PRE-FUNDED COUPON AND ZERO-COUPON BONDS: COST OF CAPITAL ANALYSIS

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ABSTRACT.

Pre-funded coupon bonds have been developed and sold by investment bankers in place of zero-coupon bonds to raise funds for companies facing cash flow problems. Additional bonds are issued and proceeds are deposited in an escrow account to finance the coupon payment. Our analysis indicates that a pre-funded coupon bond is equivalent to a zero-coupon bond only if the return from the escrow account is the same as the yield to maturity of the pre-funded issue. In reality, the escrow return is lower than the bond yield. As a result, the firm provides interest subsidy through issuing additional bonds which leads to higher leverage, greater risk and loss of value compared to a zero-coupon issue.

Keywords: zero-coupon bond, Macaulay duration, financial engineering

INTRODUCTION

Pre-funded coupon bonds were first issued in 1994 (Doherty, 1997). They were introduced as a means to raise capital for firms unable to generate cash-flow to make coupon payments, while still meeting the needs of investors to receive coupon income. With a pre-funded bond structure, additional bonds are issued and an escrow account is established to finance coupon payments over the life of the bond. In this manner, the bond is considered pre-funded. The firm is not required to generate cash flow to meet coupon obligations; it is paid out of the escrow account usually collateralized by zero-coupon Treasury securities. The risk free coupon payment allows the firm to set a lower coupon rate on the bond than the yield on a comparable zero-coupon bond. In general, the cost of funding the escrow account is greater than the return of the escrow account. This leads to an interest rate subsidy and the loss of value. In this paper, we compare zero-coupon bonds to pre-funded bonds and ascertain conditions under which the two the funding options are equivalent. A pre-funded issue simultaneously creates an asset and a liability. The net duration of the pre-funded issue is the weighted average of the asset and liability durations. The model of net duration developed in this paper incorporates increased leverage of the pre-funded issue and appropriately assess its increased risk. In spite of the fact that pre-funded bond is an interesting concept of financial engineering, there is very little academic research on this topic.

The remainder of this paper is made up of four sections. Section II discusses the options available to a firm interested in issuing debt. In Section III we derive a mathematical model for McCauley duration of the pre-funded issue to determine the interest rate risk and calculate the loss in value due to interest rate subsidy. A numerical example and its analysis are presented in Section IV. Section V concludes the paper.

FUNDING OPTIONS

A firm wants to raise funds to finance a new project. The pecking-order theory of capital structure suggests that mangers prefer internal equity to external financing (Myers, 1984). In case the internal equity (retained earnings) is not available then issuing new debt is preferred over issuing preferred or additional common stock. Further, firm would like to reduce the interest payment burden. Hence, conventional coupon bond or hybrid financing such as convertible bonds or bonds with warrants are ruled out. The available funding options are (1) zero-coupon bonds, (2) step-up bonds – initially coupon payment is set at a low value and later stepped up, (3) deferred interest bonds – initially there is no interest payment, but it is resumed in 3-7 years, (4) paid-in-kind bonds – issuer has right to pay interest in cash or with similar bonds, and (5) - pre-funded bonds. The focus of the study is to compare zero-coupon and pre-funded bonds.

Pure discount bonds are often called zero-coupon bonds. It was first issued by J.C. Penney Company Inc. in 1982 (Brigham and Daves, 2010). In recent years, other firms (e.g. IBM, GMAC, Alcoa and Martin-Marietta) have issued zero-coupon bonds. Municipalities started issuing zero-coupon bonds in 1983. These bonds are sold at a deep discount and increase in value as they approach maturity. Zero-coupon bonds do not provide interest or coupon payments at regular intervals like other types of bonds. Implicit coupons are automatically reinvested by the issuer at yield to maturity. Interest accrues over the life of the bond and a return is earned as the bond appreciates. At maturity its value equals the face value and bond holder receives the yield to maturity expected at the time of purchase. If held to maturity, the investor faces no reinvestment risk but high interest rate risk, as its market price fluctuates considerably with movements in market rates.

