# AN EXAMINATION INTO THE ADVERTISING-PRODUCTION INTERFACE: SOME THEORETICAL AND NUMERICAL RESULTS

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## ABSTRACT

The literature reveals contradiction between theoretical results (superiority of uniform policy under a concave advertising response function) and the empirical results (concavity of the advertising response function and the superiority of a pulsation policy). To reconcile the above difference, this paper offers a resolution rooted within the operations function of the enterprise based on the convexity of the shape of the production cost function. The study is seen applicable to consumer products in the mature stage of their life cycles where the manufacturing firm practices a just-in-time (JIT) philosophy resulting in near zero-inventory production.

***Key words:*** **Marketing; Operations Management; Advertising pulsation; Production cost function**

## 1. INTRODUCTION

The objective of marketing is to create customer demand while operations management focuses on the supply and fulfillment of that demand. A conflict between these two areas may lead to production inefficiencies and unsatisfied customers (Ho and Tang, 2004). A firm's marketing effort that considerably relies on advertising often requires a commitment of significant amount of resources. Therefore, the issue of whether it is best to adopt a pulsation policy of advertising or one of even-spending that costs the same is of significant interest to both academicians and practitioners. The objective of this study is to examine the impact of the shape of the production cost function on the advertising policy of the firm. It is demonstrated in this paper for the first time in the literature that, in the presence of a convex quadratic production cost function, a pulsation policy of advertising could be superior to its even-spending counterpart under a concave advertising response function.

Sasieni (1971) in his pioneering article shows that with decreasing marginal returns to advertising spending (concave advertising response function), a uniform advertising policy is superior to cyclic policies of the same cost in the long run. However, empirical evidence suggests that a pulsing advertising policy could be superior to uniform spending over time (Wells and Chnisky, 1965; Sethi, 1971; Ackoff and Emshoff, 1975; Rao and Miller, 1975; Eastlack and Rao, 1986). Due to the contradiction between theoretical and empirical findings, few models have been published with the purpose of substantiating advertising pulsation.

Unlike previous studies that attempted at reconciling the difference between the theoretical and the empirical findings in the literature through reliance upon certain mechanisms solely within the marketing function, this research offers a novel conciliation mechanism within the production function. In this regard, production researchers observe that by assigning production to the source with lowest unit cost until its capacity is fulfilled and then proceeding to the next cheapest source results in a convex production cost function which is also supported by empirical evidence (Eliashberg and Steinberg, 1993). In another research work, Eliashberg and Steinberg (1987) cite production and economics literature that employ a convex quadratic cost function.

The present study is concerned with long-term, steady-state response. Hauser and Wernerfelt (1989, p.393) argue that such focus is appropriate for strategic advertising decisions. The study is thought to be applicable to consumer products in the mature stage of their product life cycles where the manufacturing firm practices a just-in-time (JIT) philosophy resulting in near zero-inventory production.

The rest of the paper is organized as follows. The second section outlines the theoretical model. Then a comparison of alternative pulsation policies is presented in the third section. The fourth section presents numerical results in support of the theoretical findings reported herein. The fifth section summarizes and concludes the paper. Derivation of key formulas and mathematical proofs of theoretical findings are available upon request from the authors.

## 2. MODEL

The model described in this section can be applied to a business that sells a single consumer item of low level consumer involvement in a monopolistic market, where the firm's marketing efforts are mostly limited to advertising. A monopolistic environment can be approximated in a condition where a firm dominates the market while facing competition from many small firms who are too small to influence the market in a noticeable manner.

Figure 1 provides a schematic illustration of three alternative forms of practical pulsation policies from which the firm could choose. They are (a) Uniform Advertising Policy (UAP), in which the firm advertises at a constant level throughout ($x_1 = x_2$), where $x_1$ and $x_2$ indicate two different levels of advertising intensity, (b) Advertising Pulsing and Maintenance Policy (APMP), in which the firm alternates between a high level of advertising $x_1$ that lasts for a time period $t_1$, followed by a lower level $x_2$, usually a maintenance level lasting for a duration of ($T$-$t_1$), where $T$ is the length of each cycle, and (c) Advertising Pulsing Policy (APP), in which the firm alternates between high and zero levels of advertising. The relationship between $x_1$ and $x_2$ and between $t_1$ and $T$ pertaining to each policy is shown at the top of each configuration. Although the three studied pulsation policies are not by any means exhaustive, the inferiority of the uniform strategy implies the superiority of pulsing in general. The theoretical framework is constructed using the Vidale and Wolfe (1957) dynamic advertising response model and therefore, is introduced in the ensuing subsection.

