# COMPUTING THOMPSON-THRALL NONLINEAR EFFICIENCY MEASURE AGAINST LINEAR EFFICIENCY MEASURES IN DEA RELATIVE TO AN 'IDEAL REFERENCE'

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## ABSTRACT

Several researchers in the past have emphasized the importance of computing efficiency measures in Data Envelopment Analysis (DEA) relative to a best-practice benchmark. Thompson and Thrall (Thompson et al., 1995) introduced a nonlinear efficiency measure with linked-cone assurance-regions (LC-AR) in DEA. In this paper, we compute Thompson-Thrall's nonlinear measure against radial efficiency measures of CCR (Charnes et al., 1978), BCC (Banker et al., 1984), CCR/AR and BCC/AR (Thompson et al., 1992) relative to an 'ideal reference' and show a contrast in an application to a set of banks

## KEY WORDS

Data envelopment analysis (DEA), Assurance regions (AR), Linked cones (LC), LC-AR efficiency

## 1. INTRODUCTION

Since the original publication on DEA by Charnes et al. (1978) measuring the efficiency of decision making units, there has been a rapid and continuous growth in this area of study. A considerable amount of research publications have appeared, a significant portion of which focusing on DEA applications of efficiency and productivity in the banking sector. For example, a comprehensive survey of literature on bank efficiency could be found in Fethi and Pasiouras (2010). They have examined bank branch efficiencies in more than 30 studies over the period 1998-2009. All these studies are using DEA to estimate bank efficiency. Thompson et al. (1995) introduced nonlinear LC-AR efficiency measure to DEA literature, where they discussed the single-output multiple-inputs model subject to linked-cone (LC) assurance-regions (AR). In this paper, we compute LC-AR measure against the known radial measures of CCR (Charnes et al., 1978), BCC (Banker et al., 1984), CCR/AR and BCC/AR (Thompson et al. 1992), relative to an 'ideal reference'- an industrial benchmark. We demonstrate the computations and show a contrast in an application to a set of South Asian banks.

## 2. LITERATURE SURVEY

As a nonparametric technique DEA method has been used in the past to assess performance efficiency in many areas of decision science. But here, we narrow our search and focus mainly on the banking sector. Fadzlan (2010), using DEA, provided empirical evidence on the evolution of the Indonesian banking sector's efficiency during the post Asian financial crisis period of 1999-2008. His findings suggested that Indonesian banking sector's inefficiency stems largely from pure technical rather than scale. Bhattacharyya, Lovell and Sahay (1997) examined the productive efficiency of Indian commercial banks during 1986-1991 and reported a marginal increase in overall average performance after 1987. They found that the average efficiency of publicly owned banks to be much higher than in the privately owned or foreign owned banks. Sathye (2003) compared productive efficiency of publicly owned, privately owned and foreign owned banks operational in India in the year 1997/1998 and reported that private sector commercial banks as a group is paradoxically lower than that of public sector and foreign banks. In a study on Pakistan, Akhtar and Bokhari (2005) have analysed the x-efficiency, decomposed into technical and allocative efficiency, of 40 commercial banks using the DEA approach. The results of the study are showing that technical efficiency of Pakistani banks found to be lower than the allocative one. Private Banks in Pakistan emerged as efficient on both fronts i.e. technical efficiency and allocative one, compared to their counterparts, the public and foreign banks. Ataullah et al. (2004) have done a comparative analysis of commercial banks in India and Pakistan using data for 1988–1998. They applied DEA to two alternative input–output specifications to measure technical efficiency and revealed that the overall technical efficiency of the banking industry of both countries improved gradually over the years. Galagedara and Edirisuriya (2005) investigated bank efficiency using DEA methodology and productivity growth employing Malmquist index in a sample of Indian commercial banks over the period 1995-2002. Rajesh and Gaurav (2005) in another study evaluated the relative efficiency of Indian banks using DEA method and suggested that the foreign banks, as a group, have been considerably more efficient than all other bank groups, followed by the Indian private banks. In comparison, public sector banks have lagged behind their private counterparts in performance.

Das and Ghosh (2009), using DEA, examined the impact of financial deregulation on cost and profit efficiency of Indian commercial banks during the post-reform period 1992–2004.

