ABSTRACT

We examined the financial implications on supply chain performance while considering random production process failures and imperfect inspection process in a simple supply chain where defective products from a supplier may propagate and affect the production process of its downstream customers randomly. Our analysis shows that long-run average and the variance of yields of the supply chain reduces as delivery interval increases. The yield fluctuation at each tier of a manufacturing supply chain can create the defect bullwhip effect where uncertain and fluctuating upstream quality capability leads to increasing demand fluctuation as one move up a manufacturing supply chain.

KEYWORDS: Logistics, Bullwhip Effect, Just-In-Time/Kanban, Lean Manufacturing

INTRODUCTION

Many firms have implemented Toyota Production System concepts, such as just-in-time, within their productions to increase responsiveness to market demand, improve productivity, and reduce cost (Candler, 1993; Inman & Mehra, 1990; Milligan, 1999). Toyota Production System practitioners believe logistical policies with long delivery intervals reduce production responsiveness and impede the removal of wastes. Better quality management is often cited as a reason for keeping shorter delivery interval. On the other hand, management of product quality is often not a consideration for logistics managers in practice.

In this paper, we examined the financial implications on supply chain performance while considering random production process failures and imperfect inspection process in a simple manufacturing supply chain where defective products from a supplier may propagate through the supply chain and affect the production process of its downstream customers randomly. The yield fluctuation at each tier of the chain creates a defect bullwhip effect where uncertain and fluctuating upstream quality capability leads to increasing demand fluctuation as one move up the chain. We provided a numerical example to illustrate the financial implications of this effect.
Mathematical models on logistics often focus on either vehicle routing or optimal shipment size with objectives such as to maximize fleet utilization or to minimize inventory and transportation cost (e.g., Kleywegt et al., 2002; Pryor, 1999; Tersine et al., 1989). These models decouple logistics from production factors by using inventory as a buffer. As such, resulting optimal order (or shipment) size is based on tradeoffs between transportation cost and inventory holding cost.

Production modeling often uses similar assumption to isolate every manufacturer and process in a supply chain (e.g., Eppen and Schrage, 1981; Clark and Scarf, 1960; Hahm and Yano, 1995; Rosling, 1989; Sarker and Parija, 1994). Some in production research had recognized the relationship between shop-floor logistics, production and quality. Hopp and Spearman (1996) discussed the interaction between shop-floor logistics and product quality while Inman (1994) examined the benefit of production lot size reduction on improving product quality. Others (e.g., Djamaludin, Murthy, & Wilson, 1994; Khouja & Mehres, 1994) had used imperfect production quality and after sales costs, such as warranty, to justify small batch production. However, these models do not consider the potential of defects propagating down the supply chain. Consequently, quality problems of a manufacturer do not propagate beyond the boundary of its factory and the upstream suppliers’ operations, including production quality and logistics policy, are decoupled from downstream members’ operations.

The premise that one can de-couple logistics from production with inventory buffers can be misleading in practice. Errors in demand forecasting, inventory recording, production and transportation processes can cause unpredictable fluctuation in inventory level (Hau L. Lee & Billington, 1992). In supply chain management research, great efforts has been directed at the understanding and mitigation of the bullwhip effect, a distortion of demand information along a supply chain caused by demand uncertainty (e.g., Baganha & Cohen, 1998; Chen, Drezner, Ryan, & Simchi-Levi, 2000; Dejonckheere, Disney, Lambrecht, & Towill, 2003; Lee, Padmanabhan, & Whang, 1997). When each manufacturer in the supply chain tries to maintain an “optimal” inventory level or shipment size without considering the uncertain quality of production output from upstream, it takes too little of the downside cost on maintaining inventory buffer. In practice, inventory buffers can create more problems than they can solve (Davis, 1985). Holding excess inventory contributes to excessive production related costs, such as additional storage, administration, and material handling. Furthermore, excess inventory creates blind spots in the production process that reduce system responsiveness and impede the removal of wastes in the system, such as quality problems (Suzaki, 1987). Since these costs are often not considered in logistics planning because production is de-coupled from logistics, the “optimal” shipment size (thus, delivery interval) is larger than that encouraged by just-in-time logistics.

