INVENTORY SYSTEMS WITH INTERRUPTIBLE LEAD TIME – A SIMULATION STUDY

Esmail Mohebbi
College of Business, University of West Florida, Pensacola, FL 32514
e-mail: emohebbi@uwf.edu, Phone: (850) 474-2899

ABSTRACT

We consider a single-item inventory system where the lead time for a replenishment order is subject to interruptions due to randomly changing operating conditions. We consider various scenarios with respect to what happens to the disrupted outstanding order at the conclusion of the disruption period and propose a simulation model to study the system behavior.

Keywords: inventory, unreliable supplier, interruptible lead time, supply disruptions.

INTRODUCTION

One of the major challenges faced by inventory managers in a highly competitive market is to make replenishment decisions in the presence of uncertain operating conditions which may negatively affect the availability of their suppliers. Such operating conditions, ranging from equipment breakdowns, material shortages, quality issues, capacity fluctuations and labor disputes to weather conditions, natural disasters and geopolitical crises may alter the status of a supplier from 'available' to partially or completely 'unavailable' and disrupt the flow of material throughout the supply chain network. Examples of such supply disruptive events in automotive, electronics and microchip manufacturing industries, to name a few, have been cited in the literature (e.g., see [14, 16, 19, 21], among others). A relatively recent study indicated that based on a sample of 885 disruptions reported by various companies, firms that experience disruptions face on average 6.92% lower sales growth, 10.66% growth in cost, and 13.88% growth in inventories ([5, 6, 18, 19]). Furthermore, it has been suggested that a growing trend toward increasing efficiency or reducing cost may lead to an unintended negative consequence of making the supply chain brittle and susceptible to inherent risks of disruptions in the supply process ([18]). Hence, as firms are increasingly seeking effective means of managing and safeguarding their supply chains against the risk of supply disruptions - particularly in relationship with offshore suppliers – researchers are working on prescriptive and descriptive models to establish an analytical framework for better understanding of the supply disruption phenomena as well as development of effective means of disruption management . As such, while early expositions of the supply-disruption problem in the literature date back to early 1990's, in the past decade exploring this problem has emerged as a vibrant area among the research community. (See [7], among others, for a review of literature in this area.)

The vast majority of the articles published on the subject of inventory systems with supply disruptions in the literature focus on the case of instantaneous replenishment (i.e., zero lead time). As noted in [8], a typical exposition of the problem in this line of research involves an unreliable supplier whose states alternates randomly between an available (*on*) and an unavailable (*off*) state. When the supplier is in the *on* state, the system functions as an ordinary inventory system with a fully reliable (always available) supplier. However, when the supplier switches from the *on* to the *off* state, it cannot accept any orders for the duration of the *off* period. Most papers in this category focus on incorporating supply disruptions into the classic *EOQ*-type inventory setting (i.e., deterministic demand and negligible lead time) by using various forms of probability distributions to characterize the *on* and *off* periods (e.g., [1,2, 4, 14, 15, 16, 17, 20, 21]).

It should be clear that establishing a more realistic framework of analysis requires the inclusion of replenishment lead time into the problem formulation; however, due to the added complexity caused by the introduction of lead time, the field of inventory systems with non-zero lead time while facing random supply disruptions remains largely unexplored. To that end, very few analytical inventory models have appeared in the literature that address the issue of supply disruptions in the presence of a replenishment lead time ([3, 8, 9, 10, 11, 12,13]). Accordingly, it should not be surprising that papers in this category often impose a set of restrictive assumptions to ensure analytical tractability of mathematical models. For example, as discussed in [8], all these models assume that the maximum number of outstanding orders at any point in time is one. Furthermore, in all but [8,11, 12] it is assumed that a switch from the *on* to the *off* state prohibits the supplier from accepting new orders for the duration of the *off* period, but does not have any impact on the processing of an outstanding order (if any) which may already be in progress. More specifically, any order triggered while the supplier is *off* is placed on hold until such time that the supplier switches back to the *on* state. However, throughout the *off* period the supplier will continue to process and deliver any order that might have been accepted, but not delivered, prior to the start of its *off* period. Once the supplier switches back to the *on* state, any order placed on hold is immediately accepted and is subjected to a replenishment lead-time – which is formally defined as the time between the epochs of accepting an order by the supplier and receiving the ordered quantity in full in the warehouse. It should be clear that under this assumption there can be no consideration for interruptible lead times as the lead time process is not affected by the supplier's availability. To the best of our knowledge, the only analytical effort to model the impact of supplier's availability on the replenishment lead time can be found in [11] ([8,12]) where the processing of an accepted order is stopped upon the supplier's switch from the *on* to the *off* state for the duration of the *off* period and is 'resumed' ('restarted') as soon as the suppliers switches back to the *on* state.