DEA measures only relative efficiency of decision making units (DMUs) participating in the study. While highlighting this as a drawback in DEA methodology, some authors in the past have suggested incorporating best-practice into efficiency analysis by including an ‘industrial benchmark’ among the DMUs. Dharmapala et al. (2007), Cook and Zhu (2006), Zhu (1996), Thanassoulis and Dyson (1992) and Golany (1988) are among those suggested. In this paper, we measure the performance of 55 banks relative to an ‘ideal reference’- an industrial average.

## 3. METHODOLOGY

### 3.1Thompson-Thrall LC-AR efficiency model

Thompson et al. (1993) presented a new DEA theory, which did not require the use of non-Archimedean principle used in the original DEA theory developed by Charnes et al. (1978), and it allowed zero data entries. In this paper, we use Thompson et al.’s methodology as described below.

A DEA data domain consists of n decision-making units (DMUs), n input vectors (each with m inputs), and n output vectors (each with r outputs). The selected $DMU_c$ (c = 1, 2,…, n) is characterized by an input vector $X_c$ ($x_{1c}$, $x_{2c}$, …, $x_{mc}$) and an output vector $Y_c$ ($y_{1c}$, $y_{2c}$, …, $y_{rc}$ ). U-output multiplier of r unknowns ($u_k$ ; k = 1, 2,…,r ) and V-input multiplier of m unknowns ($v_i$ ; i = 1, 2,…m ) need to be determined by solving the respective nonlinear/linear programming (NLP/LP) models stated below.

Here we consider the following four inputs (m=4) and one output (r=1) for the banks:

$X_1$ – Total deposits include demand deposits, time and savings deposits, CDs, and purchased funds

$X_2$ - Fixed assets in terms of bank premises, furniture, and equipment

$X_3$ – Total non-interest expenses include employee salaries, benefits, and expenses on fixed assets

$X_4$ – Loan loss provisions. Accounting allocations to cover possible loan defaults

Y - Total loans include commercial, industrial, real-estate, and installment loans

We now formulate the General LC-AR efficiency model with nonlinear objective function (Thompson et al., 1995)

$$\text{Max } \theta_c = \left[ \sum_{k=1}^{r} u_k y_{kc} / \sum_{i=1}^{m} v_i x_{ic} \right] \quad \text{(for DMU}_c\text{)}$$

$$\sum_{k=1}^{r} u_k y_{kj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j = 1, 2, ..., n \qquad (1)$$

$$\alpha_i v_j < v_i < \beta_i v_j \qquad j < i \; ; \; j = 1, ..., m-1 \; ; \; i = 2, ..., m \qquad \text{(input cone)} \qquad (2)$$

$$\gamma_k u_j < u_k < \delta_k u_j \qquad j < k \; ; \; j = 1, ..., r-1 \; ; \; k = 2, ..., r \qquad \text{(output cone)} \qquad (3)$$

$$A_i u_k < v_i < B_i u_k \quad k \le i \text{ ; } i = 1, ..., m \text{ ; } k = 1, 2, \dots, r \quad \text{(linked cone)} \qquad (4)$$

Here the non-negative scalars $\alpha_i$, $\beta_i$, $\gamma_k$, $\delta_k$, $A_i$, and $B_i$ in constraints (2) through (4) are specified or estimated using socio-economic/market data and/or expert opinion. They are also called "price/cost" data. As illustrated in Thompson et al. (1996), input cone has $2({}^{m}C_2)$ constraints, output cone $2({}^{r}C_2)$ and the linked cone 2(m)(r), in general. In our study, with m=4 and r=1, input cone has 12, output cone has none, and linked cone has 8 constraints.

In computing $\alpha_i$, $\beta_i$, $\gamma_k$, $\delta_k$, $A_i$, and $B_i$, first we use the bounds on $v_i$ and $u_k$:

$$\text{Input multipliers: } LV_i \le v_i \le UV_i \qquad i = 1, 2, \dots, m \qquad (5)$$

$$\text{Output multipliers: } LU_k \le u_k \le UU_k \qquad k = 1, 2, \dots, r \qquad (6)$$

Notice that LV is lower and UV is upper bound for $v_i$, and LU is lower and UU is upper bound for $u_k$. Next, we compute the non-negative scalars using (5) and (6).

