THE APPLICATION OF CONSENSUS MODEL IN AUDITING DECISION MAKING

Fang Yang
College of Business Administration
University of Detroit Mercy
4001 W. McNichols Road, Detroit, MI, 48221
E-mail address: fang.yang@udmercy.edu

Lun Mo
American UN Education and Psychology Program Center

ABSTRACT

The study develops a psychometric model of Cultural Consensus Theory to aggregate auditors’ opinion in auditing tasks where many decisions are performed in group settings rather than by individuals acting alone. CCT is a statistical modeling approach to information aggregation in group settings, and it has become one of the leading methodologies for determining consensus in the areas such as cultural and medical anthropology, social networks. The present study is purporting to extend this theory to decision making in auditing, and demonstrate how a small group of auditors can reach consensus on some important issues and make an accurate alternative.

Key words: Cultural Consensus Theory, consensus, accuracy, auditing decision making
Introduction

For many auditing tasks, the accuracy of auditing decision is unobservable. Given the lack of criteria for judging the accuracy of many auditing decisions and for the purpose of internal control in organizations and corporations, consensus must be reached among a small group of auditors for an auditing decision before an audit report is released, thereby consensus among decision-makers could be a surrogate for accuracy in wide auditing practices.

Imaging a relatively small group of auditors are giving answers to the same question, and their opinions are aggregated to construct a consensus. If decision making is simply achieved by counting the number of people who have answered ‘yes’ or ‘no’, it would ignore one important fact -- some of them are more knowledgeable than others, and the opinion of more knowledgeable people should be weighted more in aggregated process. Only in this way that the final decision could be a representation to the reality that authorities have stronger voices. Meanwhile, it would be ideal if individual’s expertise level could be measured while estimating a consensus among a group of people.

Usually, it is unrealistic to expect to gain access to a large pool of auditors for a decision making task. Therefore, aggregating auditors’ opinions often involves utilizing a small sample size which raises an issue about the accuracy of aggregation. In fact, the accuracy of an aggregation is a function of both the number of auditors and the agreement among auditors. The higher degree of the agreement is, the higher the accuracy is. The challenge is to develop a model which can estimate the degree of agreement as well as individual’s knowledge level simultaneously for a small group of auditors.

Over the past 20 years, Batchelder, Romney and their colleagues (Batchelder & Romney, 1988, 1989; Romney, Weller, & Batchelder, 1986) have developed a special class of mathematical/statistical models, namely the cultural consensus theory (CCT) to aggregate opinions. CCT is a parametric statistical model for aggregating the responses of informants to a series of questions about some domains of their shared knowledge. The shared knowledge could arise from common beliefs, common education, shared politics, and the like. It is assumed that there is a consensus answer to each question based on the common experiences of the informants. However, each informant is assumed to have access to only part of the common knowledge, so they may disagree somewhat in their answers. Furthermore, these consensus answers are not known apriori, so they are treated as parameters in a model. Meanwhile, the extent to which the informants have common knowledge could be estimated in the model, and we call this trait as competence. This model has been widely applied in the field of anthropology since it was developed, and in the recent years, it was further extended to the field of medical and educational settings.

The purpose of this study is to introduce this methodology into audit decision making process. Specifically, we aim to measure the extent of agreement among auditors for certain questions based on the weighted competence of individuals who answer the questions.
Methodology

Consensus model introduction

The first step is to build up a model that expresses the probability of an auditor $i$ responding to a question $k$ with unobserved consensus answer $Z$ in terms of the person’s competency $D_i$. The competency is defined as the extent to which an auditor is able to answer questions accurately. Given the assumption that all alternatives have equal guessing probability $g$, the probability of a respondent $i$ knowing the consensus answer $z$ to a question $k$ could be a function of competence $D_i$ and guessing probability $g$ which is decided by the number of alternative answers $W$, $g=1/W$. If it is a dichotomous question, $g=1/2$.

