# A NOTE ON MODELING SERVICE CAPACITY ALLOCATION IN A HETEROGENEOUS MARKET

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A mathematical programming model is formulated and then analytically solved in this paper for the problem of allocating service capacity in a multi-segment market by a monopolistic service provider facing uncertain demand that follows an exponential probability distribution. This paper also analytically and numerically examines the impacts of changes in the segment size and related price sensitivity on both the firm's optimal expected total profit and service capacity allocation scheme.

**Keywords:** capacity allocation, pricing, market segmentation, yield management

## INTRODUCTION

This paper studies a yield management problem in which a monopolistic service provider sets the price then allocates a certain amount of service capacity to each market segment during a single selling season. The service provider aims at maximizing expected total profit from the entire multi-segment market. Three noteworthy studies are relevant to this paper. Ladany and Arbel (1991), incorporating a deterministic demand function in their modeling framework, determine the optimal segmentation of all cabins of a cruise-liner. Feng and Xiao (2000) address the allocation of substitutable commodities over a market comprised of two segments, where the demand in each segment is assumed to follow a Poisson distribution and a certain amount of service capacity is reserved for the high-price segment. An analytical solution is derived to determine when to close the low-price segment as the chance of selling more items to the high-price segment arises. In the work of Deng et al. (2008), the demands in three segments are assumed to follow independent Poisson distributions. The authors develop policies to allocate capacity to segments of higher revenue. Our paper is significantly different from the three studies cited above in at least five main aspects. First, a market comprised of $n$ heterogeneous segments is taken into account in our modeling framework. Second, the uncertain demand of each segment is modeled as a general continuous random variable. Third, the allocated capacity to a segment is treated as a decision variable. Fourth, we analytically determine the optimal scheme of capacity allocation for the case in which the uncertain demand in each segment follows an exponential probability distribution. Fifth, this study analytically and numerically examines the impacts of changes in the segment size and price sensitivity on the firm's profitability and service capacity allocation scheme.

The rest of the paper is organized as follows. In the next section, the profit function for a monopolistic firm is developed, and a mathematical programming model is formulated to find the optimal scheme of service capacity allocation. The third section introduces five propositions that highlight the optimal allocation scheme and the impacts of changes in the segment size and price sensitivity. Results of a numerical investigation of the capacity allocation problem are reported in the fourth section. Finally, the paper concludes in its fifth section with a summary of its contributions, managerial implications, limitations, and directions for future research.

## MODEL FORMULATION

In this study, we focus on a profit-maximizing monopolistic service firm facing uncertain demand at the aggregate level. It is assumed in this paper that the market segments are well sealed from one another (consumers in one segment are not allowed to purchase in other segments). The three main research questions that we attempt to address in this paper can be specifically stated as follows: (i) What is the best policy for a monopolistic firm to allocate a service capacity of $K$ identical units over a market comprised of $n$ heterogeneous segments under uncertain demand during a single selling season so that its expected total profit is maximized? (ii) What is the impact of changes in a segment's size on the firm's optimal expected total profit and service capacity allocation scheme? (iii) What is the impact of changes in a segment's price sensitivity on the firm's expected total profit and optimal allocation scheme? We make the following basic assumptions while addressing the issues of strategic significance stated above:

(i) The demand of a given segment is only affected by the price charged to that segment.

(ii) The consumers of each segment are well-informed of the price charged to that segment, which is set at the beginning of the selling season.

(iii) The demand of each segment follows an independent probability distribution conditioned by the price charged to that segment.

Let us consider a monopolistic service firm that has a service capacity of $K$ identical units to be priced and allocated over a market comprised of $n$ heterogeneous segments, which are denoted as segment $i$ ($i$ = 1, 2, …, $n$). In order to model the problem of capacity allocation, the following key notations are defined:

| | |
|---|---|
| $y_i$ | the capacity to be allocated to segment $i$ (a decision variable); |
| $P_i$ | the price per unit of capacity charged in segment $i$; |
| $C$ | the cost per unit of capacity ($C > 0$); |
| $d_i$ | the aggregate demand of consumers in segment $i$; |
| $f(x \vert P_i)$ | the probability density function (p.d.f.) of the continuous random variable $d_i$; |
| $E(d_i)$ | the expected value of $d_i$; |
| $\pi_i$ | the profit yielded from segment $i$; |
| $E(\pi_i)$ | the expected value of $\pi_i$; |
| $\pi$ | the total profit yielded from the entire $n$-segment market; |
| $E(\pi)$ | the expected value of $\pi$. |

Since buyers are assumed to be well-informed of the price charged in each segment, they would take the price into consideration while making their purchase decisions. Hence, the demand of segment $i$, $d_i$ ($i$ = 1, 2, …, $n$), may be modeled as an r.v. following a continuous probability distribution conditioned by the price charged to that segment, $P_i$.