Corporate and municipal zero-coupon are usually callable and rated as junk bonds. The financial condition of the company issuing bonds predicates the use of junk bonds, i.e. the firm is unable generate cash flows to meet coupon payments. Junk bonds are typically rated BB or lower by Standard and Poor's, or BA and lower by Moody's. Junk bonds offer a high expected return but require investors to take on higher default risk. Covenants on junk bonds are less restrictive and therefore provide alternatives for firms that may not meet the more restrictive covenants of conventional bonds. U.S. Treasury sells Zero-coupon bonds in the form of STRIPs (Separate Trading of Registered Interest and Principle of Security Program). A long term T-bond is stripped into its coupon payments and the principle. Each cash flow is sold as a zero-coupon bond. Lately, these zero-coupon Treasuries have become very popular with the investors because of low interest rate and inflation (Zeng 2008).

In raising capital with a pre-funded bond issue, additional bonds are issued and an escrow account is established. The firm is not required to generate cash flow to meet coupon obligations over the life of the bond. Bond interests are paid out of an escrow account, which is usually

collateralized by Treasury securities. In this manner, the bond is considered pre-funded. A pre-funded bond issue simultaneously creates an asset and a liability. The risk characteristics of pre-funded bond's interest payments are different from that of traditional coupon-bearing bonds because pre-funded bond's coupon payments are asset based. The default free nature of the coupon payment allows the firm to set a lower coupon rate than the yield on a comparable zero-coupon bond. In general, the cost of funding the escrow account is greater than the return from the escrow account. This spread leads to an interest rate subsidy which necessitates issuing more bonds and hence a loss of value. Greater the spread between cost of funding the escrow account and the return from the escrow account, larger the total face value of the pre-funded issue and the value loss. With a pre-funded bond issue there are additional flotation costs and cost of establishing the escrow account. However, for this analysis, we consider the escrow costs and additional flotation costs to be negligible.

Market price of pre-funded bonds fluctuates with movements in market rates, but it does not move as dramatically as zero-coupon bond prices. The reason for this difference is that zero-coupon bonds do not provide any cash flow until maturity. Coupon payments reduce the impact of interest rate changes on pre-funded bonds. Market conditions where interest rate movements are frequent and highly variable make pre-funded bond more attractive than zero-coupon bonds. The risk profiles of zero-coupon and pre-funded bonds can be summarized as follows: A zero-coupon bond has no reinvestment risk, higher price elasticity to interest rate changes and a default risk consistent with its junk bond rating. The pre-funded bond has reinvestment risk but lower price elasticity to interest rate changes. For a meaningful analysis of the interest rate risk, one must examine the combined interest rate sensitivity of the escrow asset and the bond liability. The default risk of the pre-funded issue should be decomposed into two components: the default risk of the coupon payments and the default risk of the maturity payment. The coupon payments are default free but the default risk of the maturity payments are much higher. This is due to increased leverage of the pre-funded issue compared to zero-coupon financing. In spite of the default free coupon payments, the pre-funded bonds are usually rated as junk bonds.

In the next section, the combined interest rate sensitivity of the escrow asset and the bond liability is examined. A model for the net Macaulay duration of the pre-funded issue is developed and loss of value due to interest rate subsidy is calculated.

MCCAULEY DURATION AND VALUE LOSS

In this section we calculate the total face value of the pre-funded bonds issued, initial balance of the escrow account, interest rate subsidy provided by the firm, effective cost of the pre-funded issue and resulting loss of value. Also, we derive an expression for the net Macaulay duration of the pre-funded issue, i.e. the weighted average durations of the coupon bond and the escrow asset. The face values of zero-coupon bonds issued, to raise an amount B, is

$$\mathbf{B_z = B(1 + r_z)^n} \qquad \textbf{(1)}$$

where r_z is the discount rate for the zero-coupon bond with maturity n. The Macaulay duration of zero-coupon bond is its maturity (Fabbozzi, 2010). Let B_{pf} be the face value of the pre-funded bonds issued to raise an amount B. The annual coupon payment is $B_{pf}(r_{pf})$, where r_{pf} is the pre-

funded bond yield. The initial balance in the escrow annuity account set up to meet the coupon payments is $B_{pf} - B$. Hence,

$$\mathbf{B_{pf} - B = (B_{pf} r_{pf})(PVIFA_{r_{es},n})}$$

or

$$\mathbf{B_{pf} = \frac{B}{1 - r_{pf}(PVIFA_{r_{es},n})}} \quad \textbf{(2)}$$

where PVIFA indicates present value interest factor of an annuity, n is the maturity and r_{es} is the rate of return on the escrow account. Substituting the algebraic expression for PVIFA we get

$$\mathbf{B_{pf} = \frac{r_{es}(1+r_{es})^n B}{r_{pf} - (r_{pf} - r_{es})(1+r_{es})^n}} \quad \textbf{(3)}$$

