INVENTORY RECORD INACURACY DUE TO THEFT IN PRODUCTION-INVENTORY SYSTEMS

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ABSTRACT

Inventory record inaccuracy (IRI) is a major obstacle to realizing operational excellence and lean enterprise principles in the supply chain. This paper proposes an analytical model to study the operational and economic impacts of inventory record inaccuracy stemming from theft type error on a continuous-review lost sale (s, Q) inventory system. The model builds on a Markovian inventory system and finds optimal safety stock and reorder quantity of product by minimizing the expected costs, which are composed of four components: inventory holding cost, production cost, ordering cost and lost sales cost. This production-inventory model is analyzed across two scenarios depending on which technology is deployed to optimize replenishment decisions: (1) Barcode technology; (2) RFID technology. Our numerical analysis illustrates the solution procedure and the effects of the model parameters on the inventory replenishment decisions and total costs.

Keywords: Information systems, production systems, Inventory record inaccuracy, RFID, Inventory theft

INTRODUCTION

Inventory record inaccuracy (IRI) refers to the difference between inventory records in information systems as compared to the physical inventory actually held in stock. IRI can deeply affect the performance of firms and lead to stockouts and profit losses in the supply chain.

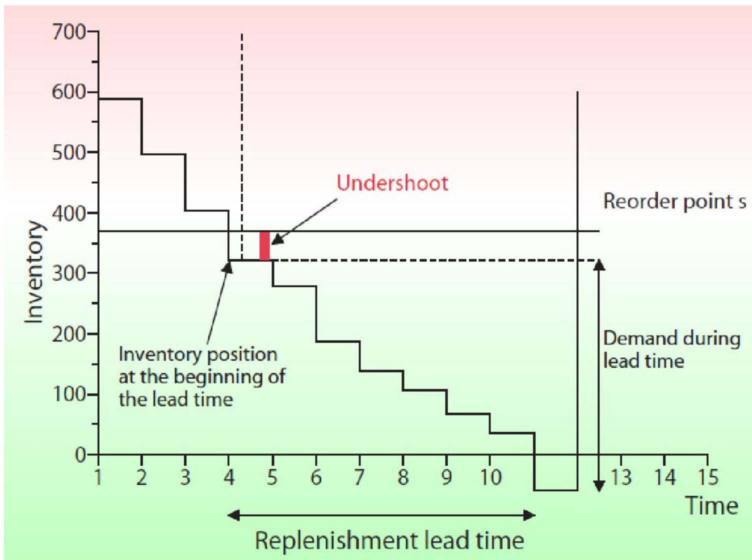
Delaunay et al. (2007) classifies four types of errors which contribute to inventory discrepancies in the supply chain. The first common error source causing inventory record inaccuracy is shrinkage errors in physical inventory due to damage, spoilage, and theft. According to a study conducted by ECR Europe (2003), shrinkage rates were 1.75% for retailers and 0.56% for manufacturers. IBM (2002) reported that the amount of inventory shrinkage rates are around 1.75% of 2001 sales in the US, Europe, and Australasia. The second common error source is temporary inventory inaccessibility errors due to misplacement, which can be defined as a type of temporary shrinkage in the physical inventory and can be corrected after inventory counting and alignment. The other two major sources of record inaccuracy are supply errors due to low product quality, imperfect inventory audits, low yield and unreliable supply process, and transaction errors stemming from scanning errors.

Experts have proposed numerous organizational and technical improvements to eliminate these types of errors. In small and medium scale enterprises (SMEs), improvement in data quality for inventory management might be achieved through inventory counting and alignment, whether manual or semi-automatic operations (e.g. scanning products using barcodes). In large size organizations, inventory record accuracy can be achieved with advanced information technologies such as enterprise resource planning (ERP) and Radio Frequency Identification (RFID) (Thiel, Hovelaque et al. 2010). RFID is a promising technology providing high supply chain visibility. Scanning products using RFID is much quicker and involves far less human intervention than other less expensive alternatives such as bar codes. RFID traceability significantly improves the efficiency of product, materials and work in process (WIP) recalls, the management of returns and warranties, the inventory shipments, or can also be useful in preventing product theft and counterfeit. RFID will enable continuous-review based stock policy to eliminate inventory count problems, allow managers to detect and prevent theft and convert thieves' unsatisfied demand to regular demand.