$$\alpha_i = LV_i / UV_j \text{ , } j < i \text{ ; } j = 1, ..., m\text{-}1 \text{ ; } i = 2, ..., m \quad \text{(for input cone)} \qquad (7)$$

$$\beta_i = UV_i / LV_j \text{ , } j < i \text{ ; } j = 1, ..., m\text{-}1 \text{ ; } i = 2, ..., m \quad \text{(for input cone)} \qquad (8)$$

$$\gamma_k = LU_k / UU_j \text{ , } j < k \text{ ; } j = 1, ..., r\text{-}1 \text{ ; } k = 2, ..., r \quad \text{(for output cone)} \qquad (9)$$

$$\delta_k = UU_k / LU_j \text{ , } j < k \text{ ; } j = 1, ..., r\text{-}1 \text{ ; } k = 2, ..., r \quad \text{(for output cone)} \qquad (10)$$

$$A_i = LV_i / UU_k \text{ , } k \le i \text{ ; } i = 1, ..., m \text{ ; } k = 1, 2, \dots, r \quad \text{(for linked cone)} \qquad (11)$$

$$B_i = UV_i / LU_k \text{ , } k \le i \text{ ; } i = 1, ..., m \text{ ; } k = 1, 2, \dots, r \quad \text{(for linked cone)} \qquad (12)$$

Then, with 1 output (r=1) and m inputs Thompson-Thrall LC-AR (TT/LC-AR) nonlinear efficiency model is the following:

$$\text{Max } \theta_c = [u_1 y_c / \sum_{i=1}^{m} v_i x_{ic}] \quad \text{(for DMU}_c\text{)}$$

$$y_j - \sum_{i=1}^{m} v_i x_{ij} \le 0 \text{ ; } j = 1, 2, ..., n \qquad (1)$$

and constraints (2) (4) (5) (7) (8) (11) (12)

### 3.2 CCR/AR and BCC/AR efficiency models

Here we recall the input-oriented CCR/AR (constant returns) and BCC/AR (variable returns) efficiency models with linear objective functions (Thompson et al., 1992).

<u>CCR/AR model</u>:

$$\text{Max } \theta = \sum_{k=1}^{r} u_k y_{kc}$$

$$\sum_{i=1}^{m} v_i x_{ic} = 1 \qquad (0)$$

$$\sum_{k=1}^{r} u_k y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1, 2, ..., n \qquad (1)$$

$\alpha_i v_j < v_i < \beta_i v_j$     j < i ; j = 1, ..., m-1 ; i = 2, ..., m     (input cone)     (2)

$\gamma_k u_j < u_k < \delta_k u_j$     j < k ; j = 1, ..., r-1 ; k = 2, ..., r     (output cone)     (3)

and constraints (5) – (10).

BCC/AR model:

$$\text{Max } \theta = \sum_{k=1}^{r} u_k y_{kc} + u^*$$

$$\sum_{i=1}^{m} v_i x_{ic} = 1 \qquad (0)$$

$$\sum_{k=1}^{r} u_k y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u^* \le 0 \quad j = 1, 2, ..., n \qquad (1)$$

$\alpha_i v_j < v_i < \beta_i v_j$     j < i ; j = 1, ..., m-1 ; i = 2, ..., m     (input cone)     (2)

$\gamma_k u_j < u_k < \delta_k u_j$     j < k ; j = 1, ..., r-1 ; k = 2, ..., r     (output cone)     (3)

and constraints (5) – (10). u* unrestricted.

We also notice that without constraints (2), (3) and (5) – (10), CCR/AR and BCC/AR reduce to CCR (Archimedean) and BCC (Archimedean) models respectively.

We now define the input and output multipliers, $v_i$ and $u_k$, using “price/cost” data.

Table 1 Multipliers and Price/Cost data

| Variable | Multiplier | Value of multiplier ( price/cost ) |
|---|---|---|
| $X_1$ | $v_1$ | Interest rate on deposits, calculated as, interest income on deposits / total deposits |
| $X_2$ | $v_2$ | Ratio of non-interest income to fixed assets |
| $X_3$ | $v_3$ | Ratio of non-interest income to non-interest expenses |
| $X_4$ | $v_4$ | Value of 1, which means 1$ of loan loss provision is worth 1$ (no interest or income earned) |
| Y | $u_1$ | Interest rate on loans, calculated as, interest income on loans / total loans |

**3.3 Formation of LC-AR constraints**

Thompson et al. (1995), when forming the assurance region bounds for multipliers $v_i$ and $u_k$ in constraints (5) and (6), used (min, max) range for $v_i$ and $u_k$. In table-2 below, we illustrate their formation of bounds for multipliers defined in table-1, and we use (min, max) range of “price/cost” data in 2007 for fifty five banks. In table-3, we lay out the bounds for multipliers.