The utility of the simplifying assumptions mentioned above should be viewed in light of the fact that analytical treatment of randomly occurring supply disruptions in inventory systems with random demand and/or random lead time is an exceedingly difficult task due to well-known issues such as "curse of dimensionality" and "orders-crossing-over" in the literature. As such, any attempt for extending the framework of analysis beyond the scope of these assumptions in order to characterize more realistic set of operating conditions calls for more flexible means of modeling such as discrete event simulation.

This paper proposes a simulation-based study plan for modeling and analysis of the random supply disruptions with interruptible lead times under fairly general operating conditions.

INVENTORY SYSTEM

Consider a single-item continuous-review inventory system with stochastic demand which replenishes its stock by ordering from a supplier whose availability alternates randomly between two states – namely, *on* (available) and *off* (unavailable) - according to an alternating renewal process; that is, the *on* and *off* periods each form sequences of *i.i.d.* random variables. The system employs the (s, S) ordering policy for replenishing it stock. According to this policy, the inventory position (= net inventory + outstanding order quantity) is reviewed continuously. As soon as the inventory position drops to or below the reorder level s, an order is triggered whose size is large enough to raise the inventory position up to the target level S ($S > s$). If the supplier is *on* at the time that an order is triggered, the order is readily accepted by the supplier; otherwise, the order is placed on hold until such time that the supplier switches back form *off* to *on* upon which the order-on-hold is released and gets accepted by the supplier immediately. It is therefore plausible for the system to release multiple orders of various sizes simultaneously - rather than only one large order- at the time of supplier's switching back to the *on* state to cover the depletion of stock during the *off* period. The replenishment lead time is comprised of an order processing time plus a transportation time. The order processing time starts from the epoch that an order is accepted by the supplier, and it consists of the time period(s) that the supplier is actively working on fulfilling the order quantity. After an order is processed in its full quantity it immediately enters the phase of transportation l time which in turn is concluded by receiving the full order quantity in stock as one replenishment. It should be clear that the processing time is subject to interruptions caused by the supplier's alternations between *on* and *off* sates whereas the transportation time is not affected by the supplier's state. Processing and transportations lead times are independent of each other, and neither depends on the order quantity nor does it depend on the number of outstanding accepted orders. There is no limit on the number of outstanding orders once they are accepted. Recall that only an order triggered while the supplier is *off* is placed on hold for as long the supplier stays in the *off* state. However, once an order is accepted it is treated as an outstanding order until such time that it is delivered to stock. Clearly, the lead time of an outstanding order may be subject to multiple interruptions in that the order processing stops at the supplier's switching epoch from the *on* to the *off* state, and remains halted throughout the *off* periods.

SIMULATION MODELING PLAN

Objective
The objective of the proposed simulation modeling effort is to expand the existing framework for analytical treatment of inventory systems with supply disruptions to include more general operational settings, and thereby gain insights into effective means of disruption management in supply chain networks.

Variables & Parameters
The simulation model will account for the following variables and input parameter:

- Inter-arrival times of demand (or demand rate)
- Demand size
- Length of supplier's *on* period
- Length of supplier's *off* period

- Order processing time
- Transportation time
- Reorder level, *s*
- Target inventory level, *S*
- Fixed ordering cost
- Selling price
- Variable ordering cost
- Inventory holding cost
- Inventory shortage cost

Performance measures

Simulation runs will be used to compute the following measures in the long run:

- Expected inventory level per unit time
- Expected shortage (backorders/lost sales) per unit time
- Expected number of orders triggered per unit time
- Expected number of orders released per unit time
- Expected number of orders delivered per unit time
- Expected length of actual order processing time
- Expected length of actual replenishment lead time
- Fill rate
- Expected profit per unit time

Experimental Scenarios

The simulation model will utilize various analytical and empirical probability distributions to describe the sequences of *i.i.d.* random variables which represent demand inter-arrival times, demand sizes, supplier's *on* and *off* periods, and order processing as well as order transportation times under the following experimental scenarios:

- The processing of a disrupted order is fully resumed upon supplier's return to the *on* state; that is, any time spent on processing the order prior to the interruption remains intact, and as such the supplier will only need to complete the remainder of the required processing time.
- The processing of a disrupted order is partially resumed upon supplier's return to the *on* state; that is, a certain portion of the time spent on processing the order prior to the interruption is lost and added to the remainder of the required processing time to be completed by the suppler.
- The processing of a disrupted order is restarted upon supplier's return to the *on* state; that is, the the time spent on processing the order prior to the interruption is completely lost, and as such the supplier must restart processing the order from the outset.