From a theoretical point of view, many studies analyze inventory inaccuracy problems with various inventory policies and management of information technologies (Kang and Gershwin 2005; Delaunay, Sahin et al. 2007; Rekik, Jemai et al. 2007; Sahin, Buzacott et al. 2008; Rekik, Sahin et al. 2009; Sahin and Dallery 2009; Thiel, Hovelaque et al. 2010; Agrawal and Sharda 2012). Within the (s, Q) inventory policy setting, many researchers emphasize uncertainties in demand, lead-times, supply and/or incident events such as machine break down and shrinkage (Petrovic and Petrovic 2001; Hnaien, Delorme et al. 2010; Schmitt, Snyder et al. 2010; Widyadana and Wee 2011; Yeo and Yuan 2011). To our knowledge, none of them address the optimal safety stock policy sensitive to theft errors in a continuous-review lost sale (s, Q) inventory system when combined with the economic and operational effects of RFID technology in a manufacturing context.

Under a continuous-review order quantity policy, the inventory position is monitored continuously. The point in time at which replenishment orders are triggered depends on the size of the reorder point s, whereas the order quantity Q is constant over time (Tempelmeier, 2011). In the past, practical use of the (s, Q) policy with continuous review was unrealistic, since in reality the inventory position was not monitored continuously and undershoot was neglected. The undershoot refers to the difference between s and the inventory position at the moment immediately before a new replenishment order is released. Ignoring this undershoot issue usually results in significant underestimation of the required safety stock in order to prevent stock out or over-estimation of the service level.

Figure 1: (s, Q) policy in discrete time- The undershot happens (Tempelmeier, 2011)



Today, information systems such as ERP systems and RFID technology enable many companies to monitor their inventory levels continuously. In other words, continuous review is practical and realistic in industry if RFID technology and information systems are deployed and well integrated. When RFID is implemented, the undershoot problem will be eliminated and theft is detected. In this paper, we assume that using RFID, thieves cannot continue to steal easily and over the long term, their demand will largely be converted to regular demand.

The research presented in this paper emphasizes that production-inventory control systems in a manufacturing context should take into consideration the disappearing inventory (e.g. shrinkage due to theft) and try to control for it. We focus on the effect of the inventory inaccuracy subject to theft type errors in production inventory problem with (s, Q) replenishment policy from an analytical modeling view point. In particular, we contribute by adjusting the Markovian inventory model to find safety stock and reorder quantity in a production-inventory system with shrinkage arising from internal or external theft errors in manufacturing sectors. With the barcode technology, because inventory accuracy is achieved through manual counting and the replenishment decisions and inventory alignment are made in discrete time intervals usually at the end of a day, the undershoot phenomenon happens and therefore a greater leadtime should be taken. When RFID is implemented over the long term, we assume thieves cannot simply carry on stealing. Presumably, only $\alpha\%$ of the theft demand is for required consumption and could be converted to regular demand. We would expect $(1-\alpha)\%$ of the theft demand to simply disappear.

LITERATURE REVIEW

A recent survey on various studies addressing analytic modeling, simulations and case studies on inventory inaccuracy problem, RFID potential benefits, the bullwhip effects experiments, ROI analyses as well as replenishment policies in supply chain management is presented by Sarac, Absi et al. (2010). Inventory theft has been defined as a combination of employee theft, shoplifting, internal and external theft, vendor fraud and administrative error (Rekik 2006; Rekik, Jemai et al. (2007). Kang and Gershwin (2005) develop an analytical and simulation model for inventory inaccuracy due to the stock loss under a single item inventory system and regular replenishment order (Q, R) policy. They compute explicit values of stockout and average inventory for different ordering parameters, demand variables, and lead time. They take a combination of demand for purchases having a normal distribution and a demand for loss described by a Poisson distribution. They show that even a small amount of recurring stock loss not reflected in the inventory information systems can disrupt the replenishment process such that revenue loss due to stockout could exceed the stock losses themselves. They also show that through RFID technology, one can achieve the best stockout-inventory compromise (the lowest inventory for any given stockout), and that the benefit of inventory accuracy provided by RFID becomes greater as the desired level of stockout becomes smaller.

Sahin, Buzacott et al. (2008) and Sahin and Dallery (2009) investigate a newsvendor type model which analytically derives the optimal policy for an inventory system in the presence of record errors.