The initial balance in the escrow account is

$$\mathbf{B_{pf} - B = \frac{r_{pf}\{(1+r_{es})^n - 1\}B}{r_{pf} - (r_{pf} - r_{es})(1+r_{es})^n}} \quad \textbf{(4)}$$

Escrow account is funded at a cost of r_{pf} and provides a return of r_{es}. Consequently, the firm is providing a pre-tax interest subsidy of $(r_{pf} B_{pf})(r_{pf} - r_{es})$ per year, which increases the cost of pre-funded issue and leads to loss of value. The loss of value is:

$$\mathbf{Value\ Loss = (r_{pf} B_{pf})(r_{pf} - r_{es})\frac{(1+r_{pf})^n - 1}{(1+r_{pf})^n}} \quad \textbf{(5)}$$

and the effective cost of the pre-funded issue is given by:

$$\mathbf{r_{eff} = \left(\frac{r_{es}(1+r_{es})^n}{r_{pf} - (r_{pf} - r_{es})(1+r_{es})^n}\right)^{\frac{1}{n}} - 1} \quad \textbf{(6)}$$

The concept of duration was introduced by Macaulay (1938) as a measure of price sensitivity of an asset or liability to a change in interest rates. Working independently, Samuelson (1945) and Redington (1952) developed the same concept about the interest rate risk of bonds. Details of duration computation can be found in any finance text (Fabbozzi, 2010). A pre-funded bond issue creates an asset, the escrow account annuity with market value $B_{pf} - B$; and a liability, coupon bonds with market value $B_{pf.}$ The net market value of the pre-funded issue is B. Let D_{es} and D_{pf} represent the duration of escrow annuity and the bond liability respectively. Duration D_{es} is the Macaulay duration of an n-year annuity with yield r_{es} and D_{pf} is the Macaulay duration of an n-year coupon bond with yield to maturity r_{pf}. The net duration of the pre-funded issue is the weighted average of the durations of the escrow account and the coupon bond. Hence

$$\mathbf{D_{net} = \frac{B_{pf}}{B} x D_{pf} - \frac{B_{pf} - B}{B} x D_{es}} \quad \textbf{(7)}$$

where $\{B_{pf} / B\}$ and $\{-(B_{pf} - B)/B\}$ are the weights of the coupon bond and the escrow annuity respectively. This definition of net duration, D_{net}, captures the increased risk due to additional leverage caused by pre funding of coupon payments and interest subsidy provided by the firm.

INUMERICAL EXAMPLE AND ANALYSIS

A firm wants to raise $10 million by issuing either zero-coupon bonds or pre-funded bonds with five or ten year maturity. We assume that transaction costs are identical for both issues and

negligible (Alternately, we can assume that all yields are net of transaction costs). Further, we assume that financial market views the zero-coupon and pre-funded bonds to be equivalent securities, and prices them with identical yields. Four different yields, 8 percent, 7 percent, 6 percent and 5 percent for zero-coupon and pre-funded bonds are considered for this analysis. Later, we modify this assumption and consider the situation where market views pre-funded bond to be safer and erroneously prices them with yields lower than the comparable zero-coupon yields by 25, 50 and 75 basis points. In doing so, market overlooks the added default risk associated with increased leverage.

Table 1 presents the face value of zero-coupon bonds issued to meet $10 million funding need. For 5-year maturity with discount rates of 8 percent, 7 percent, 6 percent, and 5 percent, the firm issues zero-coupon bonds with total face values of $14,693,281, $14,025,517, $13,382,256 and $12,762,816 respectively. These values are calculated using equation (1). The Macaulay duration of the 5-year zero-coupon bond is 5 years. For 10-year zero-coupon bonds, an 8 percent, 7 percent, 6 percent, and 5 percent discount rate leads to total face values of $21,589,250, $19,671,514, $17,908,477 and $16,288,946 respectively. The Macaulay duration of the 10-year zero-coupon bond is 10 years.