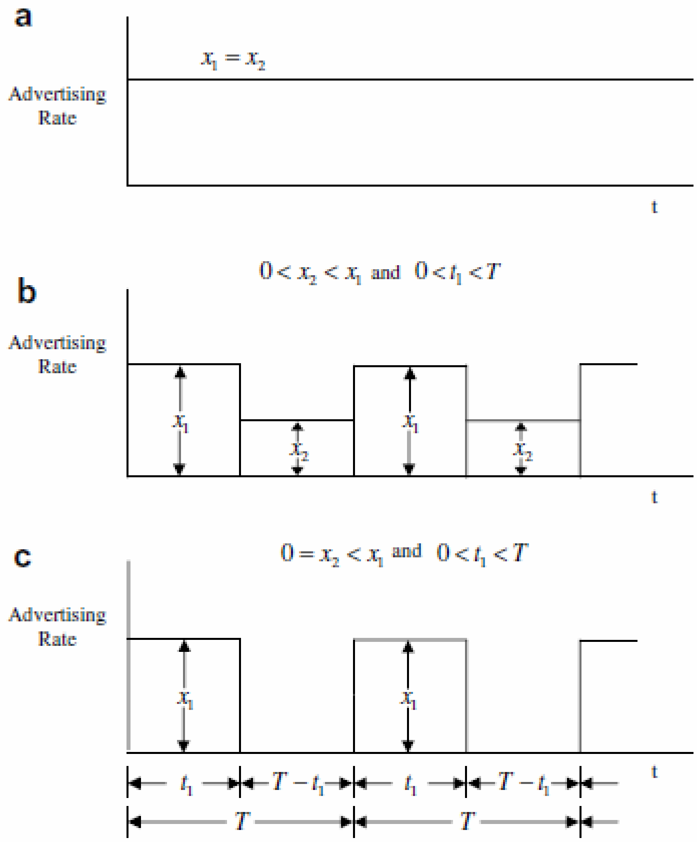


Figure 1: Different forms of advertising pulsation policies: (a) uniform advertising policy (UAP), (b) advertising pulsing and maintenance policy (APMP), and (c) advertising pulsing policy (APP).

### 2.1 Dynamic Advertising Response Model

The Vidale and Wolfe model (1957) is one of the earliest and most intensively analyzed mathematical models of dynamic advertising response (e.g. Sasieni, 1971; Mahajan and Muller, 1986). According to this model, the instantaneous change in the sales rate is given by the following first-order linear differential equation

$$\frac{dS}{dt} = bx(m - S) - aS\,, \tag{1}$$

where $S_t$ is the sales rate (\$/unit time) at time t; $x$ is the advertising rate (\$/unit time); $b$ is the advertising effectiveness parameter; $m$ is the market potential or saturation sales assumed to be constant, and $a$ is the decay constant. The steady state advertising response $S(x)$ is derived through setting $dS_t/dt = 0$, and solving equation (1) for $S$ to obtain:

$$S(x) = \frac{mbx}{a + bx}\,. \tag{2}$$

The advertising response function for the Vidale-Wolfe model (1) is linear given by $f(x) = bx$. Little (1979) proposed a modified version of the Vidale-Wolfe model for which $f(x)$ takes a power function of the form $f(x) = bx^{\delta}$, where $b$ and $\delta$ are positive constants. Thus $f(x)$ becomes linear in $x$ ( $f''(x) = 0$) for $\delta = 1$, concave in $x$ ( $f''(x) < 0$ as a sufficient condition) for $0 < \delta < 1$, and convex in $x$ ( $f''(x) > 0$ as a sufficient condition) for $\delta > 1$.