Agrawal and Sharda (2012) use simulation to investigate the effect of such loss in a (Q, R) inventory policy defined by the stock loss parameter and the frequent alignment of physical and information system inventories on the stockout and average inventory. They indicate that reduction in stockout loss can be achieved when the alignment is done monthly vs. annually. Rekik, Sahin et al. (2009) consider a finite-horizon, single-stage, single-product periodic review of store inventory with (Q, R) policy within a retail store in which inventory records are inaccurate because of theft-type errors. They also propose a comparison between different approaches based on which steps in the inventory system can be managed in the presence of theft errors. They indicate that in the RFID enabled approach, theft is eliminated and the demand for theft is totally converted into a demand for purchase. Rekik (2006) presents another formulation based on the optimization of the holding and the shortage costs. In his formulations, he assume that demand for theft is a fraction of the total demand and also dependent on the purchase demand. To make the computational results cleaner, he does not define a second random variable to describe errors. Chalasani and Sounderpandian (2004) develop an analytical model of a retailer who uses RFID to automate his reordering and shelf replenishment process.

Sounderpandian, Boppana et al. (2007) analyze the costs and benefits of implementations of RFID technologies in a supply chain that contains a manufacturer, a distributor, a retailer and consumers. They develop an analytic approach in order to estimate the load rate of RFID employment by the retailers and the cost benefits obtained through RFID applications for shelf replenishment.

Tzeng, Chen et al. (2008) propose a framework to evaluate the business value of RFID based systems to enable companies to determine its economic viability before embarking upon the implementation. Their analysis shows that delivering business value of RFID should be done through refining business processes and expanding the business model. RFID must also be included within the overall business framework with respect to issues such as workflow, supply-chain relationships and leveraging of business capabilities.

Rekik, Sahin et al. (2008) develop another analytical model to determine which tag prices make the implementation of RFID economically feasible and when RFID is cost effective when used to improve inventory accuracies in a retail store. They conclude that if errors are estimated and taken into account when optimizing the system, implementing RFID is not beneficial. However, they mention that embedding RFID tags onto products does help in controlling errors due to misplacement. de Kok, van Donselaar et al. (2008) carry out a break-even analysis to determine when it is cost effective to implement RFID tags to control pilferage. The authors conclude that this depends on the cost of the item being pilfered, extent of pilferage, and decrease in pilferage after implementing an RFID-based control system.

Çakıcı, Groenevelt et al. (2011) investigate the incremental benefits of RFID technology over barcodes for managing pharmaceutical inventories. They demonstrate that continuous review is superior to periodic review whenever accurate real-time information is available at no additional cost. They provide a cost-benefit analysis for the implementation of RFID for a radiology practice. Thiel, Hovelaque et al. (2010) investigate the relationship between the quality of service, safety stock and inventory inaccuracy under demand variations within a (Q, R) continuous-review lost-sales inventory model. Their results show that the service-level quality is a non-monotone function of the inaccuracy rate, i.e., the service-level quality increases up to an inventory record inaccuracy (IRI) level and subsequently decreases.

Although several studies look at the impact of theft on information inaccuracy in retail inventory systems, there are few studies that look at the problem of theft on manufacturing inventory management. We assume that in the RFID enabled approach, only a fraction of the theft demand is converted to regular demand. The other fraction simply disappears. Since if they cannot steal it, they will steal something else or even somewhere else, not purchase it.

PROBLEM DESCRIPTION

We consider a production inventory system in a manufacturing context where a single product is produced at a single facility. Finished products are stored in a manufacturing warehouse and retail demands are fulfilled from there. The manufacturer should take into consideration inventory record inaccuracy due to theft to decide on the appropriate production order quantity. The main objective is to determine the optimal production order and safety stock level so that the total cost of the supply chain is minimized. Customers' demand is also introduced as the existence of two separate streams: one for regular demand and one for thieves' demand. To model theft error, we suppose the unit price of that stream which indicates demand for theft equals zero. We assume only unsatisfied demand for purchase during a stockout period is lost. The demand for purchase and theft arrive as a Poisson process with different independent parameters. The lead time for the production follows an exponential distribution and is known. Under the above assumption, we solve the distributor's inventory management by using a Markov process framework (Isotupa 2006), (Bensoussan et al., 2007) in order to determine safety stock and reorder quantity of product by minimizing total cost (inventory cost, production cost, lost sales loss). Markov chain approach is the best alternative for modeling uncertain conditions compared to the classical techniques (the nature of the demand is uncertain). We analyze this production-inventory model depending on whether barcode technology is used or RFID technology is implemented.