In segment $i$ ($i$ = 1, 2, …, $n$), if the demand ($d_i$ ) exceeds the allocated capacity ($y_i$), the profit ($\pi_i$) will equal the profit per unit of capacity multiplied by the number of units sold. On the other hand, if $d_i$ is smaller than $y_i$, a portion of the allocated capacity, $y_i - d_i$, will be unsold and its cost stands as a loss to the service provider. Therefore, the profit yielded from segment $i$ ($i$ = 1, 2, …, $n$) is expressed as

$$\pi_i = \begin{cases} P_i d_i - C y_i, & d_i \le y_i, \\ (P_i - C) y_i, & d_i > y_i. \end{cases} \tag{1}$$

If $d_i$ is a continuous r.v., the expected profit yielded from segment $i$ is derived from (1) as follows:

$$E(\pi_i) = P_i \int_0^{y_i} x f_i(x|P_i)dx - C y_i \int_0^{y_i} f_i(x|P_i)dx + (P_i - C) y_i \int_{y_i}^{\infty} f_i(x|P_i)dx. \tag{2}$$

As in the works of Shah and Jha (1991), the aggregate demand of consumers in each of the $n$ segments is modeled in our study as a random variable following an exponential probability distribution. The p.d.f. of the demand of segment $i$, $d_i$, is assumed to take the following continuous form:

$$f_i(x|P_i) = \alpha_i P_i^{r_i} e^{-\alpha_i P_i^{r_i} x} \quad \text{for } x \in [0, \infty). \tag{3}$$

Based on (3), the expected demand of segment $i$ is given by

$$E(d_i) = (\alpha_i P_i^{r_i})^{-1}, \tag{4}$$

where $\alpha_i > 0$ and $r_i \in (0, 1)$. The scaling factor $\alpha_i$ in both (3) and (4) reflects the overall size of segment $i$ and the factor $r_i$ measures the degree to which demand is sensitive to price changes within that segment. Given $P_i > 0$ and $r_i \in (0, 1)$, a smaller (larger) value of $\alpha_i$ indicates a stronger (weaker) expected demand, implying a larger (smaller) segment size. A smaller (larger) value of $r_i$ indicates that the demand in segment $i$ is relatively less (more) price-sensitive. It can be conveniently proven that given $P_i > 1$ and $\alpha_i > 0$, a smaller (larger) value of $r_i$ leads to a stronger (weaker) expected demand. Empirical support for this form of demand function is found in the literature (see Pavia, 1995).

Substituting (3) into (2) yields

$$E(\pi_i) = \frac{P_i}{\alpha_i P_i^{r_i}}(1 - e^{-\alpha_i P_i^{r_i} y_i}) - C y_i. \tag{5}$$

The expected total profit from the $n$-segment market is expressed by

$$E(\pi) = \sum_{i=1}^{n} E(\pi_i). \tag{6}$$

Given a service capacity $K$ of the service firm, which is to be allocated to an $n$-segment market for sale at a predetermined price per unit of service capacity $P_i > C$, we aim at finding the optimal capacity to be allocated to segment $i$ ($i$ = 1, 2, …, $n$), $y_i^*$, to maximize the firm's expected total profit. Hence, the problem is formulated as follows:

$$\max_{y_1, y_2, \ldots, y_n} \sum_{i=1}^{n} E(\pi_i)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} y_i \le K,$$

$$y_i \ge 0 \text{ for } i = 1, 2, \ldots, n. \tag{7}$$

Next, we analytically solve model (7) and then examine the impacts of changes in the segment size and price sensitivity on the firm's profitability and service capacity allocation scheme.

## OPTIMAL CAPACITY ALLOCATION

As in the study of Lee and Ng (2001), our study only focuses on the case in which an interior solution exists such that $\sum_{i=1}^{n} y_i^* < K$. Otherwise, the results will be strictly driven by the boundary and maximum achievable profits may not be attained (Lee and Ng 2001, p. 222). Here we provide an analytical solution to model (7). As a result, five propositions are introduced below, for which the proofs are available from the authors upon request.

**Proposition 1.** *Given* $\sum_{i=1}^{n} \frac{1}{\alpha_i P_i^{r_i}} \ln(\frac{P_i}{C}) < K$, *the expected total profit E* ($\pi$) *reaches its maximal level* $E^*(\pi) = \sum_{i=1}^{n} E^*(\pi_i)$ *at* $y_i^* = \frac{1}{\alpha_i P_i^{r_i}} \ln(\frac{P_i}{C})$ *for i = 1, 2, …, n,*
*where* $E^*(\pi_i) = \frac{1}{\alpha_i P_i^{r_i}} [P_i(1 - \frac{C}{P_i}) + C \ln(\frac{C}{P_i})]$.

