DECISION SCIENCES INSTITUTE
Utilizing Box-Jenkins methodology to forecast intermittent demand
(Full Paper Submission)
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ABSTRACT

Intermittent demand presents difficulties in forecasting. A common situation is when a product demand shifts from normal demand to intermittent demand. In these conditions, it is not clear if Croston’s method (1972) provides advantages over single exponential smoothing (SES) or Box-Jenkins methodologies. This paper will conduct simulations to investigate these situations. SES with a constant smoothing constant, SES with an alpha determined each period by an ARIMA model, Croston’s method with a constant alpha and Croston’s method with an alpha determined by an ARIMA model will be used to determine forecasts in many demand situations.

KEYWORDS: Box-Jenkins, Croston’s method, Forecasting, Intermittent Demand

INTRODUCTION

The ability to develop accurate demand forecasts is fundamental to inventory management. The case of intermittent demand, often found with spare parts and slow-moving inventory, however continues to be a challenge to forecast accurately. It is not unusual for the periods of no demand to be clumped and when demands do occur, the amount can vary considerably. Croston’s method was designed to overcome some of the challenges of forecasting intermittent demand that are faced by traditional models like single exponential smoothing (SES) and Box-Jenkins methods for time series analysis. However, autoregressive integrated moving average (ARIMA) models may offer some benefits when forecasting intermittent data. This paper will investigate if Croston’s method can be adapted to utilize ARIMA models to vary the smoothing constant to achieve a superior forecast for intermittent demand.

Shenstone and Hyndman (2005) show that autocorrelation, non-stationary and a continuous sample space when using Croston’s method should be assumed and identify the ARIMA (0, 1, 1) model, which is the underlying model for SES (Box et al. 1994) as the appropriate model to compare to Croston’s method. Croston (1972) exposed many issues with SES when forecasting intermittent demand and provided an alternative. It is likely that most of the problems encountered with SES are likely problems with the similar ARIMA (0, 1, 1) models. The advantages of Croston’s method have been well documented by Willemain, Smart, Shockor, and DeSautels (1994), Johnston and Boylan (1996) and others. Many variations of Croston’s method have been developed, with a key correction for a positive bias by Syntetos and Boylan (2005) but most utilize a non-changing alpha.

LITERATURE REVIEW

Stock keepers are often faced with the choice of selecting the forecasting methodology to use for the software that is used to guide inventory decisions. When the demand is intermittent, often times special inventory modules are available, but when they are not, the choice might be between SES and an ARIMA model. This study will investigate if an advantage can be achieved by using an ARIMA model to vary the smoothing constant for a better forecast when using Croston’s method. Leven and Segerstedt have suggested a variation that in essence produces a general method when demand is slow or regular, but introduces more bias than Croston’s method (Teunter and Sani, 2009). Leven and Segerstedt’s (2004) provide a universal technique for all stocked items and can be utilized for all demand types but not for an item with distinct demand rates during two different periods.

The literature on forecasting demand for slow-moving items identify the need for categorizing demand patterns to provide guidance in forecasting and stock control. (Syntetos, Boylan, and Croston, 2005; Boylan, et al. 2006). By properly categorizing demand, the practitioner can select the optimal methodology for the forecast (Syntetos, et al 2005). Another reason for understanding the demand pattern is to help the manager pick the optimal smoothing constant. Croston’s method is generally used with small smoothing constants, but no optimal value has been identified for all situations. Ravinder (2013) provided a method for using Excel solver to determine an optimum smoothing constant. One advantage of using the ARIMA model is to avoid the risk of picking a smoothing constant that is not optimal.
This paper will examine if Croston’s method can be improved by substituting an ARIMA model for SES to forecast the demand means. Intermittent series violate many of the assumptions of an ARIMA model. While the SES model requires the practitioner to pick a smoothing constant, the ARIMA model does not require that one be identified in advance.

**Croston’s Method**

Croston’s (1972) method is described in this section. It is similar to the procedure for SES. It utilizes \( \alpha \), the smoothing constant, and like SES it assumes a constant demand mean of size of \( \mu \) taking place every \( p \) periods. Boylan and Syntetos (2007) correct for a bias using Croston’s method by multiplying the demand per period by \( 1 - \alpha^2 \). Teunter and Sani (2009) advocate that in some cases when only a limited periods have no demand, Croston’s method excels and when most periods have no demand, Syntetos and Boylan’s bias corrected technique is better. Willemain, et al (1994) can be consulted for the methodology used here for Croston’s method.

Croston’s (1972) suggested that ideal results would be attained with small alpha values between 0.1 and 0.2. Willemain, et al (1994) supports that lower values generally performed better as well. Since the data sets were designed to have no shifts in the mean and to be consistent with Croston, this work utilizes lower alpha levels. If a mean shift occurred for a data series, the forecast derived using a larger alpha would react quicker to the shift in mean and provide a forecast with less error.

**Single Exponential Smoothing**

Single Exponential Smoothing (SES) is a weighted-moving average technique conventionally used to forecast demand for inventoried items. The method is preferred because it is easy to use and not data intensive. The process uses the previous period’s smoothed value of \( Y_{t-1} \) to calculate the next period’s demand. The forecast is equal to a fraction of the previous error term plus the previous smoothed forecast. When the most recent value is more important to the prediction process a larger alpha value is typically used. SES is not recommended when trend or seasonality is noted. When a trend is present double exponential smoothing can be utilized. If trend and seasonality are both present, Winter’s Method can be applied. Winter’s method smooths the seasonal factor for every estimate, in essence a form of triple exponential smoothing. The goal in selecting an appropriate alpha is to pick a value that minimizes error. Trigg and Pitts (1962) provided guidelines for alpha selection. Later Ekern (1983) warns against “adaptive” methods that automatically adjust the constant in response to the forecast performance and identifies the case of the nonstationary demand rate as being especially difficult. Saygin (2007) considers the ability to maximize service level and minimize inventory cost in alpha selection.

