A CAPACITY CONSTRAINED CELLULAR MANUFACTURING MODEL WITH EXCEPTIONAL ELEMENTS

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ABSTRACT

This paper develops a mathematical programming model that minimizes the total costs of a cellular manufacturing system with exceptional elements including intercellular transfer, machine duplication, and subcontracting with constraints on machine capacities.

Keywords: group technology, optimization, minimization

1. INTRODUCTION

Cellular manufacturing has long been known as a way to increase manufacturing productivity and efficiency. By locating equipment in a cellular arrangement, manufacturers have been able to achieve cost savings through minimizing part movements and improving process flow. Often times, however, even the best cellular manufacturing layout will have parts that have processes outside of its cell. Bottleneck machines and exceptional parts that span two or more manufacturing cells are called exceptional elements (EEs).

Shafer et al (1992) introduced a model to deal with EEs in three ways after other efforts were exhausted: (1) intercellular transfer, (2) purchase additional machinery, or (3) subcontract the part. The model that was presented created the optimal application of intercellular transfer, machine duplication, and subcontracting based on one year of planned production. The model assumes that the cells are previously set up, but makes no mention of the number of machines contained in each cell. If the model chooses to perform an intercellular transfer, there are no capacity constraints on the receiving cell. Capacity within the cell is infinite despite the capacity constraints that the model has dictated for each machine. The capacities that are specified are used for new equipment purchases related to exceptional elements only. This paper will build upon the model presented by Shafer et al (1992) by eliminating the assumption of infinite capacities for existing machinery. Therefore, when the model looks into an intercellular transfer, constraints assure that the transferred parts cannot exceed the preset capacity of the applicable machinery in the receiving cell.

2. LITERATURE REVIEW

Most of the literature in cellular manufacturing focuses on cell formation instead of dealing with minimizing cost in a system that has already been set up. Several literature review papers exist on the numerous optimization methods for the design and formation of manufacturing cells (Singh, 1993; Offodile et al, 1994; Mansouri et al, 2000; Ghosh et al, 2011). However, often times the cost and downtime associated with moving and reorganizing machinery and redesigning the shop floor layout is unreasonable and companies need to focus on minimizing cost with the existing cellular manufacturing layout. In addition, the existing layout may still be considered optimal in the long term, but short-term manufacturing of some parts requires intercellular transfers, subcontracting the entire manufacturing process of select parts, or purchasing new equipment. This is where papers on dealing with EEs in an existing cellular manufacturing layout take over.

Burbidge (1975) observed the problem associated with EEs, which are defined as bottleneck machines and exceptional parts that span two or more manufacturing cells and suggested five approaches for eliminating EEs: (1) reroute the part, (2) modify the manufacturing process, (3) modify the part's design, (4) subcontract the part, or (5) modify the cells to accommodate the EEs.

The problems associated with EEs are common. Wemmerlöv & Hyer (1989) conducted a survey of 29 manufacturers and found that only three of these respondents did not have any instances of EEs. Dealing with EEs is an ongoing problem that is not going to go away. Part design and initial cellular formation can only do so much and EEs are an issue that companies must address as efficiently as possible.

Many methods have been proposed for dealing with EEs. As mentioned in the introduction, Shafer et al (1992) developed a mathematical programming approach for dealing with exceptional elements involving intercellular transfer, duplicating machinery, and subcontracting. Kern & Wei (1991) developed a method of creating a prioritized list of EEs for decision makers to see which actions would be the most cost-effective. Mansouri et al (2003) used a genetic algorithm for dealing with EEs in the form of a Multi-Objective Genetic Algorithm in order to minimize: (1) intercellular movements of parts, (2) total cost of machine duplication and subcontracting, (3) system under-utilization, and (4) deviations among the cells' utilization.

The model developed in this paper uses notation similar to that presented by Shafer et al (1992) and the same example, which appears several times in the literature. The next section of the paper presents the model designed to minimize the costs of the cellular manufacturing system related to EEs using intercellular transfers, machine duplication, and subcontracting while taking the predefined existing machine capacities into consideration.

3. MODEL FORMULATION

This model operates under several assumptions. Subcontracting of parts is total production subcontracting, which means the subcontractor produces the part from start to finish. Floor space exists for machine duplication. Intercellular transfer cost is set to be appropriate for the sequence of operations. A full redesign of the production layout is not feasible or appropriate at this time and, therefore, the cost minimization must retain the current cellular formation.

The capacity constrained mathematical programming model for cellular manufacturing with exceptional elements contains the following notation.

Indexing sets

- f index for machine cells.
- i index for parts.
- k index for machines.
- nf number of cells indexed by $f = 1, 2, \ldots, nf$.
- ni number of parts indexed by $i = 1, 2, \ldots, ni$.
- nk number of machines indexed by $k = 1, 2, \ldots, nk$.