Table 2 Multipliers and Thompson-Thrall bounds

| Variable | Multiplier | Lower bound of multiplier | Upper bound of multiplier |
|---|---|---|---|
| $X_1$ | $v_1$ | Minimum Interest rate on deposits | Maximum Interest rate on deposits |
| $X_2$ | $v_2$ | Minimum {non-interest income / fixed assets} | Maximum {non-interest income / fixed assets} |
| $X_3$ | $v_3$ | Minimum {non-interest income / non-interest expenses} | Maximum {non-interest income / non-interest expenses} |
| $X_4$ | $v_4$ | 1 | 1 |
| Y | $u_1$ | Minimum Interest rate on loans | Maximum Interest rate on loans |

Table 3 AR bounds of $v_1$, $v_2$, $v_3$, $v_4$, and $u_1$ (market prices in 2007)

| Multiplier | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $u_1$ |
|---|---|---|---|---|---|
| Max | 2.473564 | 12.09822 | 4.163748 | 1 | 27.59003 |
| Min | 0.000882 | 0.239849 | 0.20192 | 1 | 0.001376 |

## 4. EMPIRICAL ANALYSIS AND RESULTS

We analyze input-output data in 2007 of fifty five banks identified by their bank code. Using a MATLAB-based program, we compute their relative efficiency measures CCR, CCR/AR, BCC, BCC/AR, and TT/LC-AR, with and without ‘ideal reference’. Ideal reference carries the averages of the 4 inputs and the output, for all 55 banks. In that sense, ideal reference is an ‘industrial benchmark’, which may be a hypothetical entity. All 55 banks compete against one another and the ideal reference, which is the $56^{th}$ DMU. Relative efficiency scores of 55 banks, with and without the ideal reference, are listed below under the two cases; CCR (constant returns) and BCC (variable returns).