**ARIMA Model**

Autoregressive Integrated Moving Average (ARIMA) models are a general class of models for forecasting a time series. The general ARIMA \((p, d, q)\) model is:

\[
\Phi(L)(1 - L)^d Y_t = c + \Theta(L) \epsilon_t
\]

\( y_t \) is covariance stationary. (1)

The ARIMA \((p, d, q)\) process is a stationary and invertible ARMA \((p, q)\) after differencing \( d \) times. (Diebold, 2001). \( p \) is the number of autoregressive terms, \( d \) is the number of non-seasonal differences and \( q \) is the number of lagged errors in the forecast prediction equation. In the auto-regressive integrated moving average or ARIMA model, lags of the series differences are added to the forecasting model as autoregressive terms. Lags of the forecast errors are added as moving average terms. When a series requires differencing to be stationary it requires an integrated term. The ARIMA \((0, 1, 1)\) model is essentially an exponentially weighted moving average model and is a special case of the ARIMA model. The ARIMA \((0, 1, 1)\) process is a special case of this class and is often used as the underlying model for SES. Because it is the most studied model underlying SES forecasting, this paper will use the Gaussian ARIMA \((0, 1, 1)\) model. Using the same notation as in Croston’s method, the following approach is considered a modification of Croston’s method. Instead of using SES to forecast the demand time series an ARIMA model is fitted for the series.

**SIMULATION DESCRIPTION**

To determine the forecasting performance of exponential smoothing and Croston’s Method when the smoothing constant is unchanged and when this value is determined each period by the ARIMA model a time series was simulated over 600 periods. The first 100 periods are assigned as “slow moving” and the next 200 period as “fast” moving. Then these two sets of periods are repeated for a total of 600 periods. The probabilities of a demand occurring are assigned to the respective slow and fast moving periods as follows:
1. 0.7 for the slow and 0.9 for the regular faster demand,
2. 0.5 for the slow and 0.8 for the regular faster demand,
3. 0.4 for the slow and 0.6 for the regular faster demand, and
4. 0.9 for the slow and 0.9 for the regular faster demand.

The mean demand and standard deviation during the respective slow and fast moving periods as follows: (100, 10) and (300, 20). The smoothing constant was selected to be 0.05, 0.1, 0.2, or 0.3. The exponential smoothing model and the demand smoothing model in Croston’s procedure used the ARIMA (0, 1, 1) parameter to determine the smoothing constant. The most recent 30 periods were used to fit the ARIMA model. For the Croston’s procedure the constant smoothing parameter was used for smoothing the estimate of the time periods between demands. Forecast errors were determined in how well the forecasting procedures could predict the mean values of 100 and 300 during the “slow moving” and “fast moving” periods. The average Root Mean Squared Error (RMSE) was selected as the measure of error for this study. The results are provided in Table 1. No correction term is used for adjusting for bias in Croston’s method

<table>
<thead>
<tr>
<th>Smoothing Constant</th>
<th>Probability Demand</th>
<th>Probability Demand</th>
<th>Exponential Smoothing Constant Alpha</th>
<th>Exponential Smoothing ARIMA Alpha</th>
<th>Croston with Constant Alpha</th>
<th>Croston with ARIMA Alpha</th>
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<td>56.899</td>
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<td>0.6</td>
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<td>44.489</td>
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<td>50.831</td>
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</table>

DISCUSSION OF RESULTS AND CONCLUSIONS

ARIMA models are designed to capture autocorrelation over data points that are consecutive in nature. Since the ARIMA(0, 1, 1) model is equivalent to SES, it seems logical to determine if using the smoothing constant determined by its parameters will provide a better prediction model. SES and ARIMA were designed for stationary processes. Since the simulation study is simulating demand in a non-stationary manner that is going from “fast” to “slow” the models may not be optimal.

The results of the data reveal that if the smoothing constant is small, say 0.05 or 0.1, then Croston’s method will benefit from the changing smoothing constant that the ARIMA model provides. However, if the selected smoothing constant is larger, say, 0.2 or 0.3, there is no benefit to using the changing smoothing constant offered by the ARIMA model. This may be because a larger smoothing constant adapts more quickly to changing environments. With SES, there appears to be no benefit from using a changing smoothing constant offered by the ARIMA model. This result may be due to the fact that SES is not designed for intermittent data, whereas Croston’s procedure is. It should be recognized that forecasts created with Croston’s method with the ARIMA smoothing constant typically outperformed the SES procedure whether the same or changing smoothing constant was used. More simulations are needed to make generalizations with regard to the performance of these models. In particular, very slow data needs to be simulated to test the model robustness.

In this paper, we have investigated the performance of a modification of Croston’s method that utilizes a methodology to adjust the smoothing constant when forecasting with SES or Croston’s 2.
method. It would seem this ability would be advantageous when demand conditions are not stable. The investigated procedure attempts to gain an advantage by adjusting to current demand rates. While not without criticism, Croston’s method continues to be a prevalent technique to forecast intermittent demand and special cases need to be studied.

REFERENCES