By using the more general form for advertising response function, it can be shown after rearrangement of terms that (1) takes on the following form:

$$\frac{dS}{dt} = \alpha(x)[S(x) - S], \tag{3}$$

where

$$\alpha(x) = a + f(x), \tag{4}$$

and

$$S(x) = \frac{mf(x)}{a + f(x)}. \tag{5}$$

The decay constant, $a$, in (4) and (5) reflects the effect of advertising from the competition. Gopalakrishna and Chatterjee (1992) argue that for a firm with dominant market share and competing against a fringe of many small suppliers, each too small to influence the market, parameter $a$ could be regarded as a constant quantity.

### 2.2 Selecting a Performance Measure

An appropriate performance measure is needed to compare the effectiveness of alternative advertising policies. Mesak and Ellis (2009) quote Feinberg (1988) about the advantages and disadvantages of several performance measures, two of which are relevant to the scope of this research. They are: (i) the discounted profit over the infinite planning horizon that is sensitive to the initial sales level, and (ii) the average undiscounted profit over the infinite planning horizon that treats the profit made sooner as equal to the profit made later. Park and Hahn (1991) argue in favor of the second measure as it is independent of arbitrary initial conditions and its weakness is substantially mitigated due to the periodic nature of the advertising policies over time. Mesak (1992) shows that the second measure serves as a plausible approximation of the first one when the discount rate is small. Consequently, in this research, the second performance measure is employed.

### 2.3 Sales Response to Advertising Pulsation

Considering a general advertising pulsation policy for which $0 \leq x_2 \leq x_1$ (see Figure 2), the time axis is divided into equal similar cycles of duration $T$, in which advertising is at a high level $x_1$, over a duration of time $t_1$, and at a low level $x_2$ over a duration of time $(T-t_1)$. The first cycle starts at $t = 0$ for which the initial sales is $S_0$, the sales rate grows to $M$ at $t_1$ following the sales growth curve $g_1(t)$ while advertising is kept at its higher level. Afterwards, as the advertising level is decreased, the sales rate decays along the curve $g_2(t)$ until $T$ and the cycle repeats with new initial conditions. It can be shown (following Mesak and Ellis, 2009) that the system eventually reaches a quasi-steady state for which the steady state cycle starts and ends with the same level of sales rate as shown in Figure 2. These levels are unique and independent of the

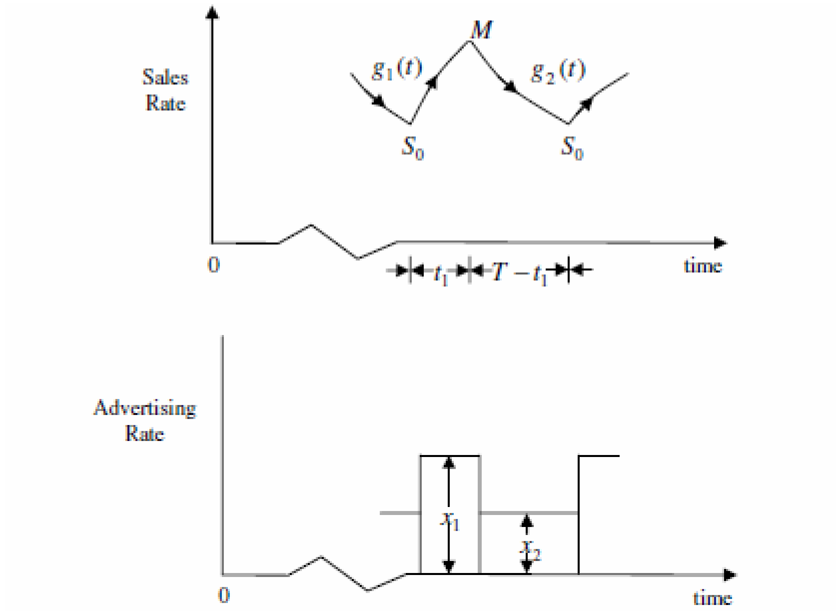

Figure 2: Sales response to advertising at the steady-state cycle

initial sales rate. For a firm practicing just-in-time (JIT) philosophy resulting in near zero-inventory products, the sales rate would equal the production rate. Referring to Figure 2 and apart from a fixed cost term for a zero production level, the following quadratic production cost function is considered in the analysis:

$$q_i(t) = vg_i(t) + \frac{1}{K} g_i^2(t)\,;\ i = 1, 2. \tag{6}$$

The above production cost function, envisioned as a Taylor expansion of order two, is composed of two terms. The first term is linear in the sales rate, measured in dollars per unit time, whereas the second term is quadratic in the same. The quantity $v$ is a positive constant fraction seen to represent the ratio of the linear production cost component to sales revenue and $K$ is another constant conceived to represent the firm's process efficiency (Eliashberg and Steinberg, 1987). The quadratic production cost function (6) is convex for $K > 0$ and concave for $K < 0$. Both shapes of the production cost function are investigated in this research.