Table 4.1 CCR, CCR-AR and TT/LC-AR, without and with Ideal Reference

| BankCode | CCR | CCR/AR | TT/LC-AR | CCR | CCR/AR | TT/LC-AR |
|---|---|---|---|---|---|---|
| 4 | 0.1958 | 0.0856 | 0 | 0.1958 | 0.0856 | 0 |
| 6 | 0.2092 | 0.1763 | 0.0172 | 0.2092 | 0.1763 | 0.0314 |
| 10 | 0.0914 | 0.0751 | 0.0751 | 0.0914 | 0.0751 | 0.0255 |
| 17 | 0.0972 | 0.068 | 0.0680 | 0.0972 | 0.068 | 0.0249 |
| 19 | 0.0873 | 0.072 | 0.0720 | 0.0873 | 0.072 | 0.0268 |
| 21 | 1 | 0.154 | 0.1540 | 1 | 0.154 | 0.0305 |
| 23 | 0.4079 | 0.0629 | 0.0629 | 0.4079 | 0.0629 | 0.0148 |
| 24 | 1 | 0.2738 | 0.2738 | 1 | 0.2738 | 0.0716 |
| 25 | 0.0007 | 0.0002 | 0.0002 | 0.0007 | 0.0002 | 0.0000 |
| 27 | 1 | 0.083 | 0.0830 | 1 | 0.083 | 0.0241 |
| 28 | 0.1211 | 0.0626 | 0.0626 | 0.1211 | 0.0626 | 0.0221 |
| 29 | 0.2279 | 0.0763 | 0.0763 | 0.2279 | 0.0763 | 0.0227 |
| 30 | 0.0644 | 0.0604 | 0.0604 | 0.0644 | 0.0604 | 0.0235 |
| 32 | 0.1246 | 0.1189 | 0.1189 | 0.1246 | 0.1189 | 0.0295 |
| 34 | 0.172 | 0.1595 | 0.1595 | 0.172 | 0.1595 | 0.0307 |
| 36 | 0.0667 | 0.065 | 0.0650 | 0.0667 | 0.065 | 0.0243 |
| 40 | 0.2908 | 0.0749 | 0.0749 | 0.2908 | 0.0749 | 0.0219 |
| 45 | 0.0972 | 0.0934 | 0.0934 | 0.0972 | 0.0934 | 0.0276 |
| 53 | 0.0633 | 0.0589 | 0.0589 | 0.0633 | 0.0589 | 0.0198 |
| 54 | 0.0386 | 0.0369 | 0.0369 | 0.0386 | 0.0369 | 0.0176 |
| 55 | 0.0642 | 0.0592 | 0.0592 | 0.0642 | 0.0592 | 0.0216 |
| 58 | 0.0866 | 0.0824 | 0.0824 | 0.0866 | 0.0824 | 0.0244 |
| 59 | 0.2274 | 0.1276 | 0.1276 | 0.2274 | 0.1276 | 0.0312 |
| 60 | 0.1054 | 0.0613 | 0.0613 | 0.1054 | 0.0613 | 0.0190 |
| 61 | 0.1671 | 0.0679 | 0.0679 | 0.1671 | 0.0679 | 0.0236 |
| 63 | 0.166 | 0.1168 | 0.1168 | 0.166 | 0.1168 | 0.0296 |
| 67 | 0.1366 | 0.0426 | 0.0426 | 0.1366 | 0.0426 | 0.0132 |
| 68 | 0.1081 | 0.1054 | 0.1054 | 0.1081 | 0.1054 | 0.0273 |
| 69 | 0.6708 | 0.1483 | 0.1483 | 0.6708 | 0.1483 | 0.0328 |
| 71 | 0.0788 | 0.0652 | 0.0652 | 0.0788 | 0.0652 | 0.0215 |
| 72 | 0.1159 | 0.1111 | 0.1111 | 0.1159 | 0.1111 | 0.0282 |
| **83** | **1** | **1** | **1** | **1** | **1** | **1** |
| 87 | 0.1682 | 0.1638 | 0.1638 | 0.1682 | 0.1638 | 0.0332 |
| 94 | 0.1296 | 0.0725 | 0.0725 | 0.1296 | 0.0725 | 0.0241 |
| 95 | 0.1488 | 0.0821 | 0.0821 | 0.1488 | 0.0821 | 0.0271 |
| 97 | 0.0741 | 0.046 | 0.0460 | 0.0741 | 0.046 | 0.0180 |
| 102 | 0.0707 | 0.0322 | 0.0322 | 0.0707 | 0.0322 | 0.0126 |
| 104 | 0.1379 | 0.0879 | 0.0879 | 0.1379 | 0.0879 | 0.0125 |
| 109 | 0.3064 | 0.1237 | 0.1237 | 0.3064 | 0.1237 | 0.0303 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 111 | 0.016 | 0.0103 | 0.0103 | 0.016 | 0.0103 | 0.0034 |
| 114 | 0.0986 | 0.042 | 0.0420 | 0.0986 | 0.042 | 0.0193 |
| 116 | 0.1862 | 0.1719 | 0.1719 | 0.1862 | 0.1719 | 0.0313 |
| 117 | 0.4483 | 0.1662 | 0.1662 | 0.4483 | 0.1662 | 0.0329 |
| 118 | 0.1545 | 0.1423 | 0.1423 | 0.1545 | 0.1423 | 0.0307 |
| 119 | 0.2124 | 0.196 | 0.1960 | 0.2124 | 0.196 | 0.0334 |
| 120 | 0.4266 | 0.194 | 0.1940 | 0.4266 | 0.194 | 0.0341 |
| 121 | 0.2499 | 0.1972 | 0.1972 | 0.2499 | 0.1972 | 0.0349 |
| 122 | 0.2021 | 0.1891 | 0.1891 | 0.2021 | 0.1891 | 0.0355 |
| 123 | 0.1346 | 0.0988 | 0.0988 | 0.1346 | 0.0988 | 0.0285 |
| 124 | 0.1394 | 0.1242 | 0.1242 | 0.1394 | 0.1242 | 0.0290 |
| 126 | 0.0961 | 0.0939 | 0.0939 | 0.0961 | 0.0939 | 0.0281 |
| 128 | 0.0766 | 0.0738 | 0.0738 | 0.0766 | 0.0738 | 0.0237 |
| 130 | 0.1145 | 0.1049 | 0.1049 | 0.1145 | 0.1049 | 0.0241 |
| 133 | 0.1089 | 0.1001 | 0.1001 | 0.1089 | 0.1001 | 0.0242 |
| 134 | 1 | 0.1111 | 0.1111 | 1 | 0.1111 | 0.0294 |
| Ideal Ref | n/a | n/a | n/a | 0.093 | 0.0738 | 0.0264 |