Sets

- EE set of all exceptional elements consisting of a part and machine combination ik where the part i needing machine k reside in different cells.
- L_{kf} set of all parts using machine k in cell f.
- N_f set of machines in cell f needed by parts outside cell f.
- T_f set of machines not in cell f needed by parts in cell f.
- EEM_{kf} set of all parts in cell f that require a machine, k, outside of cell f.
- EEP_{kf} set of all parts outside of cell f that require a machine, k, in cell f.

Decision variables

- X_i units of part i to be subcontracted
- Y_{kf} number of machines of type k to be purchased for cell f
- Z_{ik} number of intercellular transfers required by part i as a result of machine type k not being available within the part's manufacturing cell

Parameters / Other Variables

- S_i incremental cost of subcontracting a unit of part i
- I_i incremental cost for moving part i outside of a cell as opposed to moving it within the cell (this cost can also reflect the disruptive effects of having intercellular transfers)
- D_i annual forecasted demand for part i
- A_k annual cost of acquiring a machine of type k
- C_k annual capacity of machine type k
- P_{ik} processing time needed to produce part i on a machine of type k
- M_{kf} number of machines of type k needed in cell f
- B_{kf} beginning capacity for available machines of type k in cell f
- V_{kf} machine minutes available for machines of type k in cell f (includes new equipment capacity)
- R_{kf} machine minutes required for machines of type k in cell f

The capacity constrained mathematical programming model for cellular manufacturing with exceptional elements is:

$$Min \sum_{i} X_i S_i + \sum_{T_f} Y_{kf} A_k + \sum_{EE} Z_{ik} I_i \qquad (1)$$

subject to

$$X_i \le D_i \quad \text{for all } i \notin EE \qquad (2)$$

$$Z_{ik} + X_i \le D_i \quad \text{for the set of } EE \qquad (3)$$

$$M_{kf} = \frac{R_{kf}}{C_k} \quad \text{for } k \in T_f \qquad (4)$$

$$M_{kf} \le Y_{kf} \quad \text{for } k \in T_f \qquad (5)$$

$$V_{kf} = C_k Y_{kf} \quad \text{for } k \in T_f \qquad (6)$$

$$V_{kf} = B_{kf} \quad \text{for } k \in N_f \qquad (7)$$

$$R_{kf} = \sum_{i \in L_{kf}} \left(P_{ik}\left(D_i - X_i\right)\right) + \sum_{i \in EEP_{kf}} P_{ik} Z_{ik} \quad \text{for } k \in N_f \qquad (8)$$

$$R_{kf} = \sum_{i \in EEM_{kf}} \left(P_{ik}\left(D_i - X_i\right)\right) - \sum_{i \in EEM_{kf}} \left(P_{ik} Z_{ik}\right) \quad \text{for } k \in T_f \qquad (9)$$

$$V_{kf} \ge R_{kf} \quad \text{for } k \in N_f \text{ and } k \in T_f \qquad (10)$$

$$X_i, Y_{kf}, Z_{ik} \quad \text{all integer and} \ge 0 \qquad (11)$$

Equation (1) minimizes the costs associated with exceptional elements. These costs include new equipment purchases, intercellular transfers, and the incremental costs of subcontracting.

Constraint (2) states that the number of parts to be subcontracted cannot exceed the total demand for the part and while it applies to all parts, when parts span multiple cells, this constraint is redundant so this constraint is only applied to the set of parts that have all processes within their home cell.

Constraint (3) makes sure that the sum of subcontracted parts and intercellular transfers does not exceed annual demand. This constraint only has to be applied to the set of exceptional elements.

Constraint (4) sets the number of machines, *k*, not in cell *f* that are required by parts in cell *f*. It is not an integer so fractional values simply show the precise machine usage.

Constraint (5) converts any excess fractional machines into an integer value of new machines. While constraints (4) and (5) could be combined together, they are kept separate so that machine utilization can be recorded.

Constraint (6) sets the new machine hours available for machines of type *k* purchased for cell *f* equal to capacity of machine type *k* times the number of machines purchased. Since this capacity is for the newly purchased machines in cells without any previous machines of this type, the beginning capacity prior to purchases is equal to zero.

Constraint (7) simply specifies that for the set of machines in cell *f* that are required by parts outside of cell *f*, the machine hours available for machines of type *k* in that cell equals the beginning capacity. This is because if a machine were to be purchased, it would naturally be placed in the cell that would not require an intercellular transfer.

Constraint (8) captures the machine hours required for machines of type *k* for the set of machines in cell *f* that are required by parts outside of cell *f*. The constraint counts the machine hours for parts within the cell that are not subcontracted and adds in the machine hours needed for any parts that are transferred in from other cells.