Considering the steady state cycle, the expressions for average sales revenue per unit time ($R$), the average production cost per unit time ($PC$), and the related average profit per unit time ($PRO$) are given by expressions (7), (8) and (9), respectively.

$$R = \frac{1}{T}\left( \int_0^{t_1} g_1(t)dt + \int_0^{T-t_1} g_2(t)dt \right). \tag{7}$$

$$PC = vR + \frac{1}{KT}\left( \int_0^{t_1} g_1^2(t)dt + \int_0^{T-t_1} g_2^2(t)dt \right). \tag{8}$$

The first term in (8) is designated by $PC1$ whereas its second term is designated by $PC2$. Combining equations (7) and (8) and introducing $\gamma$, with a given average rate of advertising spending $x$ assumed to have been determined exogenously, net profit per unit time is given by

$$PRO = \frac{1}{T}\left\{\gamma\left(\int_0^{t_1} g_1(t)dt + \int_0^{T-t_1} g_2(t)dt\right) - \frac{1}{K}\left(\int_0^{t_1} g_1^2(t)dt + \int_0^{T-t_1} g_2^2(t)dt\right)\right\} - x \tag{9}$$

where $\gamma$ is a fraction less than 1, given by $\gamma = 1 - \varepsilon$, and $\varepsilon > v > 0$ represents the ratio of costs (other than those related to the nonlinear production cost and the advertising expenditure, such as the linear production cost and the physical distribution cost) to sales revenue which is assumed to be constant (Nicholson, 1983).

It can be shown that expression (7) takes the following form:

$$R = \frac{1}{T}\left[S(x_1)t_1 + S(x_2)(T - t_1) + \{S(x_1) - S(x_2)\}\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_1}\right)\frac{\left(1 - e^{-\alpha_1 t_1}\right)\left(1 - e^{-\alpha_2 (T-t_1)}\right)}{\left(1 - e^{-(\alpha_1 t_1 + \alpha_2 (T-t_1))}\right)}\right], \tag{10}$$

while the second term in expression (8), $PC2$, takes the following form:

$$PC2 = \frac{1}{KT}\left[\begin{array}{l} S^2(x_1)t_1 + S^2(x_2)(T - t_1) + 2\{S(x_1) - S(x_2)\}\left(\dfrac{S(x_2)}{\alpha_2} - \dfrac{S(x_1)}{\alpha_1}\right) \\ \dfrac{\left(1 - e^{-\alpha_1 t_1}\right)\left(1 - e^{-\alpha_2 (T-t_1)}\right)}{\left(1 - e^{-(\alpha_1 t_1 + \alpha_2 (T-t_1))}\right)} + \dfrac{\{S(x_1) - S(x_2)\}^2}{2\left(1 - e^{-(\alpha_1 t_1 + \alpha_2 (T-t_1))}\right)^2} \\ \left\{\dfrac{\left(1 - e^{-2\alpha_1 t_1}\right)\left(1 - e^{-\alpha_2 (T-t_1)}\right)^2}{\alpha_1} + \dfrac{\left(1 - e^{-\alpha_1 t_1}\right)^2\left(1 - e^{-2\alpha_2 (T-t_1)}\right)}{\alpha_2}\right\} \end{array}\right], \tag{11}$$

where $S(x_1) = mf(x_1)/(a + f(x_1))$; $S(x_2) = mf(x_2)/(a + f(x_2))$; $\alpha_1 = a + f(x_1)$; and $\alpha_2 = a + f(x_2)$.

Therefore, the net profit per unit time is given from expression (9) by

$$PRO = \gamma R - PC2 - x. \tag{12}$$

## 3. COMPARISON OF ALTERNATIVE PULSATION POLICIES

Based on expressions (10) – (12), we are in a position to introduce seven results of which proofs are available upon request from the authors.

