its maximal level $E^*(\pi) = \sum_{i=1}^{n} E^*(\pi_i) \omega_i$ for $i = 1, 2, \ldots, n$, where $E^*(\pi_i) = \left(1/2\right) \left(P_i - C\right) \omega_i$.

Proposition 6 advocates that in Case 2, the service provider should allocate the capacity exactly equal to twice the expected demand in each segment to maximize its expected total profit (see expression (4)).

Proposition 7. Given $P_i > 2C$ ($i = 1, 2, \ldots, n$) and all other things being equal:

(i) If $P_i > \frac{2\alpha_i}{\alpha_i - 1} - C$, then $E^*(\pi)$ is monotonically decreasing in $P_i$.

(ii) If $P_i < \frac{2\alpha_i}{\alpha_i - 1} - C$, then $E^*(\pi)$ is monotonically increasing in $P_i$.

Given $C < P_i < 2C$ ($i = 1, 2, \ldots, n$) and all other things being equal, $E^*(\pi)$ is monotonically increasing in $P_i$.

Proposition 8. Given $P_i > 2C$ ($i = 1, 2, \ldots, n$) and all other things being equal:

(i) If $P_i > 1$, then $E^*(\pi)$ is monotonically decreasing in $\alpha_i$.

(ii) If $P_i < 1$, then $E^*(\pi)$ is monotonically increasing in $\alpha_i$.

(iii) If $P_i = 1$, then $E^*(\pi)$ is constant in $\alpha_i$.

Given $C < P_i < 2C$ ($i = 1, 2, \ldots, n$) and all other things being equal:

(i) If $P_i > 1$, then $E^*(\pi)$ is monotonically increasing in $\alpha_i$.

(ii) If $P_i < 1$, then $E^*(\pi)$ is monotonically decreasing in $\alpha_i$.

(iii) If $P_i = 1$, then $E^*(\pi)$ is constant in $\alpha_i$. 


Given \( P_i = 2C \) \( (i = 1, 2, ..., n) \) and all other things being equal, then \( E^*(\pi) \) equals zero for any value of \( \alpha_e \).

Propositions 7 and 8 suggest that, as in Case 1, there exist opportunities dictated by the various conditions for the firm in Case 2 to enhance its profitability by raising or lowering the price charged or causing the parameter \( \alpha_i \) to change in each segment (Proposition 7 indicates that the expected total profit reaches the highest level if the price charged in segment \( i \) is set at \( \frac{2\alpha_i}{\alpha_i - 1} C \)).

Proposition 9. Given \( P_i > C \) \( (i = 1, 2, ..., n) \) and all other things being equal, \( y_i^* \) is monotonically decreasing in \( P_i \).

Proposition 10. Given \( P_i > C \) \( (i = 1, 2, ..., n) \) and all other things being equal:

(i) If \( P_i > 1 \), then \( y_i^* \) is monotonically decreasing in \( \alpha_i \).

(ii) If \( P_i < 1 \), then \( y_i^* \) is monotonically increasing in \( \alpha_i \).

(iii) If \( P_i = 1 \), then \( y_i^* \) is constant in \( \alpha_i \).

Guidelines are offered by Propositions 9 and 10 for the service provider in Case 2 to adjust its optimal scheme of capacity allocation in response to changes in the price \( P_i \) and the parameter \( \alpha_i \), respectively.

**NUMERICAL ILLUSTRATIONS**

This section presents a numerical study to (i) show eight schemes of the optimal capacity allocation for the service provider, each of which is determined for one of five pricing options, and (ii) explore the impacts of the pricing options and two values of each \( \alpha_i \) upon the optimal scheme of capacity allocation and the optimal expected total profit. For illustrative purposes, we examine both Cases 1 and 2 discussed in the third section for the service provider that allocates its capacity in a market comprised of segment \( i \) \( (i = 1, 2, 3, 4, 5) \). Five pricing options of the firm, \((P_1, P_2, P_3, P_4, P_5) = (500, 1500, 2500, 3500, 4500)\), \((P_1, P_2, P_3, P_4, P_5) = (3300, 3300, 3300, 3300, 3300)\), \((P_1, P_2, P_3, P_4, P_5) = (1800, 1800, 1800, 1800, 1800)\), \((P_1, P_2, P_3, P_4, P_5) = (2100, 2100, 2100, 2100, 2100)\), are compared in this experiment. The values of the other model parameters are selected as follows: \( C = 300; \alpha_i = 1.2 \) or \( 1.4; \beta_i = 1 \) million \( (i = 1, 2, 3, 4, 5) \).