‘bold’ indicates robustly efficient bank in all categories; n/a – not available

In tab. 4.1, we notice that efficiency scores of CCR and CCR/AR have not changed in the presence of ideal reference, which indicates that CCR and CCR/AR efficiencies are not sensitive towards the ideal reference. But TT/LC-AR scores have changed significantly; the presence of ideal reference has lowered the score for all banks except no.6. Also noticeable is that TT/LC-AR scores are lower than CCR and CCR/AR scores, which indicates that the nonlinear measure is stricter than the two linear measures. If we define an ‘order of strictness’ for efficiency ‘>>’, then the order would be TT/LC-AR >> CCR/AR >> CCR. As for the ideal reference itself, the three efficiency scores are very low, and the order ‘>>’ applies well. And bank 83 emerged as the only efficient bank, overall.

Now we prove our assertion, “the presence of ideal reference lowers TT/LC-AR score for all banks,” by conducting the nonparametric Wilcoxon signed rank test for the difference of TT/LC-AR scores without and with ideal reference. (Difference between columns 4 and 7 in table 4.1)

$H_0$: Median of difference $\leq 0$ vs. $H_a$: Median of difference $> 0$

MINITAB reported a p-value 0.000 with Wilcoxon test statistic 1428.0 (estimated median 0.0661), which allows us to reject $H_0$ and justify our assertion.