In this paper, we explicitly considered the interdependency among supply chain members and the impact of local quality problems on the entire supply chain. In the following sections, we started with a simplified scenario that describes the propagation of defective products in a supply chain. We then derived the equations for the expected yields under random production process failures. Finally, we analyzed the long-run behavior of the yields as a function of delivery interval. We also provided a numerical example to illustrate the financial implications.

**MODELING AND ANALYSIS OF PROPAGATION OF DEFECTS IN A SUPPLY CHAIN**

We considered a simple tandem supply chain relationship with one supplier and its buyer where parts from the supplier’s production process are inspected and stored in the outgoing inventory location before being delivered to the buyer at regular intervals. Parts received by the buyer are
stored in the incoming inventory location. These parts are inspected by the buyer before entering its production process.

We modeled each linkage between the supplier and its buyer as four sequentially linked subprocesses and locations; the production process of the supplier (P), the outgoing inspection process of the supplier (S), the overall inventory between the supplier and its buyer (L), and the incoming inspection process of the buyer (R). The overall inventory (L) is a combination of the outgoing inventory of the supplier, the inventory in transit, and the incoming inventory of the buyer.

The production process fails randomly. When this happens, the quality level of the parts from the production process (P) are lowered, i.e. the percentage of conforming parts produced is higher. Defective parts are discarded if detected. However, inspection processes are imperfect so some defective parts pass outgoing inspection (S). Without loss of generality, we assumed that the probability of rejecting a conforming part is negligible and that the outgoing inspection process detects defective products with a constant probability. We also assumed that the processes and inventory locations are First-In-First-Out.

We defined a non-conforming (defective) product as ones that do not meet the producer's or the subsequent buyers' physical or performance specifications. The parts flowing through linkage \( n \) may either enter the next linkage or be removed at the inspection processes, S and R, as illustrated in Figure 1.

Let \( p \) be the probability that non-conforming parts are produced by the production process. Let \( s \) and \( r \) be the probability that non-conforming parts are scrapped by the outgoing and incoming inspection processes, respectively. We defined the apparent yield of the \( n \)th linkage, i.e. the apparent rate that the input is converted into output by linkage \( n \), as \( e(n) \). We define the actual yield of the \( n \)th linkage, i.e. the actual rate that the input is converted into conforming output by linkage \( n \), as \( g(n) \).

A part may pass through a linkage under one of three states. In state 1, parts are produced while production process is in-control with low probability of being defective, \( p_1 \), and inspected with low rejection probability, \( r_1 \). In state 2, parts are produced while production process is out-of-control with high probability of being defective, \( p_2 \), and inspected with low rejection probability, \( r_2 \). In state 3, parts are produced while production process is out-of-control with high probability of being defective, \( p_3 \), and inspected with high rejection probability, \( r_3 \). For simplicity, we define \( p_2 = p_3 \) and \( r_1 = r_2 \). Thus, \( 1 \geq p_3 = p_2 > p_1 \geq 0 \) and \( 1 > r_1 > r_2 = r_3 \geq 0 \). Since we assumed imperfect outgoing inspection with a constant rejection rate \( s \), \( 1 > s_1 = s_2 = s_3 \geq 0 \). Thus, the values of apparent yield and actual yield from a linkage cycle through three states.
Let $P_k$ be the long-run proportion of time in each state $k$. These states follow a Semi-Markov process, so 

$$P_k = E[T_k]/E\left[\sum_{i=1}^{3} T_i\right],$$

where $T_k$ is the duration in state $k$. 

$$\tau_{FE} = T_E/T_R$$

The long-run time-weighted average of apparent yield and actual yield can be expressed as 

$$\overline{e} = \sum_{k=1}^{3} P_k e_k$$

and 

$$\overline{g} = \sum_{k=1}^{3} P_k g_k,$$

respectively. The long-run variance of apparent yield and actual yield are 

$$\sigma^2_e = \left(\sum_{k=1}^{3} P_k e_k^2\right) - \overline{e}^2$$

and 

$$\sigma^2_g = \left(\sum_{k=1}^{3} P_k g_k^2\right) - \overline{g}^2,$$

respectively.

The time varying aspect of apparent yield introduces variance on the production quantity of the supplier even if its buyer faces constant downstream demand. The impacts of logistical policy on production translate into impacts on financial performance when the cost structure associated with a unit product is defined. We grouped these costs into operations cost and liability cost. The former is directly related to the apparent yield while the latter is related to the actual yield.