Table 1

Zero-Coupon Bond

		Discount Rate, r_z			
		8%	7%	6%	5%
Maturity, n 5 years	Funds needed, B	$ 10,000,000	$ 10,000,000	$ 10,000,000	$ 10,000,000
	Face value of bonds issued, B_z	$ 14,693,281	$ 14,025,517	$ 13,382,256	$ 12,762,816
	Market value of bonds issued	$ 10,000,000	$ 10,000,000	$ 10,000,000	$ 10,000,000
	Duration, D_z	5 years	5 years	5 years	5 years
Maturity, n 10 years	Funds needed, B	$ 10,000,000	$ 10,000,000	$ 10,000,000	$ 10,000,000
	Face value of bonds issued, B_z	$ 21,589,250	$ 19,671,514	$ 17,908,477	$ 16,288,946
	Market value of bonds issued	$ 10,000,000	$ 10,000,000	$ 10,000,000	$ 10,000,000
	Duration, D_z	10 years	10 years	10 years	10 years

$B_z = B(1 + r_z)^n$ and $D_z = n$

In Table 2 we present the total face value of the pre-funded issue, amount of annual coupon payment disbursed from escrow account and the effective cost of pre-funded issue. It provides following important inferences. First, when the pre-funded bond yield, r_{pf}, is the same as the escrow account return, r_{es}, then (i) total face value of the pre-funded issued is the as same the total face value of the zero-coupon bonds and (ii) the effective cost of pre-funded issue, r_{eff}, is the same as the yield to maturity of the zero-coupon bond, r_z. Second, increase in the spread between r_{pf} and r_{es} increases the total face value of the bonds issued and its effective cost. Finally, for a given spread the total face value of the bonds issued and its effective cost increases with maturity.

Table 2

Total Face Value and Effective Cost of Pre-funded Issue

$$B_{pf} = \frac{r_{es}(1+r_{es})^n B}{r_{pf} - (r_{pf} - r_{es})(1+r_{es})^n} \text{ and } r_{eff} = \left(\frac{r_{es}(1+r_{es})^n}{r_{pf} - (r_{pf} - r_{es})(1+r_{es})^n} \right)^{\frac{1}{n}} - 1$$

	Escrow Return, r_{es}		**Pre-funded Bond Yield, r_{pf}** 8%	7%	6%	5%
Maturity, n 5 years	8%	Face value, B_{pf}	$ 14,693,281			
		Escrow payment	$ 1,175,462			
		Effective Cost, r_{eff}	8.000%			
	7%	Face value, B_{pf}	$ 14,881,302	$ 14, 025,517		
		Escrow payment	$ 1,190,504	$ 981,786		
		Effective Cost, r_{eff}	8.275%	7.000%		
	6%	Face value, B_{pf}	$ 15,082,708	$ 14,181,691	$ 13,382,256	
		Escrow payment	$ 1,206,617	$ 992,718	$ 802,935	
		Effective Cost, r_{eff}	8.567%	7.237%	6.000%	
	5%	Face value, B_{pf}	$ 15,298,893	$ 14,368,507	$ 13,509,289	$ 12,762,816
		Escrow payment	$ 1,223,912	$ 1,004,395	$ 810,557	$ 638,141
		Effective Cost, r_{eff}	8.876%	7.518%	6.201%	5.000%
Maturity, n 10 years	8%	Face value, B_{pf}	$ 21,589,250			
		Escrow payment	$ 1,727,140			
		Effective Cost, r_{eff}	8.000%			
	7%	Face value, B_{pf}	$ 22,825,137	$ 19,671,514		
		Escrow payment	$ 1,826,011	$ 1,377,006		
		Effective Cost, r_{eff}	8.603%	7.000%		
	6%	Face value, B_{pf}	$ 24,319,478	$ 20,627,322	$ 17,098,477	
		Escrow payment	$ 1,945,558	$ 1,443,913	$ 1,074,509	
		Effective Cost, r_{eff}	9.294%	7.509%	6.000%	
	5%	Face value, B_{pf}	$ 26,160,123	$ 21,763,801	$ 18,632,525	$ 16,288,946
		Escrow payment	$ 2,092,810	$ 1,523,466	$ 1,117,952	$ 814,447
		Effective Cost, r_{eff}	10.094%	8.087%	6.421%	5.000%