Table 4.2 BCC, BCC-AR and TT/LC-AR, without and with Ideal Reference

| BankCode | BCC | BCC/AR | TT/LC-AR | BCC | BCC/AR | TT/LC-AR |
|---|---|---|---|---|---|---|
| 4 | 0.7757 | 0.7349 | 0 | 0.7757 | 0.7349 | 0 |
| 6 | 0.7643 | 0.6946 | 0.0172 | 0.7643 | 0.6946 | 0.0314 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 10 | 0.6685 | 0.6677 | 0.0751 | 0.6685 | 0.6677 | 0.0255 |
| 17 | 0.9231 | 0.9171 | 0.0680 | 0.9231 | 0.9171 | 0.0249 |
| 19 | 1 | 1 | 0.0720 | 1 | 1 | 0.0268 |
| 21 | 1 | 0.5561 | 0.1540 | 1 | 0.5561 | 0.0305 |
| 23 | 0.5267 | 0.0905 | 0.0629 | 0.5267 | 0.0905 | 0.0148 |
| 24 | 1 | 1 | 0.2738 | 1 | 1 | 0.0716 |
| 25 | 1 | 1 | 0.0002 | 1 | 1 | 0.0000 |
| 27 | 1 | 0.8969 | 0.0830 | 1 | 0.8969 | 0.0241 |
| 28 | 0.1953 | 0.1953 | 0.0626 | 0.1953 | 0.1953 | 0.0221 |
| 29 | 0.9311 | 0.8553 | 0.0763 | 0.9311 | 0.8553 | 0.0227 |
| 30 | 0.533 | 0.5314 | 0.0604 | 0.533 | 0.5314 | 0.0235 |
| 32 | 0.3166 | 0.3007 | 0.1189 | 0.3166 | 0.3007 | 0.0295 |
| 34 | 0.6878 | 0.6834 | 0.1595 | 0.6878 | 0.6834 | 0.0307 |
| 36 | 0.1908 | 0.1885 | 0.0650 | 0.1908 | 0.1885 | 0.0243 |
| 40 | 0.3404 | 0.3184 | 0.0749 | 0.3404 | 0.3184 | 0.0219 |
| 45 | 0.0975 | 0.0941 | 0.0934 | 0.0975 | 0.0941 | 0.0276 |
| 53 | 0.5702 | 0.569 | 0.0589 | 0.5702 | 0.569 | 0.0198 |
| 54 | 0.3205 | 0.319 | 0.0369 | 0.3205 | 0.319 | 0.0176 |
| 55 | 1 | 1 | 0.0592 | 1 | 1 | 0.0216 |
| 58 | 0.4795 | 0.4758 | 0.0824 | 0.4795 | 0.4758 | 0.0244 |
| 59 | 1 | 0.9616 | 0.1276 | 1 | 0.9616 | 0.0312 |
| 60 | 0.1076 | 0.0674 | 0.0613 | 0.1076 | 0.0674 | 0.0190 |
| 61 | 0.1838 | 0.0755 | 0.0679 | 0.1838 | 0.0755 | 0.0236 |
| 63 | 0.1698 | 0.1177 | 0.1168 | 0.1698 | 0.1177 | 0.0296 |
| 67 | 1 | 1 | 0.0426 | 1 | 1 | 0.0132 |
| 68 | 0.1474 | 0.1439 | 0.1054 | 0.1474 | 0.1439 | 0.0273 |
| 69 | 0.8824 | 0.2041 | 0.1483 | 0.8824 | 0.2041 | 0.0328 |
| 71 | 0.243 | 0.2385 | 0.0652 | 0.243 | 0.2385 | 0.0215 |
| 72 | 0.3335 | 0.3202 | 0.1111 | 0.3335 | 0.3202 | 0.0282 |
| **83** | **1** | **1** | **1** | **1** | **1** | **1** |
| 87 | 1 | 1 | 0.1638 | 1 | 1 | 0.0332 |
| 94 | 0.1356 | 0.0731 | 0.0725 | 0.1356 | 0.0731 | 0.0241 |
| 95 | 1 | 1 | 0.0821 | 1 | 1 | 0.0271 |
| 97 | 0.8857 | 0.8847 | 0.0460 | 0.8857 | 0.8847 | 0.0180 |
| 102 | 0.5654 | 0.5461 | 0.0322 | 0.5654 | 0.5461 | 0.0126 |
| 104 | 1 | 1 | 0.0879 | 1 | 1 | 0.0125 |
| 109 | 0.344 | 0.1612 | 0.1237 | 0.344 | 0.1612 | 0.0303 |
| 111 | 0.1336 | 0.1136 | 0.0103 | 0.1336 | 0.1136 | 0.0034 |
| 114 | 0.5068 | 0.4925 | 0.0420 | 0.5068 | 0.4925 | 0.0193 |
| 116 | 0.4665 | 0.423 | 0.1719 | 0.4665 | 0.423 | 0.0313 |
| 117 | 1 | 0.8272 | 0.1662 | 1 | 0.8272 | 0.0329 |
| 118 | 1 | 1 | 0.1423 | 1 | 1 | 0.0307 |
| 119 | 0.2148 | 0.201 | 0.1960 | 0.2148 | 0.201 | 0.0334 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 120 | 0.4542 | 0.2992 | 0.1940 | 0.4542 | 0.2992 | 0.0341 |
| 121 | 0.9524 | 0.8637 | 0.1972 | 0.9524 | 0.8637 | 0.0349 |
| 122 | 0.5842 | 0.5798 | 0.1891 | 0.5842 | 0.5798 | 0.0355 |
| 123 | 0.8632 | 0.8593 | 0.0988 | 0.8632 | 0.8593 | 0.0285 |
| 124 | 0.2883 | 0.2771 | 0.1242 | 0.2883 | 0.2771 | 0.0290 |
| 126 | 0.7822 | 0.7774 | 0.0939 | 0.7822 | 0.7774 | 0.0281 |
| 128 | 0.9034 | 0.8875 | 0.0738 | 0.9034 | 0.8875 | 0.0237 |
| 130 | 0.3629 | 0.3489 | 0.1049 | 0.3629 | 0.3489 | 0.0241 |
| 133 | 0.1144 | 0.107 | 0.1001 | 0.1144 | 0.107 | 0.0242 |
| 134 | 1 | 0.1656 | 0.1111 | 1 | 0.1656 | 0.0294 |
| Ideal Ref | n/a | n/a | n/a | 0.5781 | 0.5777 | 0.0264 |

‘bold’ indicates robustly efficient bank in all categories; n/a – not available

What we observed in tab. 4.1 repeats in tab. 4.2, with BCC scores being higher than CCR scores. The order of strictness ‘>>’ applies here as well, and bank 83 is again overall efficient.