Measurement of individual competencies

Under the circumstance of only one auditor being involved in the task, the decision process can be depicted in Figure 1. Correspondingly, the probability of answering “yes” or “no” can be expressed as:

$$P(\text{yes}) = D_i + (1 - D_i)g$$
$$P(\text{no}) = (1 - D_i)(1 - g)$$

![Figure 1](attachment:tree.png)

Figure 1  Tree structure for one person answering one question
Figure 2. Tree structure for one pair of persons answering one question

For two auditors, there are four possible outcomes as below:

a. Both auditor i and j know the answer with probability $D_iD_j$

b. i knows answer but j answers correctly by guessing with probability $D_i(1 - D_j)g$

c. i answers correctly by guessing and j knows answer with the probability $(1 - D_i)gD_j$

d. Both i and j give the same answer by guessing with the probability $(1 - D_i)(1 - D_j)g^2 + (1 - D_i)(1 - D_j)(1 - g)^2$

Now, a random variable for matches is defined as

$M = \begin{cases} 1, & \text{matched} \\ 0, & \text{otherwise} \end{cases}$

The probability that i and j match on a question with dichotomous choices:

$$P(M_{ij} = 1) = D_iD_j + D_i(1 - D_j)/2 + (1 - D_i)D_j/2 + (1 - D_i)(1 - D_j)/2$$

Algebraically reduced as $P(M_{ij} = 1) = D_iD_j + (1 - D_iD_j)/2 \quad (1)$

So $D_iD_j = 2P(M_{ij} = 1) - 1 \quad (2)$

Of course, the matched number between auditor i and j can be empirically observed and can be put in Equation (2) in place of $P(M_{ij} = 1)$, then solve for $D_iD_j$ to obtain an estimate. We
choose estimation approach of least square which minimize the sum of the squared discrepancies between observed and predicted proportion of matched number.

For the dichotomous data with Bernoulli distribution,
\[ E(D_iD_j) = D_iD_j \]
\[ SS(D) = \sum \sum (D_iD_j - D_iD_j)^2 \]

There is an independent equation (1) for each pair of auditor, in total there are \( N(N-1)/2 \) equations, thus we are able to obtain estimates for individual Ds when \( N \geq 3 \).

**Measurement of consensus answer**

Based on the individual competencies estimated above, the next step is to estimate their consensus. We imagine three auditors A, B, C making a judgment with dichotomous answers 1 (yes) and 0 (no), and their competencies are \( D_A \), \( D_B \) and \( D_C \), respectively. There are possible eight response patterns \( X_A X_B X_C \), for example \([1 \ 1 \ 0]\), which means, the first two auditors give answer “yes”, and third one give answer “no”. If we assume that “yes” is the finally unobserved consensus answer, then \( P(<X_A X_B X_C > | 1) \) is the conditional probability of response pattern \([1 \ 1 \ 0]\) given consensus answer being “yes”, and computed as the product of the probability of the first two auditors giving answer “yes” and third auditor giving answer “no”. Similarly, \( P(<X_A X_B X_C > | 0) \) is defined as conditional probability of response pattern \([1 \ 1 \ 0]\) given consensus answer being “no.” \( P(1) \) and \( P(0) \) are prior probabilities of answering “yes” and “no.” According to the Bayes’ law, we have the posterior probability of giving consensus answer:

\[
P(1|X_A X_B X_C) = \frac{P(<X_A X_B X_C > | 1)P(1)}{P(<X_A X_B X_C > | 1)P(1) + P(<X_A X_B X_C > | 0)P(0)} \tag{3}
\]

If the posterior probability is greater than 0.5, then we can conclude that “yes” is the consensus answer; otherwise, “no” is the consensus answer. Now we illustrate this model by imagining that three auditors have answered 30 questions, and response data are presented below:

\[
\begin{array}{cccccccccccc}
1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
\end{array}
\]