Based on Proposition 1 developed for Case 1 (see the previous section), the optimal allocation scheme \( y_i^* \) and the corresponding expected profit from segment \( i \) \( (i = 1, 2, 3, 4, 5) \), \( E^*(\pi_i) \), are calculated and reported in Tables 1 and 2 for each combination of the selected values of model parameters. Similarly, the two series \( \{y_i^*\} \) and \( \{E^*(\pi_i)\} \) in Tables 3 and 4 are calculated based on Proposition 6 developed for Case 2. In Tables 1 – 4, the expected total profit \( E^*(\pi) \) is calculated by expression (7). The five market segments are characterized in each of Tables 1 – 4 by identical values of the two parameters \( \alpha_i \) and \( \beta_i \), respectively. As shown in Table 1, for example, given \((P_1, P_2, P_3, P_4, P_5) = (500, 1500, 2500, 3500, 4500)\), \( C = 300; \alpha_i = 1.2 \) and \( \beta_i = 1 \) million \( (i = 1, 2, 3, 4, 5) \), the firm should allocate 230.83, 123.53, 73.61, 51.07 and 38.56 units of capacity to segment \( i \) \( (i = 1, 2, 3, 4, 5) \), respectively, and as a result, its expected total profit would be $340,879.72.
According to Proposition 1, when \( y_i < \omega_i \) in segment \( i \) (\( i = 1, 2, 3, 4, 5 \)), the expected total profit \( E(\pi) \) reaches the highest level if the price charged in segment \( i \), \( P_i \), is set at \( \frac{\alpha_i + 1}{\alpha_i - 1} C \). For example, given \( \frac{\alpha_i + 1}{\alpha_i - 1} C = $3300 \) (\( i = 1, 2, 3, 4, 5 \)) in Table 1, the pricing option, \( (P_1, P_2, P_3, P_4, P_5) \) = \( ($3300, $3300, $3300, $3300, $3300) \), yields an expected total profit of \$408,744.45, much higher than the one of \$340,879.72 yielded by the other option, \( (P_1, P_2, P_3, P_4, P_5) = ($500, $1500, $2500, $3500, $4500) \). Similarly, Table 2 shows that the pricing option, \( (P_1, P_2, P_3, P_4, P_5) \) = \( ($1800, $1800, $1800, $1800, $1800) \), yields a higher expected total profit.

According to Proposition 7, when \( y_i \geq \omega_i \) and \( P_i > 2C \) in segment \( i \) (\( i = 1, 2, 3, 4, 5 \)), the expected total profit \( E(\pi) \) reaches the highest level if the price charged in segment \( i \), \( P_i \), is set at \( \frac{2\alpha_i}{\alpha_i - 1} C \). As shown in Table 3, the pricing option, \( (P_1, P_2, P_3, P_4, P_5) = ($3600, $3600, $3600, $3600, $3600) \), yields a higher profit than the other, \( (P_1, P_2, P_3, P_4, P_5) = ($500, $1500, $2500, $3500, $4500) \) in profit enhancement. Table 4 reveals that the pricing option, \( (P_1, P_2, P_3, P_4, P_5) = ($2100, $2100, $2100, $2100, $2100) \), yields a higher profit.

As noted in Tables 1 – 4, given the values of the model parameters and the five pricing options of the service provider, the smaller value of \( \alpha_i \) tends to be favorable for the firm to improve its profitability. It is also noted that if the firm charges a lower price in a segment, then the optimal amount of capacity allocated to that segment will be getting larger as expected.

**CONCLUSIONS**

This paper tackles the problem of optimally allocating and pricing service capacity by a service provider in an \( n \)-segment monopolistic market under uncertain demand following a uniform probability distribution. A mathematical programming model is developed and then analytically solved to determine the optimal scheme of capacity allocation and the optimal pricing option. In addition, the impacts of changes in the price charged by the focal firm and the price sensitivity of the market are analytically and numerically investigated.