Empty cell represent the improbable case of $r_{pf} < r_z$.
If $r_{es} = r_{pf}$ then $B_{pf} = B(1 + r_{pf})^n = B_z$ and $r_{eff} = r$

For example, consider the case when both r_{pf} and r_{es} are equal to 8 percent and firm wants to issue bonds 5-year maturity bonds to raise $10 million. It can issue either zero-coupon bonds or pre-funded coupon bonds with $14,693,281 face value and 8 percent effective costs. For 10-year maturity, it will have to issue $21,589,250 zero-coupon or pre-funded bonds. However, with a 3 percent spread, i.e. r_{pf} = 8 percent and r_{es} = 5 percent, the firm will have to issue $15,298,893 coupon bonds with maturity 5 years or $26,160,132 coupon bonds with maturity 10 years. The effective cost of 5-year and 10-year pre-funded issues will rise to 8.876 percent and 10.094 percent respectively. Examples of net duration of pre-funded issue, i.e. the weighted average durations of the escrow asset and coupon bond liability are presented in Tables 3 and 4. In Table 3 we present a 5-year bond issue without spread i.e. both r_{pf} and r_{es} are equal to 8 percent. Firm issues $14,693,281 bonds with annual coupon payment of $1,175,462. Coupon payments are disbursed out of an escrow account with $4,693,281 initial balance. Panel A of Table 3 shows that duration of the coupon bond, D_{pf}, is 4.3121 years. Panel B of Table 3 shows that the duration of the escrow annuity, D_{es}, is 2.8465 years. Panel C of Table 3 shows that the weights of bond liability and escrow asset are 1.469 and -.469 respectively. Hence, the net duration, D_{net}, of the pre-funded issue is 5 years, which is identical to the duration of a zero-coupon bond. The result is understandable because firm has no net cash outflow for years 1 to 4, the only cash outflow of $14,693,281 is in year 5.

Table 3

Net Duration of the Pre-Funded Issue

$$\mathbf{D_{net} = \frac{B_{pf}}{B} x D_{pf} - \frac{B_{pf} - B}{B} x D_{es}}$$

Panel A: Bonds Issued

Time, t	Cash Outflow, CF	$PVIF_{8\%,5}$	CF*PVIF	t*CF*PVIF	Duration, D_{pf}
1	$ 1,175,462	0.9259	$ 1,088,391	$ 1,088,391	
2	1,175,462	0.8573	1,007,769	2,015,538	
3	1,175,462	0.7938	933,120	2,799,359	
4	1,175,462	0.7350	864,000	3,455,999	
5	15,868,743	0.6806	10,800,000	53,999,999	
			$14,693,280	$63,359,286	4.3121

Panel A: Escrow Annuity

Time, t	Cash Inflow, CF	$PVIF_{8\%,5}$	CF*PVIF	t*CF*PVIF	Duration, D_{es}
1	$ 1,175,462	0.9259	$ 1,088,391	$ 1,088,391	
2	1,175,462	0.8573	1,007,769	2,015,538	
3	1,175,462	0.7938	933,120	2,799,359	
4	1,175,462	0.7350	864,000	3,455,999	
5	1,175,462	0.6806	800,000	3,999,998	

			$ 4,693,280	$13,359,282	2.8465

Panel C: Net Durations

Fund raised, B	$10,000,000	Escrow amount, B_{pf} –B	$ 4,693,281
Face value of bond, B_{pf}	$14,693,281	Escrow return, r_{es}	8%
Bond yield, r_{pf}	8.00%	Escrow weight, $(B_{pf}–B)/B$	0.469
Bond weight, B_{pf}/B	1.469	Escrow duration, D_{es}	2.847
Bond duration, D_{pf}	4.312	Net duration, D_{net}	5.000

If escrow return equals the bond yield, i.e. $r_{es} = r_{pf}$, then the net duration equals the maturity.