PROBLEM FORMULATION

We assume that items are produced one single unit at a time with exponentially distributed production times with mean 1/μ. In other words, the leadtime for the production order is exponentially distributed with rate $\mu(>0)$. The manufacturer can produce ahead of demand in a make-to-stock fashion. However, a holding cost *h* is then incurred per unit in inventory per unit time. Regular customers place orders continuously over time according to a Poisson process with rate $\lambda_r > 0$. The demand for theft comes as another Poisson process with rate $\lambda_t > 0$. Since theft is not subject to control, we consider that both types of demands have the same priority and are satisfied as long as there is physical inventory available in stock. That means, upon arrival, an order is either fulfilled from inventory, if available, or rejected (lost).

The notations we use throughout the paper are summarized in Table 1 and 2.

Table 1: List of parameters used in the model

Parameter	Description
λ_r	Regular customer's demand arrival rate
λ_t	Thieves' demand arrival rate
μ	Production rate
h	Inventory holding cost
K	Setup cost of product
c	Production cost per unit of product
f	Fixed cost per order placed for barcode or RFID system ($f_{Barcode}$ or f_{RFID})
g_r	Cost per unit shortage of product for regular demand
β	Price unit of product
T	RFID tag price per unit of product
α	Percentage thieves' unsatisfied demand due to required consumption

Table 2: List of variables and functions used in the model

Variables	Description
s	Safety stock (reorder point)
Q	Order quantity
R	Reorder rate (order cycle)
I	Inventory level
Γ	Shortage level

Once the inventory level drops off to the safety stock level s , an order for Q units is placed. Thus the maximum on hand inventory is $Q+s(=S)$. The condition $Q>s$ guarantees that there are no perpetual shortages. If $Q\leq s$ and inventory level reaches zero, then the system will be in shortage until the discrepancy is noted and repaired. As orders are usually placed when stock levels get low, demands for purchase and theft that arrive still are served. As soon as inventory level reaches zero, demand due to purchase type are assumed to be lost.

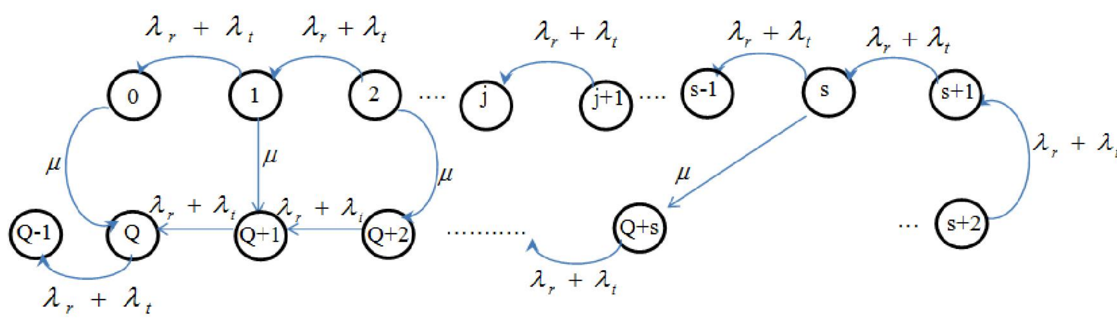
Let *I(t)* denote the on-hand inventory level at time *t*, $\{I(t);t\geq 0\}$, with state space $E=\{0,1,2,...,s,..,Q,...,Q+s\}$ is a Markov process. Let

$$P(i,j,t)=\Pr[I(t)=j\,|\,I(0)=i] \quad i,j\in E \tag{1}$$

Equation (1) shows the probability of having *j* items at time *t* where the inventory level originally was *i*. In the steady state, let $P(j)=\lim_{t\to\infty}P(i,j,t)$.

To build the balance equations, transition diagrams are obtained subject to the Markov process properties of inventory (Figure 2). We equate the sum of the arrows (multiplied by the probability of the states where they originate) coming into a state with the sum of the arrows (multiplied by the probability of the state) going out of that state.