This exploratory study suggests some possibilities for future research. First, our study provides analytical solutions to the problem of capacity allocation under uncertain demand following a uniform probability distribution. A rather challenging direction for future research would be to analytically solve the problem under a demand following other probability distributions. Second, one main assumption made in this paper is that the price charged in each segment is the only independent variable that affects the aggregate demand in that segment. This assumption could be relaxed for capacity allocation and pricing in a multi-segment market, where the demand in a segment is also affected by the prices charged in other segments. Third, empirical studies could be conducted to explore the probability distributions of the uncertain demand of each segment and find out their structures.
### Table 1  Optimal schemes of capacity allocation and the expected profits in Case 1

\( C = $300, \ \alpha_i = 1.2, \ \frac{\alpha_i + 1}{\alpha_i - 1} \)  
\( C = $3300, \ \beta_i = 1 \text{ million}, \ i = 1, 2, 3, 4, 5 \)

<table>
<thead>
<tr>
<th>( y_1^* )</th>
<th>( y_2^* )</th>
<th>( y_3^* )</th>
<th>( y_4^* )</th>
<th>( y_5^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>230.83</td>
<td>123.53</td>
<td>73.61</td>
<td>51.07</td>
<td>38.56</td>
</tr>
</tbody>
</table>

\( E^*(\pi_1)($) \)  
\( E^*(\pi_2)($) \)  
\( E^*(\pi_3)($) \)  
\( E^*(\pi_4)($) \)  
\( E^*(\pi_5)($) \)  
\( E^*(\pi)($) \)

<table>
<thead>
<tr>
<th>( P_1($) )</th>
<th>( P_2($) )</th>
<th>( P_3($) )</th>
<th>( P_4($) )</th>
<th>( P_5($) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1500</td>
<td>2500</td>
<td>3500</td>
<td>4500</td>
</tr>
</tbody>
</table>

### Table 2  Optimal schemes of capacity allocation and the expected profits in Case 1

\( C = $300, \ \alpha_i = 1.4, \ \frac{\alpha_i + 1}{\alpha_i - 1} \)  
\( C = $1800, \ \beta_i = 1 \text{ million}, \ i = 1, 2, 3, 4, 5 \)

<table>
<thead>
<tr>
<th>( y_1^* )</th>
<th>( y_2^* )</th>
<th>( y_3^* )</th>
<th>( y_4^* )</th>
<th>( y_5^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>54.50</td>
<td>54.50</td>
<td>54.50</td>
<td>54.50</td>
<td>54.50</td>
</tr>
</tbody>
</table>

\( E^*(\pi_1)($) \)  
\( E^*(\pi_2)($) \)  
\( E^*(\pi_3)($) \)  
\( E^*(\pi_4)($) \)  
\( E^*(\pi_5)($) \)  
\( E^*(\pi)($) \)

<table>
<thead>
<tr>
<th>( P_1($) )</th>
<th>( P_2($) )</th>
<th>( P_3($) )</th>
<th>( P_4($) )</th>
<th>( P_5($) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3300</td>
<td>3300</td>
<td>3300</td>
<td>3300</td>
<td>3300</td>
</tr>
</tbody>
</table>

### Table 3  Optimal schemes of capacity allocation and the expected profits in Case 1

\( C = $300, \ \alpha_i = 1.6, \ \frac{\alpha_i + 1}{\alpha_i - 1} \)  
\( C = $1800, \ \beta_i = 1 \text{ million}, \ i = 1, 2, 3, 4, 5 \)

<table>
<thead>
<tr>
<th>( y_1^* )</th>
<th>( y_2^* )</th>
<th>( y_3^* )</th>
<th>( y_4^* )</th>
<th>( y_5^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.47</td>
<td>33.47</td>
<td>33.47</td>
<td>33.47</td>
<td>33.47</td>
</tr>
</tbody>
</table>

\( E^*(\pi_1)($) \)  
\( E^*(\pi_2)($) \)  
\( E^*(\pi_3)($) \)  
\( E^*(\pi_4)($) \)  
\( E^*(\pi_5)($) \)  
\( E^*(\pi)($) \)

<table>
<thead>
<tr>
<th>( P_1($) )</th>
<th>( P_2($) )</th>
<th>( P_3($) )</th>
<th>( P_4($) )</th>
<th>( P_5($) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
</tr>
</tbody>
</table>