In Table 4 we present an example of 5-year pre-funded bond issue with 3 percent spread i.e. r_{pf} = 8percent and r_{es} = 5percent. Firm issues $15,298,250 bonds with annual coupon payment of $1,223,912. Coupon payments are disbursed out of an escrow account with $5,298,250 initial balance. Firm provides the interest subsidy by issuing additional bond compared to Table 3 example. Panel A of Table 4 shows that the duration of the coupon bond, D_{pf}, is 4.3121 years, same as the Table 3 example. But the duration of the escrow annuity, D_{es}, increases to 2.9025 years. The weights of bond liability and escrow asset, reported in Panel C of Table 4, are 1.530 and -.530 respectively. The net duration, D_{net}, of the pre-funded issue increases to 5.059 years. The interest subsidy creates the additional leverage and which stretches the duration beyond its maturity.[1] Because interest subsidy is a realistic condition, the pre-funded bond issue has greater interest rate risk than the comparable zero-coupon bond.

Table 4

Net Duration of the Pre-Funded Issue

$$\mathbf{D_{net} = \frac{B_{pf}}{B} x D_{pf} - \frac{B_{pf} - B}{B} x D_{es}}$$

Panel A: Bonds Issued

ne, t	Cash Outflow, CF	$PVIF_{8\%,5}$	CF*PVIF	t*CF*PVIF	Duration, D_{pf}
1	$ 1,223,912	0.9259	$ 1,133,252	$ 1,133,252	
2	1,223,912	0.8573	1,049,307	2,098,615	
3	1,223,912	0.7938	971,581	2,914,742	
4	1,223,912	0.7350	899,612	3,598,447	
5	16,522,162	0.6806	11,244,706	56,223,529	
			$15,298,458	$65,968,585	4.3121

Panel A: Escrow Annuity

Time, t	Cash Inflow, CF	$PVIF_{5\%,5}$	CF*PVIF	t*CF*PVIF	Duration, D_{es}
1	$ 1,223912	0.9524	$ 1,165,630	$ 1,165,630	
2	1,223,912	0.9070	1,110,124	2,220,249	
3	1,223,912	0.8638	1,057,261	3,171,784	
4	1,223,912	0.8227	1,006,915	4,027,662	
5	1,223,912	0.7835	958,967	4,794,835	
			$ 5,298,897	$15,380,160	2.9025

Panel C: Net Durations

Fund raised, B	$ 10,000,000	Escrow amount, B_{pf} –B	$ 5,298,250
Face value of bond, B_{pf}	$ 15,298,250	Escrow return, r_{es}	5%
Bond yield, r_{pf}	8.00%	Escrow weight, $(B_{pf}–B)/B$	0.530
Bond weight, B_{pf}/B	1.530	Escrow duration, D_{es}	2.903
Bond duration, D_{pf}	4.312	Net duration, D_{net}	5.059

If escrow return is less than the bond yield, i.e. $r_{es} < r_{pf}$, then the net duration exceeds the maturity.

Table 5 presents net duration, interest subsidy and loss of value associated with a pre-funded bond issue for different bond yields and escrow returns. When $r_{pf} = r_{es}$ then there is no interest subsidy or loss of value and the net duration of the pre-funded issue is equal to bond maturity. The net duration, interest subsidy and loss of value increases with the increase in the spread, $r_{pf} = r_{es}$.

Table 5

Net Duration, Interest Subsidy and Value Loss of Pre-Funded Bonds

Pre-Tax Interest Subsidy = $(r_{pf} B_{pf})(r_{pf} – r_{es})$ per year

$$\textbf{Value Loss} = (r_{pf} B_{pf})(r_{pf} - r_{es})\frac{(1+r_{pf})^n - 1}{(1+r_{pf})^n}$$

	Escrow Return, r_{es}		**Pre-funded Bond Yield, r_{pf}** 8%	7%	6%	5%
	8%	Net duration, yrs	5			
		Interest subsidy	0			