Figure 2: Transition diagram



$$(\lambda_r+\lambda_t)P(Q+s)=\mu P(s) \qquad (2)$$
$$(\lambda_r+\lambda_t)P(j)=(\lambda_r+\lambda_t)P(j+1)+\mu P(j-Q) \qquad Q\le j\le Q+s-1 \qquad (3)$$
$$(\lambda_r+\lambda_t)P(j)=(\lambda_r+\lambda_t)P(j+1) \qquad s+1\le j\le Q-1 \qquad (4)$$
$$(\lambda_r+\lambda_t+\mu)P(s)=(\lambda_r+\lambda_t)P(s+1) \qquad (5)$$
$$(\lambda_r+\lambda_t+\mu)P(j)=(\lambda_r+\lambda_t)P(j+1) \qquad 1\le j\le s-1 \qquad (6)$$
$$\mu P(0)=(\lambda_r+\lambda_t)P(1) \qquad (7)$$

On solving the above balance equations (2, 3, 4, 5, 6 and 7), the following results are obtained:

$$P(j)=(1+\frac{\mu}{\lambda_r+\lambda_t})^{j-1}\frac{\mu}{\lambda_r+\lambda_t}P(0) \qquad 1\le j\le s \qquad (8)$$

$$P(j)=(1+\frac{\mu}{\lambda_r+\lambda_t})^{s}\frac{\mu}{\lambda_r+\lambda_t}P(0) \qquad s+1\le j\le Q \qquad (9)$$

$$P(j)=\left[\left(1+\frac{\mu}{\lambda_r+\lambda_t}\right)^{s}-\left(1+\frac{\mu}{\lambda_r+\lambda_t}\right)^{j-Q-1}\right]\frac{\mu}{\lambda_r+\lambda_t}P(0) \qquad Q+1\le j\le Q+s \qquad (10)$$

As $\sum_{j=0}^{S=Q+s}P(j)=1$ from the above equations, we have

$$P(0)=\frac{\lambda_r+\lambda_t}{\lambda_r+(Q\mu)(1+\frac{\mu}{\lambda_r+\lambda_t})^{s}} \qquad (11)$$

Then the average inventory level $\overline{I}$ is given by

$$\overline{I}=\left(1+\frac{\mu}{\lambda_r+\lambda_t}\right)^{s}\left[\frac{\mu Q(Q+2s+1)}{2(\lambda_r+\lambda_t)}-Q\right]P(0)+QP(0) \qquad (12)$$

The mean reorder rate *R*, and the mean shortage rate for regular customers Γ_r are given by

$$R=\left(\lambda_r+\lambda_t\right)P\left(s+1\right)=\mu\left(1+\frac{\mu}{\lambda_r+\lambda_t}\right)^{s}P(0) \qquad (13)$$
$$\Gamma_r=\lambda_r P(0) \qquad (14)$$

THE STRUCTURE OF THE OPTIMAL POLICY

In this section, we describe the structure of the optimal policy. The objective cost function comprises four components mentioned earlier, namely: 1) inventory holding cost, 2) production cost, 3) ordering cost, and 4) lost sales cost. The mathematical model's objective is to minimize the total cost of the system. Therefore, the total expected cost structure for product is given by:

$$C(s,Q) = h\overline{I} + (K + f + cQ)R + g_r\Gamma_r \tag{15}$$

By definition of g_r, the cost of product c should be less than g_r and by definition of f, the fixed cost per order placed for barcode and RFID is given by $f_{Barcode}$ and f_{RFID} respectively. According to Çakıcı, Groenevelt et al. (2011), RFID removes the need for labor-intensive inventory counts and filling in paperwork due to automatic counting and results in a lower fixed ordering cost (f). Fixed ordering cost includes also cost of the technology installation and the infrastructure (whether RFID or barcode). We assume that technology deployment cost (whether barcode or RFID) occurs depending on order frequency (reorder rate) and $f_{Barcode} - f_{RFID}$ is interpreted as a manual counting cost avoided by the RFID deployment.