Constraint (9) captures the machine hours required for machines of type *k* for the set of bottleneck machines outside of cell *f* that are required by parts in cell *f*. The constraint deals with new machine purchases. It counts the machine hours for parts where new equipment has been purchased for the cell to avoid intercellular transfer.

Constraint (10) assures that the machine capacity available is greater than or equal to the machine hours required for each machine of type *k* in cell *f*. This applies to both the set of bottleneck machines outside cell *f* required by parts in cell *f* and the set of machines in cell *f* required by parts outside cell *f*.

Constraint (11) simply specifies that all decision variables are integer values and greater than or equal to 0.

4. A NUMERICAL EXAMPLE

While this model could be applied to any new example, the example introduced by Shafer et al (1992) is used to demonstrate the differences that would result in this particular problem when existing machine capacities are predefined and taken into consideration. The results are then compared. Figure I shows the data that is used in this example. Manufacturing cells are boxed and greyed in with cell one, two, and three going left to right. While A_k, C_k, S_i, D_i, and I_i are pretty clearly defined, the numbers within the cells are the manufacturing time, P_{ik}, in minutes for part *i* and machine *k*. Note that while these part processing times are shown in minutes, the machine capacities are shown in hours.

Figure I. Data for Numerical Example

Machines \ Parts	1	2	3	4	5	6	7	8	9	10	A_k	C_k
1	2.95		2.20							4.61	50784	2000
2	2.76	5.18	1.89	3.89		5.14					67053	2000
3	5.54	4.29									43944	2000
4	2.91			1.97	2.59	4.01		2.70			67345	2000
5				4.28		4.51					42414	2000
6	1.92						2.23		5.52		75225	2000
7					3.40		1.16	4.72		2.49	52741	2000
8		5.32						3.75	3.85		63523	2000
9							4.04			1.83	50632	2000
S_i	4.2	4.3	3.5	4.4	5.0	3.9	4.4	4.6	5.0	5.0		
D_i	32128	27598	20651	11340	18707	17040	46196	45384	16409	22000		
I_i	3.7	2.8	2.8	3.3	2.8	3.5	2.8	2.6	3.4	3.2		

Figure I can be used to identify the exceptional elements for both parts and machines. For example, the set of bottleneck machines for cell 3 are machines 1 and 4 due to parts 8 and 10 in cell 3 requiring these machines, which reside outside of cell 3. The set of exceptional parts associated with cell 3 are parts 1, 2, and 5 because in the given cell formation, these parts require machines 6, 7, and 8, which reside in cell 3. The set of exceptional elements in part-machine, *ik*, format are 14, 16, 28, 42, 57, 62, 84, and 101.

When the original cellular formation is examined, the minimum machines required by each cell are given in Figure II. As you can see, the machine times required for the parts coupled with the machine capacities results in multiple machines of each type residing in each cell. For the purposes of this example, we assume that there are only enough machines available to satisfy current demand requirements within the cell. This is the assumed starting point of the example.

Figure II. Within Cell Problem Scenario

			Within Cell Machine Utilization		
Cell Number	Machine Number	Machine Capacity (Hours)	Machine Hours Needed	Number of Machines	Cellular Machine Utilization
1	1	2000	2336.8	2	58.4%
1	2	2000	4511.0	3	75.2%
1	3	2000	4939.7	3	82.3%
2	4	2000	2318.7	2	58.0%
2	5	2000	2089.8	2	52.2%
3	6	2000	3226.6	2	80.7%
3	7	2000	5376.3	3	89.6%
3	8	2000	3889.4	2	97.2%
3	9	2000	3781.5	2	94.5%

This problem scenario will demonstrate where the problems with existing machines become problematic. In the optimal solution without existing machine capacity constraints as presented by Shafer et al (1992), we naturally run into problems when we take away the assumption of infinite capacities of existing machines. If we assume that the company does not have excess machines in each cell just collecting dust, then we can see that the available hours for each machine in each cell are finite. For example, cell number three's machine number eight has 4000 hours of machine time available, but only 3889.4 hours are needed. That would theoretically leave 110.6 hours of additional machine capacity available for utilization. As stated previously, the model presented by Shafer et al (1992) assumes infinite capacities for existing machinery and as a result the optimal solution has problems when you add capacities for existing machines. Figure III shows the optimal solution of Shafer et al (1992) with the resulting intercellular moves, subcontracted units, and new equipment purchases all factored in. In addition, unplanned machine purchases due to capacity violations are added in.