<u>*Result 1*</u>: For a concave advertising response function and considering the performance measure R given by (10), advertising pulsing/maintenance policy (APMP) dominates advertising pulsing policy (APP) but is dominated by uniform advertising policy (UAP).

<u>*Result 2:*</u> For a concave advertising response function, the absolute value of the quadratic term of the production cost function ($PC2$) attains its maximum value for an UAP.

Based on results 1 and 2, Result 3 is introduced.

*Result 3:* For an Advertising Pulsing Policy (APP),

$$Sign\ of\ \left.\frac{d(PRO)}{dt_1}\right|_{t_1 \to T}\ is\ given\ by: Sign\ of\ \left(\gamma - \frac{2S(x)}{K}\right).Sign\ of\left\{f(x) - xf'(x)\right\}. \qquad (14)$$

From the above result, and Figure 2 , the following observations are made:

(i) The sign of $[f(x) - xf'(x)]$ is positive for a concave advertising response function,

(ii) It is observed from (2) that the maximum value that $S(x)$ could take for a finite value of $x$ is less than $m$. Therefore, if a solution of the equation $\gamma = 2S(x)/K$ does not exist ($\gamma K/2 \geq m$), the sign of $[\gamma - 2S(x)/K]$ is always positive, and

(iii) If a solution of the equation $\gamma = 2S(x)/K$ does exist ($\gamma K/2 < m$), the solution would be unique at a value $x_s > 0$. In this case, the sign of $[\gamma - 2S(x)/K]$ is positive for advertising budgets $x < x_s$ and the sign of $[\gamma - 2S(x)/K]$ is negative for $x > x_s$.

Result 3 implies that if the sign of expression (14) is negative, then APP dominates UAP as there will be at least one APP policy for which $t_1$ is smaller but close enough to $T$ for which *PRO* is larger than the only unique UAP counterpart that costs the same. Based on Result 3 and its related discussions, Results 4 and 5 are introduced.

*Result 4:* For a concave advertising response function and a concave quadratic production cost function, APMP dominates APP but is dominated by UAP.

Result 4 broadens the scope of applicability of known findings in the literature (e.g. Sasieni, 1971; Mesak and Darrat, 1992) to the situation of a concave quadratic production cost function that has not been examined before in the literature.

*Result 5:* For a firm of a convex quadratic production cost function,

(i) If a solution to the equation $\gamma = 2S(x)/K$ does not exist, UAP is optimal under a concave advertising response function.

(ii) If a solution to the equation $\gamma = 2S(x)/K$ exists at $x_s > 0$, then in the presence of a concave advertising response function, APP and APMP dominate UAP for all advertising budgets $x > x_s$.

The findings depicted in Result 5(ii) are in contradiction with previous theoretical research findings reported in the literature. The Result 5(ii) is inconsistent with the findings of Sasieni (1971) who only considers a linear production cost function in the modeling effort and asserts the optimality of UAP for a concave advertising response function. Results 4 and 5 taken together attribute the potential superiority of a pulsation policy in the presence of concavity in the advertising response function to the convexity of the production cost function (Results 5). From Result 3, such possibility cannot materialize for a linear production cost function ($K = \infty$) or a concave one ($K < 0$).

While Result 5(ii) indicates that there exists a pulsation policy that is superior to its UAP counterpart in the presence of a concave advertising response function when $x > x_s$, it does not identify such policy. In addition, when $x \leq x_s$, the optimal advertising policy is not disclosed. Result 6 sheds light on such issues.

Before introducing Result 6, it would be advantageous to define the policy parameter $\lambda$ ($0 \leq \lambda \leq 1$) as $\lambda = x_2 / x$, so that the different advertising pulsation policies would be characterized in terms of $\lambda$ in the following manner: UAP is characterized by $\lambda = 1$ (Figure 1a), APMP is characterized by $0 < \lambda < 1$ (Figure 1b), and APP is characterized by $\lambda = 0$ (Figure 1c). Thus for any given value of $\lambda$, the high advertising level $x_1$ can be uniquely determined from (15) upon replacing $x_2$ by $\lambda x$

$$x_1 t_1 + x_2 (T - t_1) = xT. \tag{15}$$

Expression (15) implies that the advertising pulsation policies UAP, APMP, and APP cost the same.