**Operations Cost and Liability Cost**

We assumed that the supplier and the buyer follow a replenishment rule that keeps the total inventory level between them constant. In this case, the total inventory holding cost depends on the total amount of inventories in the block, which is constant. The operation cost per part sold in a linkage in each period is

$$c = \left[\frac{C_P + \beta C_{IR}}{e} + \frac{\beta \gamma C_k (N + M - 1)}{e_3} + \frac{C_T}{d}\left[\frac{a(t)}{K}\right]\right],$$

where $d$ is the downstream demand faced by linkage $n$ and 

$$\beta = 1 - (1 - g + pg)s.$$
value of unit failure cost, $C_F$, the present value of the unit warranty cost with warranty period $w$ is $C_W = C_F[gR_g + (1 - g)R_b]$, where

- $C_W$: present value of unit warranty cost with product warranty period $w$
- $C_F$: present value of unit failure cost for each failure incidence that resulted in a warranty expenses
- $R_g, R_b$: number of failure incidents among the conforming and non-conforming parts, respectively, within the warranty period

**Long-Run Average and Variance of Profitability**

We define profit as the difference between the revenue from parts sold and the cost of all manufacturing activities required to meet these sales. We also define unit profit and unit cost based on parts sold, not on parts produced. Thus, return on investment per unit (unit return) is the ratio of unit profit to unit cost. Under the scenario of random production process failure and imperfect inspection process, the production quantity in linkage $n$ is random even if downstream demand is constant. Therefore, the total profit generated by block $n$ is fixed, while the total cost necessary to generate this profit is random. As a result, the unit return is random.

Let $C_R$ be the unit revenue, and $\theta_N(t)$ be the unit profit from a linkage with delivery interval $N$ at period $t$. It can be shown that $\bar{\theta}_N$, the long-run average of unit profit per part sold with a delivery interval of $N$, is bounded by $\gamma_1 - \gamma_2 \bar{\sigma} + \gamma_3 \bar{g} \geq \bar{\theta}_N \geq \gamma_1 - C_T / Nd - \gamma_2 \bar{\sigma} + \gamma_3 \bar{g}$, where

$\gamma_1 = C_R - C_F R_b - (\beta_3 C_h(N + M - 1)/e_3), \gamma_2 = C_p + (C_{IR} + C_T/K)(1 - s), \gamma_3 = C_F (R_b - R_g) - (C_{IR} + C_T/K)s$. $\gamma_2$ and $\gamma_3$ are independent of the delivery interval while $\gamma_1$ decreases with respect to an increasing delivery interval. $\bar{\sigma}$ increases and $\bar{g}$ decreases as $N$ increases.

Since $\bar{g} < \bar{\sigma}$, $\bar{\theta}_N$ decreases as the delivery interval increases.

The long-run variance of the unit profit is $VAR[\theta_N(t)] = \gamma_2^2 VAR[1/e] + \gamma_3^2 VAR[g]$. $VAR[1/e]$ and $VAR[g]$ decrease to zero as the delivery interval becomes very large or very small. That is, the uncertainty of long-run unit profit decreases as the delivery interval becomes very large or very small.

**NUMERICAL EXAMPLE OF FINANCIAL PERFORMANCE ANALYSIS**

The following numerical example illustrates the decreasing profit and varying risk on profit as the delivery interval increases. Suppose that for a particular linkage in a supply chain, all inbound parts are defect-free ($g = 1$). The supplier produces parts with no defects when its production process is in-control, and half of all parts produced are defective when the production process is out-of-control ($p_1 = 0, p_2 = p_3 = 0.5$). The supplier has a 50% chance of detecting a defective part ($s = 0.5$). Suppose also that the buyer usually does not inspect inbound parts unless it detects a quality spill event from its supplier. When this occurs, the supplier inspects all inbound parts but only has a 50% chance of detecting a defective part ($r_1 = r_2 = 0, r_3 = 0.5$).

Let the supplier’s mean time between production process failures be $\tau_F = 100$ period and the buyer’s mean time to detection be $\tau_D = 1$ period. Assume that the delivery lead time is simply
$M = 1$ period. The derived parameters of this particular linkage are: $e_1 = 1$, $e_2 = 0.75$, $e_3 = 0.625$, $g_1 = 1.00$, $g_2 = 0.67$, $g_3 = 0.80$, $\gamma_2 = 25.05$ and $\gamma_3 = 6.95$.