Maturity, n 5 years		Value loss	0			
	7%	Net duration, yrs	5.019	5		
		Interest subsidy	$ 11,905	0		
		Value loss	($ 47,533)	0		
	6%	Net duration, yrs	5.038	5.016	5	
		Interest subsidy	$ 24,132	$ 9,927	0	
		Value loss	($ 96,353)	($ 40,703)	0	
	5%	Net duration, yrs	5.059	5.037	5.013	5
		Interest subsidy	$ 36,717	$ 20,088	$ 8,106	0
		Value loss	($ 146,602)	($ 82,364)	($ 34,144)	0
Maturity, n 10 years	8%	Net duration, yrs	10			
		Interest subsidy	0			
		Value loss	0			
	7%	Net duration, yrs	10.198	10		
		Interest subsidy	$ 18,260	0		
		Value loss	($ 122,527)	0		
	6%	Net duration, yrs	10.433	10.165	10	
		Interest subsidy	$ 38,911	$ 14,439	0	
		Value loss	($ 261,097)	($ 101,414)	0	
	5%	Net duration, yrs	10.715	10.358	10.135	10
		Interest subsidy	$ 62,784	$ 30,469	$ 11,180	0
		Value loss	($421,288)	($214,004)	($ 82,282)	0

Empty cell represent the improbable case of $r_{pf} < r_z$.
Zero-coupon and pre-funded bonds are priced by market as equivalent securities.

Table 6 presents the case when pre-funded bonds are priced to yield lower than the zero-coupons. The asset-based coupon payments of the pre-funded issue are default free, thus market lowers the yield by 25, 50 or 75 basis points from the comparable zero-coupon yield. We recalculate the total face value, net duration, interest subsidy and loss of value under these conditions. Results in Table 6 indicate that impact of the spread, $r_{pf} - r_{es}$ is still dominant. The total face value and net duration of the pre-funded issue is greater than corresponding values for the zero-coupon bond.

Table 6

Face Value, Net Duration, Interest Subsidy and Value Loss of Pre-Funded Bonds

Pre-Funded Bonds are Priced to Yield Lower than Comparable Zero-Coupon Bonds

Discount Rate on Zero-Coupon Bonds, r_z	**Face Value of Zero-Coupon Bonds, B_z**	**Pre-Funded Bond Yield, B_{pf}**			
		r_z	**r_z - .25%**	**r_z - .50%**	**r_z - .75%**

8%	$21,589,250	Face Value of Pre-Funded, B_{pf} Duration, D_{net} Interest Subsidy Value Loss	$26,160,123 10.718 yrs $62,784 ($421,288)	$24,902,535 10.611 yrs $53,074 ($360,179)	$23,760,313 10.516 yrs $44,551 ($305,799)	$22,718,277 10.432 yrs $37,059 ($257,307)
7%	$19,671,514	Face Value of Pre-Funded, B_{pf} Duration, D_{net} Interest Subsidy Value Loss	$21,763,801 10.358 yrs $30,469 ($214,004)	$20,886,293 10.291 yrs $24,672 ($175,306)	$20,076,805 10.266 yrs $19,575 ($140,721)	$19,327,721 10.181 yrs $15,100 ($109,831)
6%	$13,382,256	Face Value of Pre-Funded, B_{pf} Duration, D_{net} Interest Subsidy Value Loss	$18,632,525 10.135 yrs $11,180 ($82,282)	$17,985,604 10.094 yrs $7,756 ($57,769)	$17,382,097 10.058 yrs $4,780 ($36,030)	$16,817,777 10.027 yrs $2,207 ($16,839)

Maturity = 10 years and escrow account yield = 5%

CONCLUSION

Pre-funded coupon bonds have been developed and sold by investment bankers in place of zero-coupon bonds to raise funds for companies facing cash flow problems. Additional bonds are issued and proceeds are deposited in an escrow account to finance the coupon payment. Our analysis indicates that when the pre-funded bond yield is the same as the escrow return then total face value of the pre-funded issued is the same as the total face value of the zero-coupon bonds and the effective cost of pre-funded issue is the same as the yield to maturity of the zero-coupon bond. Also, increase in the spread between pre-funded bond yield and zero-coupon yield increases the total face value of the bonds issued and its effective cost. The interest subsidy creates additional leverage, which stretches the net duration of the pre-funded issue beyond its maturity. Increase in the spread between pre-funded bond yield and zero-coupon yield increases net duration, interest subsidy and loss of value. Even when pre-funded bonds are priced to yield lower than the zero-coupons, impact of the spread is dominant -- total face value and net duration of the pre-funded issue is still greater than corresponding values for the zero-coupon bond.

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