In addition to performing comparative analyses of the system performance measures for the cased listed above, the model will also be used to experiment with a range of control policy and other system parameter values to investigate the influence of releasing multiple orders while allowing for orders to cross on the overall performance of the inventory system.

Software
The model will be developed using an off-the-shelf simulation software package.

Expected Outcomes
If successful, the proposed simulation modeling plan is expected to yield the following outcomes:

- It will enhance our understanding of the intricacies and dynamics of a complex problem known as randomly interruptible lead times in inventory control.
- It will produce an analytical tool for exploring effective means of disruption management in uncertain environments.
- It will provide managerial insights.

REFERENCES

[1] Balcioglu, B., & Gurler, U. On the Use of Phase-Type Distributions for Inventory Management with Supply Disruptions. Working paper, Mechanical and Industrial Engineering, University of Toronto, Canada.

[2] Berk, E., & Arreola-Risa, A. (1994). Note on ‘Future Supply Uncertainty in EOQ Models. *Naval Research Logistics*, 41, 129-132.

[3] Gupta, D. (1996). The (*Q,r*) Inventory System with an Unreliable Supplier. *INFOR*, 34, 59-76.

[4] Gurler, U., & Parlar, M. (1997). An Inventory Problem with Two Randomly Available Suppliers. *Operations Research*, 45, 904-918.

[5] Hendricks, K.B., & Singhal, V.R. (2005). Association between Supply Chain Glitches and Operating Performance. *Management Science*, 51, 695-711.

[6] Hendricks, K.B. & Singhal, V.R. (2003). The Effect of Supply Chain Glitches on Shareholder Wealth. *Journal of Operations Management*, 21, 501-522.

[7] Mohebbi, E. A Focused Survey of Inventory Control Models with Supply Disruptions. *Proceedings of the* 41th *Annual Meeting of the Decision Sciences Institute* (DSI), San Diego, CA, November 2010.

[8] Mohebbi, E., & Hao, D. Stock Replenishment through an Unreliable Supply Channel. *Proceedings of the* 39th *Annual Meeting of the Decision Sciences Institute* (DSI), Baltimore, MD, November 2008.

[9] Mohebbi E. (2003). Supply Interruptions in a Lost-Sales Inventory System with Random Lead Time. *Computers and Operations Research*, 30, 411-426.

[10] Mohebbi, E. (2004). A Replenishment Model for the Supply Uncertainty Problem. *International Journal of Production economics*, 87, 25-37.

[11] Mohebbi, E. & Hao, D. (2006). When Supplier’s Availability Affects the Replenishment Lead Time – An Extension of the Supply-Interruption Problem. *European Journal of Operational Research*, 175, 992-1008.

[12] Mohebbi, E., & Hao, D. (2008). An Inventory Model with Non-Resuming Randomly Interruptible Lead Time. *International Journal of Production economics*, 114, 755-768.

[13] Parlar, M. (1997). Continuous-Review Inventory Problem with Random Supply Interruptions. *European Journal of Operational Research*, 99, 366-385.

[14] Parlar, M. (2000). Probabilistic Analysis of Renewal Cycles: An Application to a Non-Markovian Inventory Problem with Multiple Objectives. *Operations Research*, 48, 243-255.
[15] Parlar, M., & Berkin, D. (1991). Future Supply Uncertainty in EOQ Models. *Naval Research Logistics*, 38, 107-121.
[16] Parlar, M., & Perry, D. (1995). Optimal (*Q, r, T*) Policies in Deterministic and Random Yield Models with Uncertain Future Supply. *European Journal of Operational Research*, 84, 431-443.
[17] Parlar, M., & Perry, D. (1996). Inventory Models of Future Supply Uncertainty with Single and Multiple Suppliers. *Naval Research Logistics*, 43, 192-210.
[18] Singhal, V. The effect of supply chain disruptions on operating performance. http://www.alba.edu.gr/uploads/singhal_abstarct.pdf.
[19] Singhal, V.R., & Hendricks, K.B. (2002). How Supply Chain Glitches Torpedo Shareholder Value. *Supply Chain Management Review*, January/February, 18-24.
[20] Snyder, L.. A Tight Approximation for a Continuous-Review Inventory Model with Supply Disruptions. Technical report, Industrial and Systems Engineering, Lehigh University. USA.
[21] Weiss, H.J., & Rosenthal, E.C. (1992). Optimal Ordering Policies when Anticipating a Disruption in Supply or Demand. *European Journal of Operational Research*, 59, 370-382.