By substituting $\overline{I}, R, \Gamma_r$ from equations 12, 13 and 14 in $C(s,Q)$ we have:

$$\begin{aligned} Min \quad C(s,Q) = {} & hQP(0) + h\left(1+\frac{\mu}{\lambda_r+\lambda_t}\right)^s \left[\frac{\mu Q(Q+2s+1)}{2(\lambda_r+\lambda_t)} - Q\right]P(0) \\ & + \left(K+f+CQ\right)\mu\left(1+\frac{\mu}{\lambda_r+\lambda_t}\right)^s P(0) + g_r\lambda_r P(0) \end{aligned} \tag{16}$$

Production quantity and safety stock are the outputs of the above optimization problem that we discuss in appendix 1. Finally, the total expected profit of the production-inventory system will be calculated from the difference between the revenue and the cost:

Total profit $=$ Total revenue $-$ Total cost

$$\begin{aligned} &= (\beta - T)\left(RQ - \Gamma_r\right) - \left(h\overline{I} + (K+f+cQ)R + g_r\Gamma_r\right) \\ &= (\beta - T)\left(RQ - \Gamma_r\right) - \left[\begin{array}{l} hQP(0) + h\left(1+\frac{\mu}{\lambda_r+\lambda_t}\right)^s \left[\frac{\mu Q(Q+2s+1)}{2(\lambda_r+\lambda_t)} - Q\right]P(0) \\ \quad + \left(K+f+cQ\right)\mu\left(1+\frac{\mu}{\lambda_r+\lambda_t}\right)^s P(0) + g_r\lambda_r P(0) \end{array}\right] \end{aligned} \tag{17}$$

which β is the unit price and T is the RFID tag price per unit of product.

Our model is an adaptation of a Markovian framework developed by Isotupa (2006), (Benjaafar, ElHafsi et al. 2010), and (Stratos 2011). They extend a Markovian single stage system producing a single item which satisfies demand from two different customer classes (ordinary and priority) with exponentially distributed leadtime. In our paper, the regular customer's unsatisfied demand is considered a lost sales opportunity during a stockout, while the unsatisfied demand for theft is not considered a lost sale. Therefore, their results for the special case of $\lambda_1 = \lambda_{r} + \lambda_t$ and $\lambda_2 = 0$ support our model. We adjust the solution methodology developed (Isotupa 2006) in order to find the optimal values of Q and s. (See appendix 1). The model in this paper was implemented using *Matlab* software.

NUMERICAL ANALYSIS

As mentioned earlier, our analysis throughout the paper is based on the comparison of two different scenarios. In the first scenario (i.e. barcode), the manufacturer is either not aware of theft that takes place in the warehouse or if he knows, he chooses to ignore it. To align physical and information inventory, manual inventory counting is done frequently. If the inventory position drops to the order point *s*, an order quantity *Q* is released. However, when inventory accuracy is achieved through manual counting, theft demand is not detected on time. In other words, management simply aligns physical inventory and information system inventory, and thieves still continue their activities. In addition, since the replenishment decisions and inventory alignment are made in discrete time intervals usually at the end of a day, the undershoot phenomenon happens and therefore a greater lead time must be used. Within the second setting, RFID scenario, the manufacturer is aware of theft and the benefits of having RFID technology to get real time visibility and traceability to prevent it. When RFID is implemented, the undershoot problem will be eliminated and also theft is largely detected. Over the long term, we assume when using RFID, thieves cannot simply continue to steal and $\alpha\%$ of their theft demand is then available for consumption and could be converted to regular demand. We would expect $(1-\alpha)\%$ of the theft demand to simply disappear. These scenarios are summarized in Table 3.

Table 3: Scenarios throughout the analysis $(\mu_{RFID} < \mu_{Barcode})$

Scenario	Counting	Theft demand	Parameters setting
Barcode	Manual	Yes	$\lambda_{r_{Barcode}}, \lambda_{t_{Barcode}} \neq 0, \mu_{Barcode}$
RFID	Automatic	No	$\lambda_{t_{RFID}} = 0, \lambda_{r_{RFID}} = \lambda_{r_{Barcode}} + \alpha\lambda_{t_{Barcode}}, \mu_{RFID}$

In order to illustrate the above solution procedure across these scenarios, let us consider hypothetical inventory systems with the following exemplary data (Table 4). As mentioned earlier, the shrinkage rate is 0.56% for manufacturers based on the studies conducted by ECR Europe (2003) and IBM (2002). Therefore, we take into account their statistics to design our numerical examples. Depending on production rate and factory size, theft issue will be definitely significant. Also, we consider that $\alpha = 70\%$ of theft demand is for required consumption and could be converted into regular demand when RFID is in place. Moreover, according to an empirical study done by the center for coordination science at MIT (Subirana et al. 2003), the cost saving and value opportunity enabled by RFID is over 80% compared to traditional systems.