### Table 4  Optimal schemes of capacity allocation and the expected profits in Case 1

\( C = $300, \ \alpha_i = 2, \ \frac{\alpha_i + 1}{\alpha_i - 1} \)  
\( C = $1800, \ \beta_i = 1 \text{ million}, \ i = 1, 2, 3, 4, 5 \)

<table>
<thead>
<tr>
<th>( y_1^* )</th>
<th>( y_2^* )</th>
<th>( y_3^* )</th>
<th>( y_4^* )</th>
<th>( y_5^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.09</td>
<td>23.09</td>
<td>23.09</td>
<td>23.09</td>
<td>23.09</td>
</tr>
</tbody>
</table>

\( E^*(\pi_1)($) \)  
\( E^*(\pi_2)($) \)  
\( E^*(\pi_3)($) \)  
\( E^*(\pi_4)($) \)  
\( E^*(\pi_5)($) \)  
\( E^*(\pi)($) \)

<table>
<thead>
<tr>
<th>( P_1($) )</th>
<th>( P_2($) )</th>
<th>( P_3($) )</th>
<th>( P_4($) )</th>
<th>( P_5($) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>17318.03</td>
<td>17318.03</td>
<td>17318.03</td>
<td>17318.03</td>
<td>17318.03</td>
</tr>
</tbody>
</table>
Table 3  Optimal schemes of capacity allocation and the expected profits in Case 2

\( (C = $300, \ \alpha_i = 1.2, \ \frac{2\alpha_i}{\alpha_i - 1} C = $3600, \ \beta_i = 1 \text{ million}, \ i = 1, 2, 3, 4, 5) \)

<table>
<thead>
<tr>
<th>( P_1 ) ($)</th>
<th>( P_2 ) ($)</th>
<th>( P_3 ) ($)</th>
<th>( P_4 ) ($)</th>
<th>( P_5 ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1500</td>
<td>2500</td>
<td>3500</td>
<td>4500</td>
</tr>
</tbody>
</table>

\( y_1^* \quad y_2^* \quad y_3^* \quad y_4^* \quad y_5^* \)

577.08 154.42 83.65 55.86 41.32

\( E^*(\pi_1) ($) \quad E^*(\pi_2) ($) \quad E^*(\pi_3) ($) \quad E^*(\pi_4) ($) \quad E^*(\pi_5) ($) \quad E^*(\pi) ($) \)

-28854.00 69486.91 79468.61 81000.25 80571.16 241040.02

---

Table 4  Optimal schemes of capacity allocation and the expected profits in Case 2

\( (C = $300, \ \alpha_i = 1.4, \ \frac{2\alpha_i}{\alpha_i - 1} C = $2100, \ \beta_i = 1 \text{ million}, \ i = 1, 2, 3, 4, 5) \)

<table>
<thead>
<tr>
<th>( P_1 ) ($)</th>
<th>( P_2 ) ($)</th>
<th>( P_3 ) ($)</th>
<th>( P_4 ) ($)</th>
<th>( P_5 ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3600</td>
<td>3600</td>
<td>3600</td>
<td>3600</td>
<td>3600</td>
</tr>
</tbody>
</table>

\( y_1^* \quad y_2^* \quad y_3^* \quad y_4^* \quad y_5^* \)

54.50 54.50 54.50 54.50 54.50

\( E^*(\pi_1) ($) \quad E^*(\pi_2) ($) \quad E^*(\pi_3) ($) \quad E^*(\pi_4) ($) \quad E^*(\pi_5) ($) \quad E^*(\pi) ($) \)

81008.06 81008.06 81008.06 81008.06 81008.06 405040.30

---

<table>
<thead>
<tr>
<th>( P_1 ) ($)</th>
<th>( P_2 ) ($)</th>
<th>( P_3 ) ($)</th>
<th>( P_4 ) ($)</th>
<th>( P_5 ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2100</td>
<td>2100</td>
<td>2100</td>
<td>2100</td>
<td>2100</td>
</tr>
</tbody>
</table>

\( y_1^* \quad y_2^* \quad y_3^* \quad y_4^* \quad y_5^* \)

22.33 22.33 22.33 22.33 22.33

\( E^*(\pi_1) ($) \quad E^*(\pi_2) ($) \quad E^*(\pi_3) ($) \quad E^*(\pi_4) ($) \quad E^*(\pi_5) ($) \quad E^*(\pi) ($) \)

16747.66 16747.66 16747.66 16747.66 16747.66 83738.30
REFERENCES