Proposition 1 shows that the firm could optimize its profitability by properly allocating its capacity to each of the *n* segments. The optimal capacity allocated to segment *i*, $y_i^*$, is determined by several factors, namely, the size and price sensitivity of segment *i*, the price charged in the segment, and the cost per unit of capacity.

**Proposition 2.** *Given* $P_i > C$ *and all other things equal, the optimal expected total profit,* $E^*(\pi)$, *is a monotonically decreasing function of the scaling factor* $\alpha_i$ *of segment i* ($i = 1, 2, \ldots, n$).

**Proposition 3.** *Given* $P_i > C$ *and* $P_i > 1$ *and all other things equal, the optimal expected total profit,* $E^*(\pi)$, *is a monotonically decreasing function of the price-sensitivity factor* $r_i$ *of segment i* ($i = 1, 2, \ldots, n$).

**Proposition 4.** *Given* $P_i > C$ *and all other things equal, the optimal capacity allocated to segment i,* $y_i^*$, *is a monotonically decreasing function of the scaling factor* $\alpha_i$ *of segment i* ($i = 1, 2, \ldots, n$).

**Proposition 5.** *Given $P_i > C$ and $P_i > 1$ and all other things equal, the optimal capacity allocated to segment i, $y_i^*$, is a monotonically decreasing function of the price-sensitivity factor $r_i$ of segment i* ($i = 1, 2, \ldots, n$).

Propositions 2 and 3 advocate increasing the size of each market segment (i.e., smaller $\alpha_i$) and reducing the consumers' price-sensitivity (i.e., smaller $r_i$) to enhance the firm's profitability. Proposition 2, in particular, supports the recommendation of designing purchase options that expand the market segment to which the firm sells (Lieberman, 2011). In order to alter the segment size and price sensitivity, the firm may consider options such as upgrading the service, advertising and sales promotion. Propositions 4 and 5 suggest that the firm might take a preferential approach to capacity management, allocating larger portions of capacity to those market segments of larger sizes and lower price-sensitivity.

## NUMERICAL ILLUSTRATIONS

This section presents a numerical study to (i) show the various typical patterns of the optimal allocation scheme, and (ii) investigate the impacts of changes in the segment size and price-sensitivity on the service provider's expected total profit and optimal allocation scheme. We consider a four-segment market in the presence of price discrimination (He and Sun, 2006; Anderson and Dana, 2009). It is assumed in the numerical examples that cost-plus pricing is adopted by the service provider. In a survey of service firms, Zeithaml et al. (1985) report that 63% of the firms base their prices primarily on costs.

With price discrimination and cost-plus pricing implemented across the four market segments by the firm, the prices are set at $P_i = (1 + 0.3i)C$ ($i = 1, 2, 3, 4$), which indicates that the price charged in each segment exceeds the cost per unit of capacity and the price in segment $i$+1 is higher than that in segment $i$ for $i = 1, 2, 3$. For illustrative purposes, the cost per unit of capacity is selected as $C$ = \$20,000. The values of both $\alpha_i$ and $r_i$ ($i = 1, 2, 3, 4$) chosen to examine the impacts of changes in each segment's size and price sensitivity are given in Table 1. The vectors ($\alpha_1, \alpha_2, \alpha_3, \alpha_4$) and ($r_1, r_2, r_3, r_4$) are denoted by $\alpha$ and $r$, respectively. Based on Proposition 1, the optimal allocation scheme $\{y_i^*\}$ ($i = 1, 2, 3, 4$) and the corresponding expected total profit $E^*(\pi)$ are calculated for each combination of the selected values of the model parameters. Our computational results are only partially reported in Table 2 for illustrative purposes.

Table 2 reports, for $C$ = \$20,000 and the considered prices $\{P_i\}$, the optimal allocation scheme $\{y_i^*\}$ and the optimal expected total profit $E^*(\pi)$. As shown in Table 2, for example, given $\alpha$ = (0.05, 0.05, 0.05, 0.05) and $r$ = (0.08, 0.06, 0.04, 0.02), the firm should allocate 2.327, 5.045, 8.419, and 12.733 units of capacity to segment $i$ ($i = 1, 2, 3, 4$), respectively, and as a result, the expected total profit would be \$235,227.61.

The four market segments in Table 2 are characterized by identical values of the segment-size parameter but different values of the price-sensitivity parameter. It is noted in Table 2 and from our computational results (not reported here) that charging higher prices to less price-sensitive segments would generate a higher expected total profit, while an increase in the price-sensitivity across the four segments reduces the expected total profit. Our simulation experiments reveal that larger segment sizes enhance the firm's profitability.