Figure III. Result of Applying Optimal Solution without Constraints on Capacity

			Within Cell Machine Utilization				
Cell Number	Machine Number	Machine Capacity (Hours)	Machine Hours Needed	Number of Machines	Cellular Machine Utilization	Machines Now Idle	Additional Machines Needed
1	1	2000	757.2	2	18.9%	1	
1	2	2000	3228.1	3	53.8%	1	
1	3	2000	1973.3	3	32.9%	2	
2	2*	2000	2000.0	1	100.0%		
2	4	2000	2361.0	2	59.0%		
2	5	2000	2089.8	2	52.2%		
3	1*	2000	1690.3	1	84.5%		
3	4*	2000	2000.0	1	100.0%		
3	6	2000	3226.6	2	80.7%		
3	7	2000	6436.4	3	107.3%		1**
3	8	2000	6336.4	2	158.4%		2**
3	9	2000	3781.5	2	94.5%		

* Planned machinery purchase

** Unplanned machinery purchase due to infinite capacity assumption

The last column in Figure IV shows the problems that result from ignoring machine capacity. It should be noted that in cell one, the subcontracting decision for part one has meant that the machine utilization in cell one has dropped dramatically for each of its three machines. There are now idle machines not being utilized. However, the costs dictate this so it is not a huge concern. This was done because it was more economically feasible to subcontract the part because of its need for two machines not residing in cell one. The movement costs were such that it made more fiscal sense to subcontract the parts compared to making them in house. One might look at four of eight machines residing in cell one being turned off and wonder about the accounting practices that were involved in the creation of the costs. Cost accounting notwithstanding, dealing with manufacturing cell machine utilization is an area for further research.

The larger, more glaring problem comes in cell three. The intercellular moves that were requested have resulted in too many machine hours needed for the available capacities. This infinite cellular capacity assumption is problematic. Would additional machines need to be purchased to meet these requirements? If so, these costs need to be factored in. According to the example, the cost of purchasing these new machines would total $179,787. This would bring the total cost related to exceptional elements to $639,970.60. Hence, when you look at the problem with pre-existing capacities in mind, this is no longer the optimal solution.

That result can now be compared to the solution generated by the model in this paper, which was found using LINGO solver and is represented in Figure IV. Notice the differences in the solution when machine capacities in each manufacturing cell are taken into consideration. The global optimal solution has a cost of $466,603.70. When additional machines that are needed due to capacity violations are added to the optimal solution that was solved with no constraints on capacity, the total cost becomes $639,970.60. Therefore, the cost savings using the presented capacity constrained model is $173,366.90 ($639,970.60 - $466,603.70).

Figure IV. Capacity Constrained Optimization Result

			Within Cell Machine Utilization				
Cell Number	Machine Number	Machine Capacity (Hours)	Machine Hours Needed	Number of Machines	Cellular Machine Utilization	Machines Now Idle	Additional Machines Needed
1	1	2000	757.2	2	18.9%	1	
1	2	2000	2900.5	3	48.3%	1	
1	3	2000	1701.9	3	28.4%	2	
1	8*	2000	2000.0	**1**	100.0%		
2	2*	2000	2000.0	**1**	100.0%		
2	4	2000	2361.0	2	59.0%		
2	5	2000	2089.8	2	52.2%		
2	7*	2000	1060.1	**1**	53.0%		
3	1*	2000	1690.3	**1**	84.5%		
3	4*	2000	2000.0	**1**	100.0%		
3	6	2000	3226.6	2	80.7%		
3	7	2000	5376.3	3	89.6%		
3	8	2000	4000.0	2	100.0%		
3	9	2000	3781.5	2	94.5%		

* Planned machinery purchase

Figure V breaks down the costs associated with EEs by the type of action taken. The results of the solution without capacity constraints show the machine costs associated with planned equipment purchases and unaccounted for equipment purchases that were the result of the infinite capacity assumption for existing machinery.

Figure V. Cost Breakdown Comparison

Cost	No Capacity Constraints	Capacity Constraints
Subcontracting	$134,937.60	$151,256.10
Intercellular Transfer	$140,064.00	$13,901.60
New Machines:		
Planned Purchases	$185,182.00	$301,446.00
Due to Capacity Violation	$179,787.00	$0.00
Total Cost	**$639,970.60**	**$466,603.70**

5. CONCLUSION

This paper has presented a capacity constrained mathematical programming model for cellular manufacturing with exceptional elements. This model shows how the capacity of existing machines impacts a cellular manufacturing optimization with exceptional elements. A comparison was made to an optimization with no capacity constraints on existing machinery to highlight the importance of including all capacities as constraints in a mathematical programming model. The model could be easily adapted to find the minimum cost of any pre-existing cellular manufacturing system with EEs utilizing intercellular transfer, machine duplication, and subcontracting. An additional customization that might be of interest would be to create scenarios where the same machine exists in multiple cells and therefore any intercellular transfers involving those machines requires choosing the appropriate cell to transfer to given associated capacities and costs.

Future research could include adding more complexities such as sequencing of machine-part processes. In addition, subcontracting of individual processes instead of total part processing could be explored and added as an additional option if it is appropriate in the industry of interest.

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