## 5. SUMMARY AND CONCLUSIONS

The following conclusions are in order, as we summarize the above results and findings:

- The presence of ‘ideal reference’ has no effect on the linear efficiency scores, but it does have an effect on the nonlinear score
- Wilcoxon signed rank test confirms that the presence of ‘ideal reference’ lowers the nonlinear TT/LC-AR score
- Thompson-Thrall nonlinear measure is stricter than the linear measures
- Out of all fifty five banks, only bank 83 is overall efficient

## REFERENCES


Akhtar, M.H. & Bokhari, H. (2005) X-Efficiency Analysis of Commercial Banks in Pakistan: A Preliminary Investigation, *European Journal of Scientific Research*, 10, 78-91

Ataullah, A., Cockerill, T. & Hang Le (2004) Financial liberalization and bank efficiency: a comparative analysis of India and Pakistan, *Applied Economics*, 36, 1915–1924

Banker, R.D., Charnes, A. & Cooper, W.W. (1984) Some models for estimating technical and scale efficiencies in data envelopment analysis, *Management Science,* 30, 1078-1092

Bhattacharya, A., Lovell, C.A.K., & Sahay, P. (1997) The impact of liberalization on the productive efficiency of Indian Commercial Bank, *European Journal of Operational Research,* 98, 332-345

Charnes, A., Cooper, W.W., & Rhodes, E. (1978) Measuring the efficiency of decision-making units', *European Journal of Operational Research*, 2, 429–444

Cook, W. & Zhu, J. (2006) Incorporating multi-process performance standards into the DEA framework, *Operations Research*, 54(4), 656-665

Das, A. & Ghosh, S. (2009) Financial deregulation and profit efficiency: A nonparametric analysis of Indian banks, *Journal of Economics and Business*, 61, 509–528

Dharmapala, P.S., Ghosh, J.B. & Saber, H.M. (2007) Market- and Merit-based adjustment of faculty salaries, *Asia-Pacific Journal of Operational Research*, 24(1), 1-19

Fadzlan, S. (2010) Evolution in the efficiency of the Indonesian banking sector: a DEA approach, *International Journal of Applied Management Science*, 2(4), 388 - 414

Fethi, M. & Pasiouras, F. (2010) Assessing bank efficiency and performance with operational research and artificial intelligence techniques: A survey, *European Journal of Operational Research*, 204, 189-198

Galagedara, U.A. & Edirisuriya, P. (2005) Performance of Indian commercial banks, (1995-2002), *South Asia Journal of Management*, 12, 52-74

Golany, B. (1988) An interactive MOLP procedure for the extension of DEA to Effectiveness Analysis, *Journal of Operational Research Society*, 39, 725-734

Horta, I.M., Ana S., Camanho, A.S. & Jorge Moreira Da Costa, J.M. (2010) Performance Assessment of Construction Companies Integrating Key Performance Indicators and Data Envelopment Analysis, *Journal of construction engineering and management*, 136, 581-594

Juan-Carlos, M. & Concepcion, R. (2010) Evaluating the service quality of major air carriers: a DEA approach, *International Journal of Applied Management Science*, 2, No.4, 351 – 371

Rajesh, C. & Gaurav, C. (2005) Bank efficiency in India since the reforms - An assessment, *Money & Finance*, 2(22–23), 31-48

Sathye, M. (2003) Efficiency of banks in a developing economy: The case of India, *European Journal of Operational Research*, 148, 662–666

Thanassoulis, E. and Dyson, R. (1992) Estimating preferred target input-output levels using Data Envelopment Analysis, *European Journal of Operational Research*, 56, 80-97

Thompson, R. G., Lee, E. & Thrall, R.M. (1992) DEA/AR efficiency of U.S. independent oil/gas producers over time, *Computers and Operations Research*, 19(5), 377-391

Thompson, R. G., Dharmapala, P.S. & Thrall, R.M. (1993) Importance for DEA of zeros in data and multipliers, *Journal of Productivity Analysis,* 4, 337-348

Thompson, R. G., Dharmapala, P.S. & Thrall, R.M. (1995) Linked-cone DEA profit ratios and technical efficiency with application to Illinois coal mines, *International Journal of Production Economics*, 39, 99-115

Thompson, R. G., Dharmapala, P.S., Humphrey, D.B., Taylor, W.M. and Thrall, R.M. (1996) Computing DEA/AR efficiency and profit ratio measures with an illustrative bank application, *Annals of Operations Research*, 68, 303-327

Zhu, J. (1996) Data envelopment analysis with preference structure, *Journal of Operational Research Society*, 47, 136-150