**Table 1** Parameter settings for $r = (r_1, r_2, r_3, r_4)$ and $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ in the simulation experiments

| | |
|---|---|
| $r$ = (0.02, 0.04, 0.06, 0.08) with | $r$ = (0.08, 0.06, 0.04, 0.02) with |
| Example 1.1. $\alpha$ = (0.05, 0.05, 0.05, 0.05) | Example 2.1. $\alpha$ = (0.05, 0.05, 0.05, 0.05) |
| Example 1.2. $\alpha$ = (0.10, 0.10, 0.10, 0.10) | Example 2.2. $\alpha$ = (0.10, 0.10, 0.10, 0.10) |
| Example 1.3. $\alpha$ = (0.15, 0.15, 015, 0.15) | Example 2.3. $\alpha$ = (0.15, 0.15, 0.15, 0.15) |
| $r$ = (0.03, 0.05, 0.07, 0.09) with | $r$ = (0.09, 0.07, 0.05, 0.03) with |
| Example 3.1. $\alpha$ = (0.05, 0.05, 0.05, 0.05) | Example 4.1. $\alpha$ = (0.05, 0.05, 0.05, 0.05) |
| Example 3.2. $\alpha$ = (0.10, 0.10, 0.10, 0.10) | Example 4.2. $\alpha$ = (0.10, 0.10, 0.10, 0.10) |
| Example 3.3. $\alpha$ = (0.15, 0.15, 015, 0.15) | Example 4.3. $\alpha$ = (0.15, 0.15, 0.15, 0.15) |

**Table 2** Optimal schemes of capacity allocation ($C$ = \$20,000, $\alpha$ = (0.05, 0.05, 0.05, 0.05))

| $P_1$(\$) | $P_2$(\$) | $P_3$(\$) | $P_4$(\$) | |
|---|---|---|---|---|
| 26,000 | 32,000 | 38,000 | 44,000 | |
| $r_1$ | $r_2$ | $r_3$ | $r_4$ | |
| 0.02 | 0.04 | 0.06 | 0.08 | |
| $y_1^*$ | $y_2^*$ | $y_3^*$ | $y_4^*$ | $E^*(\pi)$ (\$) |
| 4.282 | 6.208 | 6.818 | 6.704 | 171,452.58 |
| $r_1$ | $r_2$ | $r_3$ | $r_4$ | |
| 0.08 | 0.06 | 0.04 | 0.02 | |
| $y_1^*$ | $y_2^*$ | $y_3^*$ | $y_4^*$ | $E^*(\pi)$ (\$) |
| 2.327 | 5.045 | 8.419 | 12.733 | 235,227.61 |
| $r_1$ | $r_2$ | $r_3$ | $r_4$ | |
| 0.03 | 0.05 | 0.07 | 0.09 | |
| $y_1^*$ | $y_2^*$ | $y_3^*$ | $y_4^*$ | $E^*(\pi)$ (\$) |
| 3.868 | 5.596 | 6.136 | 6.024 | 154,295.90 |
| $r_1$ | $r_2$ | $r_3$ | $r_4$ | |
| 0.09 | 0.07 | 0.05 | 0.03 | |
| $y_1^*$ | $y_2^*$ | $y_3^*$ | $y_4^*$ | $E^*(\pi)$ (\$) |
| 2.102 | 4.547 | 7.577 | 11.442 | 211,575.97 |

## CONCLUDING REMARKS

This paper tackles the problem of optimally allocating a service capacity by a monopolistic firm over a general *n*-segment market under uncertain demand. A mathematical programming model is developed and then analytically solved to determine the optimal scheme of capacity allocation**.**

In addition to the methodological contribution, simulation experiments are performed in this paper to numerically study the impacts of changes in the segment size and price sensitivity. Our findings suggest that less price-sensitive segments should be charged higher prices and receive larger proportions of service capacity; everything else being constant, more capacity should be allocated to larger segments. The firm's efforts to upgrade the service and promotional activities such as advertising could be geared toward enlarging the segment size and reducing the price sensitivity of consumers to improve profitability.

This exploratory study suggests some possibilities for future research. First, in this paper we have examined the situation of a monopolistic service firm that aims at determining its optimal scheme of capacity allocation over a multiple-segment market. An interesting research direction for the future would be to take competition into account and formulate the problem from a game-theory point of view. Second, our study has only provided an analytical solution to the problem of capacity allocation for the uncertain demand following an exponential probability distribution. A rather challenging direction for future research would be to analytically solve the problem for a demand following other probability distributions. Third, empirical studies could be conducted to explore the probability distributions of the uncertain demand of each segment and find their structures. Addressing these unanswered questions will undoubtedly broaden the horizon of research on the allocation of service capacity in a heterogeneous market.

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