Other case studies, (Lee and Özer 2007) and (Çakıcı, Groenevelt et al. 2011), report a 75% overall cost saving resulted from using RFID technology and redesigning its business processes. According to Çakıcı, Groenevelt et al. (2011), RFID removes the need for labor-intensive inventory counts and filling in paperwork due to automatic counting and results in a lower fixed ordering cost (f). Fixed ordering cost includes also cost of the technology installation and the infrastructure (whether RFID or barcode). It means technology deployment cost occurs depending on order frequency (reorder rate). Therefore, we assume that automatic counting cost per unit (f_{RFID}) is less than manual counting cost per unit $(f_{Barcode})$. To be accurate about the relationship between costs of RFID and barcode systems, we consider 80% cost saving as a reference. Note that if the resulting value of $\lambda_{theft_{Barcode}}$ and $\lambda_{regular_{RFID}}$ are non-integer, we round them up.

Table 4: Input parameters for numerical example

$$\{\lambda_{theft_{Barcode}} \gg (\%0.56)\lambda_{regular_{Barcode}}\} And \{\lambda_{regular_{RFID}} = \lambda_{regular_{Barcode}} + \%70\lambda_{theft_{Barcode}}\} And \{f_{RFID} = (\%80) f_{Barcode}\}$$

	$\lambda_{r_{Barcode}}$	$\lambda_{t_{Barcode}}$	$\lambda_{r_{RFID}}$	μ	K	$f_{Barcode}$	f_{RFID}	c	g_1	h
1	100	1	101	50	3	10	2	40	100	0.3
2	200	2	202	50	3	10	2	40	100	0.3
3	500	3	503	50	3	10	2	40	100	0.3
4	800	5	804	50	3	10	2	40	100	0.3
5	1000	6	1005	50	3	10	2	40	100	0.3
6	2000	12	2009	50	3	10	2	40	100	0.3
7	5000	28	5020	50	3	10	2	40	100	0.3

Table 5 shows outputs of the production-inventory model.

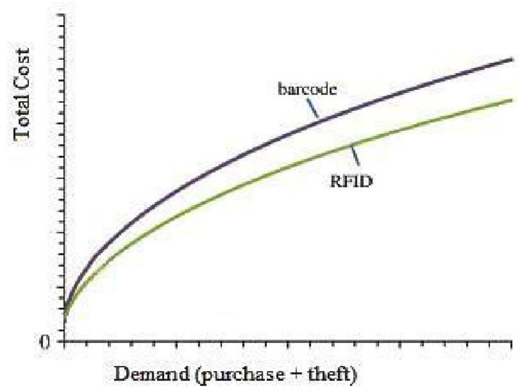
Table 5 Outputs of the Markovian production-inventory model

		s	Q	R	$\bar{I}$	Γ_r	c_{Total}
1	RFID	13.4248	61	1.6555	42.4076	0.0151	4062
	Barcode	12.2721	96	1.0519	58.7578	0.0153	4072
2	RFID	24.6698	87	2.3215	66.7118	0.0253	8113
	Barcode	24.6933	137	1.4743	89.6194	0.0255	8199
3	RFID	68.3936	140	3.5925	128.8348	0.0553	20180
	Barcode	63.5479	220	2.2861	163.9942	0.0558	20202

4	RFID	111.6128	178	4.4470	187.6113	0.0851	32279
	Barcode	104.2504	281	2.8645	229.1561	0.0862	32311
5	RFID	140.9502	201	4.9068	221.9281	0.1054	40312
	Barcode	131.8909	317	3.1732	270.7759	0.1063	40369
6	RFID	291.8877	300	6.6396	401.5372	0.1524	80514
	Barcode	274.6211	460	4.3735	464.8809	0.2079	80689
7	RFRD	728.9307	723	6.8682	921.3245	0.4283	201370
	Barcode	520.6647	1126	4.4585	1030.1245	0.6099	201597

Numerical examples illustrate that for the same cost, RFID scenario yields lower average inventory in hand, lower order quantities, higher reorder point due to actual demand, adjusted/corrected safety stock, less shortage and finally, lower total cost for different ordering parameters and demand variables. Figure 3 shows the difference of total supply chain costs when RFID or barcode system is deployed.