The observed matched proportion for each pair of three auditors is

\[
\begin{bmatrix}
1 & 2 & 3 \\
1 & \\
2 & 0.267 & \\
3 & 0.60 & 0.533
\end{bmatrix}
\]

By using least square method, estimated competency value are \{0.658, 0.573, and 0.919\}. Now we turn to consensus answer estimates. For any question, there are eight possible response patterns for three auditors. For response pattern \([1 \ 1 \ 1]\), \( P(<X_A X_B X_C > | 1) \) can be computed as \((0.659 + (1-0.659)/2)*(0.573 + (1-0.573)/2)*(0.919 + (1-0.919)/2)\); \( P(<X_A X_B X_C > | 0) = 1 – \)
Yang, Mo

\[ P(\{X_A \geq X_B \geq X_C\} > \{1\}) = P(1) = P(0) = 1/2. \] Based on Equation (3), posterior probability \[ P(1|X_A \geq X_B \geq X_C) \] is obtained, which is, \[ 0|X_A \geq X_B \geq X_C = 1 - P(1|X_A \geq X_B \geq X_C). \] If posterior probability of 1 is greater than 0.5, then inferred consensus answer is “yes”; Vice versa, if posterior probability of 0 is greater than 0.5, then the inferred answer is “no”.

Table 1 presents posterior probability for each response pattern and their inferred consensus answer. For example, for pattern \{0 0 1\}, although the first two auditors have answered “no”, only the last auditor answered “yes”, but because the third person is more competent (0.919, compared to other two auditors, 0.658 and 0.573), the inferred answer is “yes” rather than “no.” Therefore, the weighted competency value has a significant impact on the estimation of consensus answer.

Table 1 Posterior Probability and Inferred Consensus Answers

<table>
<thead>
<tr>
<th>Response pattern</th>
<th>Posterior prob if consensus answer is</th>
<th>Inferred answer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>111</td>
<td>0.998</td>
<td>0.002</td>
</tr>
<tr>
<td>110</td>
<td>0.430</td>
<td>0.565</td>
</tr>
<tr>
<td>101</td>
<td>0.983</td>
<td>0.034</td>
</tr>
<tr>
<td>100</td>
<td>0.069</td>
<td>0.963</td>
</tr>
<tr>
<td>011</td>
<td>0.963</td>
<td>0.069</td>
</tr>
<tr>
<td>010</td>
<td>0.034</td>
<td>0.983</td>
</tr>
<tr>
<td>001</td>
<td>0.565</td>
<td>0.430</td>
</tr>
<tr>
<td>000</td>
<td>0.002</td>
<td>0.998</td>
</tr>
</tbody>
</table>

**Conclusion**

In the world of auditing, consensus is often used as a surrogate for accuracy. However, there is a variety of approaches to form a consensus. The present study introduces a psychometric CCT
model to the area of audit decision making, because it has demonstrated several advantages over other approaches.

First, this model weights individual auditor’s competency as well as aggregated responses simultaneously. Pincus (1990) proposed one consensus model to examine the relationship of the consensus-accuracy which assumes all auditors have the same level of competencies, and effects of relaxation of the simplifying assumption are analyzed. However, its assumption is not consistent with the reality that auditors differ from one another in terms of their expertise, experience, judgment ability. In contrast, the present model respects the individual difference in their competencies and is capable of estimating all individual competencies. Secondly, estimation in this model is purely based on the original responses rather than performance data which are converted from response data through answer key. In many auditing cases, people do not know answers to auditing questions, and there is no way to obtain performance data. Thus this model seems very suitable for various auditing tasks. Finally, this model provides a quantitative consensus answer for each auditing question, so consensus among auditing experts could be used as a surrogate for accuracy. Based on these demonstrated advantages, we believe that this new model can be very helpful in thinking about the way consensus is used in audit practice, and it is a valuable addition to the auditing decision-making studies.

Reference