We define return as the ratio of unit profit over the unit cost, i.e. $\frac{\bar{\theta}}{C_R - \bar{\theta}}$. Figure 2 shows the plot of return versus risk using $\max \bar{\theta}$, the upper bound on unit profit. Figure 3 shows the plot of return versus risk using $\min \bar{\theta}$, the lower bound on unit profit. The return on the vertical axis reflects the level of financial performance of the linkage. The risk on the horizontal axis represents the corresponding uncertainty of that performance. The arrow in each figure shows the direction of increasing response time in units of $1/\text{mean time between failure}$ along the line. We will divide each plot into three sections for separate analysis.

The bottom half in Figures 2 and 3, where both return and risk decrease as the response time increases, corresponds to conventional logistical policy with long delivery intervals. As the delivery interval increases and inventory level increases (while holding mean time between failure constant), risk is reduced. This reduction is the result of smaller fluctuations of production quantity necessary to meet the demand. However, apparent yield decreases as the delivery interval increases. Thus, return decreases due to greater materials loss as the delivery interval increases.

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The upper-right-hand quadrant in Figures 2 and 3 correspond to the claims of just-in-time logistical policy. As the delivery interval decreases and inventory level decreases, return is improved while risk is reduced. Unit return improves since there are fewer material losses (increasing apparent yield) as delivery interval decreases. The reduction in risk is the result of smaller fluctuations of production quantity since the apparent yield’s variance is smaller as the delivery interval decreases.

In Figure 3, the return for the Lower Bound on Unit Profit line in the upper-left-hand quadrant decreases (decreasing value along the vertical axis) as the delivery interval decreases. Here, the effect of a higher unit transportation cost due to underutilized truck capacity outweighed the savings from improving product quality. The benefit of reducing the length of delivery intervals can only be fully realized if other aspects of logistical operations are modified as well, e.g. switching to smaller trucks or implementing milk-runs to consolidate small shipments from multiple suppliers. Unit return will continue to improve as the delivery interval decreases if truck capacity can be fully utilized even as delivery interval decreases, as shown in Figure 2.

**DISCUSSION AND CONCLUSIONS**

Under the condition of imperfect product quality and imperfect detection of non-conforming products, logistical decisions can affect the product quality of a supply chain. A longer delivery interval means a slower response time, thus potentially reducing the quality of the final product. Without a systematic view on cost reduction efforts, savings in logistics can become additional costs in production operation. This can lead to higher overall costs for a manufacturer and other supply chain members.

When random production process failure and imperfect detection of non-conforming products are taken into consideration, the profitability of manufacturing operations becomes uncertain.
This is true even under the assumption of constant downstream demand. This uncertainty is also influenced by the length of delivery interval. These results agree with the benefits of just-in-time delivery policy as claimed by its practitioners. By combining the analyses on average return and its uncertainty, we demonstrate at least one possible way to reconcile the just-in-time and conventional logistical policies (Figures 2 and 3).

Product quality at the end of a supply chain, and the overall cost of the chain, are affected by the amount of inventory, stationary or en-route, between chain members. This effect is minimized only when a constant and minimum inventory level can only be achieved when the supplier and the buyer’s production are matched. Without such synchronization, the inventory level will be random and the average level will increase because all demands of the buyers must be met on time. Increasing the inventory level will decrease the long-run average of yields, and the long-run average of return. Randomness of the inventory level will increase the variance of the yields, and the uncertainty of return. Thus, collaboration between suppliers and buyers to reduce inventory stock carries financial benefits beyond inventory holding cost.

There are some limitations in our approach. First, our model does not consider rework, though this should not change significantly our conclusions. While some defective parts may be salvageable, there is additional resource requirement for reworks. Furthermore, the possibility of rework and the associated delay introduce more variability to the system. Thus though rework may improve the long-run average return, its variance may increase. Second, we model liability cost based on historical warranty cost, thus ignore the possibility of failure due to previously unknown or new sources of non-conformance. Some product liabilities may appear long after warranty has expired. This means the actual financial impact will be stronger than shown by our model, particularly since potential litigation cost is ignored. Future research may extend on our model to examine the specific effect of rework and litigation cost.

REFERENCES

References available upon request