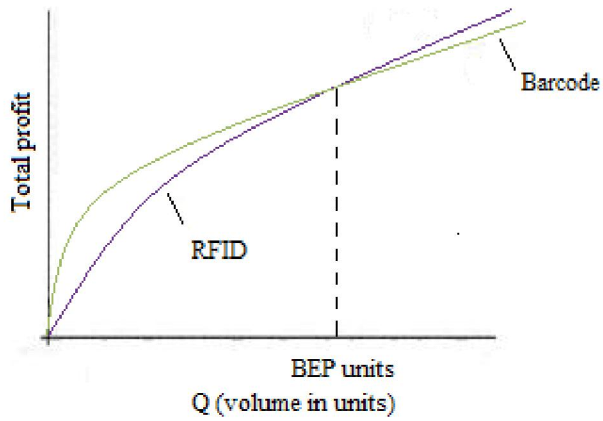
Figure 3: Comparison of RFID and Barcode's total cost



Since the benefits of RFID technology for the product manufacturer is less clear than for retailers and the product manufacturer must pay for the RFID tags, finding the break-even point on the production quantity can help the manufacture to answer who should bear the RFID tags' price and determine when the RFID system is cost effective. However, the manufacturer could pass the cost of the RFID tag onto the customer by increasing the price of the product; this might not work in a competitive market. Figure 4 shows our analysis on total profit to determine break-even point for Q using equation (17) and example data of the product price $\beta = \$100$ and the RFID tag price $T = \$0.15$.

As a result, continuous review is practical, beneficial and realistic in industry when it integrates with RFID technology and ERP systems. For managers, these results highlight the importance of deploying RFID, differentiating between customer demand classes, and detecting thefts. The corresponding cost savings due to theft errors and stockout can be significant.

Figure 4: Break-even point on production quantity



CONCUSION

This paper was motivated by the potential of RFID technology to address inventory inaccuracy caused by theft. We studied the impact of information inaccuracy stemming from theft type error on a continuous-review production inventory-system with a single product and two separate demand streams where lost sales are allowed. We formulated the problem as a Markov decision process and used that process to characterize the structure of the optimal policy.

The results showed that continuous review is only realistic in industry after RFID technology is deployed. When RFID is in place, theft becomes more difficult and consequently, thieves' unsatisfied demand converts to regular demand. The results of the numerical examples pointed out that for the same cost, RFID alternative yields lower reorder quantities, higher reorder point, lower average inventory, less shortage and adjusted/corrected reorder point.

One extension of this study will include the creation of a stochastic inventory model for stock-out situations in which a fraction of the regular demand can be backordered. Another study could examine the effects of inaccuracy records and RFID on inventory-transportation problems. Finally, since theft often takes place during resale or returns, it will be also useful to consider inventory systems used with return and resale policies.

Our numerical analysis is limited to the hypothetical data we used in the model. Conducting manufacturing case studies in order to collect real data to test our model can show a better image of the model implications and benefits.

APPENDIX

SOLUTION METODOLOGY

In this section, we mention the following three lemmas which are based on pseudo-convexity properties in order to reduce the computational time required finding the optimal values of Q and s.

Lemma 1: For a fixed Q, the expected cost rate is pseudo-convex in s.

Lemma 2: Whenever $Q > \frac{(g_1 - c)(\lambda_r + \lambda_t)^2}{h\mu}$, $C(s,Q)$ is an increasing function of s and hence $s^* = 0$.

Lemma 3: For a fixed s, the long-run expected cost rate is pseudo-convex in Q.

A proof of following Lemmas can be found in Isotupa (2006).

Under following heuristic algorithm we find optimum s and Q :

Step 1: Find $Q_{\min} = \frac{(g_r - c)(\lambda_r + \lambda_t)^2}{h\mu}$

Step 2: Find s, Q so that

$C(s_0, Q_0) \leq C(s,Q) \qquad 0 < s < Q,\ s < Q < Q_{\min}$

To find optimum s for each $Q < Q_{\min}$ solve the equation below:

$C(s+1, Q) - C(s,Q) = 0$

Step 3- find Q_1 so that

$C(0, Q_1) \leq -C(s,Q),\ \ Q_{\min} \leq Q, Q_1 \leq \infty.$

To find optimum $Q_{\min} \leq Q$ that minimizes total costs solve

$C(0, Q+1) - C(0,Q) = 0$

Step 4- If $C(0,Q_1) < C(s_0,Q_0)$ then $Q^* = Q_1, s^* = 0$ and end.

Step 5- If $C(0,Q_1) \geq C(s_0,Q_0)$ then $s^* = s_0, Q^* = Q_0$.

Step 6- Stop.

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