*Result 6:* In the presence of a concave advertising response function and for a firm of a convex quadratic production cost function

(i) If a solution to the equation $\gamma = 2S(x)/K$ exists at $x_s > 0$ such that $x \leq x_s$, the optimal advertising policy is UAP of a policy parameter $\lambda^* = 1$.

(ii) If a solution to the equation $\gamma = 2S(x)/K$ exists at $x_s > 0$ such that $x > x_s$, the optimal policy parameter is given by Max $\{0, \lambda^*\}$ and $\lambda^* < 1$ satisfies the equation $(2m - \gamma K)(f(x_1)t_1 + f(x_2)(T-t_1)) - \gamma KaT = 0$, where $x_1 = (x/t_1)(T - \lambda(T-t_1))$ and $x_2 = \lambda x$.

Result 6 indicates that an optimal advertising pulsation policy does not only require a convex production function but also a relatively large advertising budget $x > x_s$. Although the advertising budget x is determined exogenously in this exploratory study, it is advantageous to relate x to the optimum advertising expenditure $x^*$ that maximizes the firm's profit in a monopolistic setting.

*Result 7:* In the presence of a concave advertising response function and for a convex quadratic production cost function, the optimum advertising expenditure $x^*$ that maximizes the firm's profit is smaller than $x_s$ provided that $\gamma K/2 < m$.

The condition $\gamma K/2 < m$ implies that $x_s$ exists by observation (iii) related to Result 3. A monopolistic firm usually sets its advertising budget x at a level significantly higher than $x^*$ under the threat of competitive entry (Gupta and Di Bendetto, 2007). Bagwell (2007) provides a comprehensive review of models that aid in determining the equilibrium advertising budget x prior of competitive entry. Therefore, an advertising pulsation policy is of theoretical and practical relevance when a monopolist over-advertises to deter entry of potential rivals.

## 4. A NUMERICAL INVESTIGATION

The main purpose of the numerical investigation of this section is to provide illustrative examples in support of the statements of Results 1 through 7 for both a convex and concave quadratic production cost functions in the presence of a concave advertising response function. The employed values of model parameters have been influenced a great deal by the results of the empirical estimation of modified versions of the Vidale-Wolfe model found in the studies of Mesak and Darrat (1992) and Mesak and Ellis (2009).

For $T$ = 0.50, $t_1$ = 0.25, $\gamma$ = 0.4, $b$ = 0.01, $a$ = 0.20, $m$ = 16,000, $K$ = 40,000, and f(0) = 0, the modified Vidale-Wolfe model (3) of a concave advertising response function, $f(x) = 0.01\sqrt{x}$, takes the form

$$dS/dt = \left(0.20 + 0.01\sqrt{x}\right)\left[S(x) - S\right], \text{ and}$$
$$S(x) = (16{,}000)\frac{0.01\sqrt{x}}{0.2 + 0.01\sqrt{x}}.$$

Referring to Result 5, it can be shown that the solution of the equation $\gamma = 2S(x)/K$ is given by $x_s$ = 400. Table 1 reports the quantities $R$, $PC2$, $PRO1$ and $PRO2$ as a function of the policy parameter $\lambda$ for two values of the advertising budgets $x = 300 < x_s$ and $x = 500 > x_s$. Consistent also with Result 5, optimal $PRO1$ occurs at a policy parameter $\lambda$ less than 1 for $x$ = 500 (Result 6 predicts a value of $\lambda^*$=0.20) whereas optimal $PRO1$ occurs at $\lambda$ = 1 for $x$ = 300. Consistent with Results 1,2 and 4, the optimal values of $R$, $PC2$, and $PRO2$ occur at $\lambda^*$=1.00. The profit PRO1 for $\lambda^*$=0.20 when x = 500 is smaller than the profit PRO1 for $\lambda^*$=1.00 when x = 300 is indicative of over advertising as predicted by Result 7 (the first derivative of PRO1 with respect to x at x = 300 and at x = 500 are both negative).

**Table 1**

Numerical findings in support of Results 4 and 5 – Concave response function

| | $x$ = 300 | | | | $x$ = 500 | | | |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $R$ | $PC2$ | $PRO1$ | $PRO2$ | $R$ | $PC2$ | $PRO1$ | $PRO2$ |
| 0.0 | 6075.959 | 923.124 | 1207.260 | 3053.508 | 7063.132 | 1247.456 | 1077.797 | 3572.709 |
| 0.1 | 6771.295 | 1146.355 | 1262.163 | 3554.873 | 7783.012 | 1514.506 | 1098.699 | 4127.710 |
| 0.2 | 6983.585 | 1219.325 | 1274.109 | 3712.759 | 7999.583 | 1599.917 | **1099.917** | 4299.750 |
| 0.3 | 7119.548 | 1267.243 | 1280.576 | 3815.062 | 8137.513 | 1655.535 | 1099.470 | 4410.541 |
| 0.4 | 7216.298 | 1301.904 | 1284.615 | 3888.423 | 8235.296 | 1695.542 | 1098.577 | 4489.660 |
| 0.5 | 7287.762 | 1327.807 | 1287.298 | 3942.911 | 8307.328 | 1725.318 | 1097.613 | 4548.250 |
| 0.6 | 7340.799 | 1347.196 | 1289.124 | 3983.515 | 8360.682 | 1747.541 | 1096.732 | 4591.813 |
| 0.7 | 7379.255 | 1361.342 | 1290.360 | 4013.044 | 8399.310 | 1763.719 | 1096.005 | 4623.443 |
| 0.8 | 7405.414 | 1371.007 | 1291.159 | 4033.172 | 8425.559 | 1774.755 | 1095.469 | 4644.978 |
| 0.9 | 7420.629 | 1376.644 | 1291.608 | 4044.896 | 8440.816 | 1781.185 | 1095.141 | 4657.512 |
| 1.0 | **7425.625** | **1378.498** | **1291.752** | **4048.748** | **8445.824** | **1783.299** | 1095.031 | **4661.628** |

[a] $PRO1 = \gamma R - PC2 - x$ (profit under a convex quadratic production cost function)
[b] $PRO2 = \gamma R + PC2 - x$ (profit under a concave quadratic production cost function)

## 5. SUMMARY AND CONCLUSIONS

The literature reveals a contradiction between the theoretical results and the empirical research findings on the issue of whether a firm should advertise at a constant rate or in a cyclic fashion in order to maximize its performance. Sasieni (1971) in his pioneering theoretical analysis shows that under linear or concave advertising response function, a pulsation or cyclic advertising policy can never be superior to a uniform advertising policy in the long run when the total cost of both policies are the same. On the other hand, several empirical findings conclude that the advertising response function is concave and an advertising pulsation policy is superior to a policy of uniform spending over time that costs the same. While a few research attempts to resolve the above said difference is presented in the literature almost exclusively within the marketing domain, this study offers a resolution within the operations function of the enterprise.

For our model (sections 2 through 4), this paper attributes the potential superiority of pulsation in the presence of concavity in the advertising response function to the convexity of the production cost function and the magnitude of the advertising budget (Result 5). To substantiate the superiority of an advertising policy of pulsation, the convexity of the shape of the production cost function is envisioned to be a central requirement; otherwise the uniform advertising policy becomes optimal (Results 4 and 5). To that end, the theoretical and numerical analysis employ a dynamic advertising response model (modified Vidale-Wolfe model) in a just-in-time environment.

Results 5 and 6, guided by the findings of Result 3, provide management with simple rules to assess the superiority of an advertising pulsation policy in the presence of a convex quadratic cost function for advertising budgets lying in the concavity region of the advertising response function. The implementation of such rules and their consequences are illustrated by the results of a numerical investigation associated with the analyzed model in the fourth section (Table 1).

The modeling effort developed in this paper is exploratory, revealing many possibilities for future research. This study assumes price to be exogenous. A challenging task would be to devise the optimal pricing and advertising policies in a unified framework. In this research work we have made aggregation of advertising over all media. An interesting area for future research would be to consider different advertising media of different effectiveness and examine synergy effects among them. In addition, the modeling effort in this study considers sales-advertising relationship to be deterministic and deals with stationary markets for which the parameters of sales response are assumed to remain the same over different cycles. Relaxing these assumptions by introducing appropriate stochastic and non-stationary mechanisms would offer additional topics for future research. Furthermore, the current study is confined to non-seasonal consumer products for which advertising is the major element of the firm's marketing efforts. It would be interesting to examine seasonal products in future research. In particular, extending the modeling effort to competitive markets should be advantageous.

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