EMPIRICAL VALIDATION OF A DEMAND FORECASTING TECHNIQUE USING CORRELATED PRODUCT DEMAND FOR A NATIONAL RETAILER

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ABSTRACT

A modification of Croston's (1972) method to forecast demand for slow-moving products with correlated demand for a national retailer is empirically tested in this paper. The method is applied to retail items that generally experience demand when the related item experiences a demand. Simulations by the authors previously showed that an improved forecast could be obtained by utilizing additional demand information from complementary products compared to using single exponential smoothing (SES) and Croston's method on each product individually. Several products from a national retailer are used to explore the accuracy of the method on data collected over a two year period in one outlet and validate the proposed technique.

Keywords: Slow-moving inventory, Croston's method, Simulation

INTRODUCTION

The increased attention on forecasting demand for slow-moving inventory over the past 10 years has not been limited to just academic research, but has included the retailers and companies that deal with items that move slowly or have intermittent demand (Boylan, J. E., & Syntetos, A. A., 2007; Shale, E. A., Boylan, J. E., & Johnston, F. R., 2006; Syntetos, A. A., & Boylan, J. E., 2001, 2005, 2006; Willemain, T. R., Smart, C. N., & Schwarz, H. F., 2004). This is evident in the addition of modules or add-ins to popular demand forecasting packages to handle series with "sparse" data. Oracle's Retail Auto ES, Forecast Pro XE and Pro Basic and MaxQ Technologies Demand Planning are widely used packages that have added modules to forecast items with intermittent demand. While no method has been shown to be superior for all cases of slow-moving inventory, classified as items in which 20% to 70% of the periods have zero demand by Altay, Rudisill and Litteral (2008), Croston's method (Croston, 1972) has been the theoretical and practical benchmark for inventory models and forecasts (Teunter, Syntetos and Babai, 2010). Croston's method differs from previous SES forecasts in that it uses two forecasts – one for the size of only positive demands and one for the periods between positive demands.

Additional information is generally considered to be an advantage when creating forecasts such as a longer data series or an associated product. This paper investigates the use of another product that has correlated demand by examining data from a national retailer with over 6000 outlets. About 600 products, classified as slow-moving and common to most of the outlets were studied. While Croston's method has been shown to improve forecasts for intermittent demand, this paper attempts to empirically validate a modification of Croston's method that can be used for items with correlated demand. A correlation analysis was performed for the identified slow-moving products to find SKU's with correlated demand over a two year period. Several pairs of SKU's with correlation coefficients over 50% and related descriptions were selected from one typical outlet.

Review of Forecasting Methodology and Description of Modified Croston's Method

Traditionally, SES is a popular statistical technique often used forecasting intermittent demand (Willemain, 2004). Croston (1972)'s procedure of using separate forecasts for the time between nonzero demands and for the positive demand sizes typically assumes no trend in the data and that the time between nonzero demands and size of demands are independent. Motivated by the common supply chain management problems with products having a high proportion of zero values in their demand history, this paper examines a special case of forecasting intermittent demand for two related slow-moving products. SES is an elegant weighted-moving average technique traditionally used to forecast demand for items held in inventory. The method is explained in most texts on forecasting. Willemain et al. (1994) should be consulted for a review of Croston's method, notation and implementation methodology.

Croston Assumptions and Violations

Croston used SES to adjust p_t", the expected mean interval between demands which is assumed to be a constant value and does not change. Typically, the demand sizes are assumed to be normally distributed; demand is random and has a 1/p probability of occurring. The demand for the product is also assumed to be independent of other demands (future intervals are independent, the size of future demands are independent, as well). In reality the demand is not always independent of other demands. Lindsey and Pavur (2008) investigated the violation of the demand rate being constant, that is, with an increasing or decreasing demand rate. Many items have complementary demands and are not independent. The complimentary products could both be slow-moving items or one could be slow-moving and the other could experience a regular demand rate. Or they could both have regular demands.

Proposed Methodology for Two Products

This study computes demand forecasts for two complementary items. One assumption is that both products will only experience a high demand size when there is simultaneous demand. In the "Two-Product Croston" method that is being validated, three unique equations are used. The first answers the question "How many periods will go by before a demand is experienced for both products?" The second answers "How many periods will go by before a demand is experienced for product 1 when there is no demand for product 2?" The third answers "How many periods will go by before a demand is experienced for product 2 when there is no demand for product 1?" We are then computing demand forecasts using the two-product method that combines the demands of the complementary products to create a forecast. For the two-product method, the results are compared to SES and the percent improvement in the MSE of the forecasts are computed for each forecast. The same comparison is made using Croston's original method. An alpha value of 0.3 is used as recommended by Willemain et al (1994).

The Two-Product Croston model is illustrated using the following notation. Separate exponential smoothing estimates are made for the average demand of Product 1 and Product 2 when demand occurs simultaneously, for the average demand of Product 1 when there is no demand for Product 2, for the average demand of Product 2 when there is no demand for Product 1. The interval between the demands for simultaneous demands and single demands are also computed.

X_{11} = demand binary indicator at time t when Product 1 &2 demand occur in same Period.
X_{10} =demand binary indicator at time t for Product 1 with no demand for 2 in the same Period.
X_{01} = demand binary indicator at time t for Product 2 when no demand for 1 in the same Period.
X_{00} = demand binary indicator at time t with no demand for Product 1 or 2 in the same Period.
Z_{1t} = size of demand for Product 1 in period t
$X11Z_{1t}$ = Product 1demand size with positive demand for Product 1 & 2 occur in the same period.
$X10Z_{1t}$ = size of demand for Product 1, positive demand for Product 1, none for Product 2.
Z_{2t} = size of demand for Product 2 in period t
$X11Z_{1t}$ = size of demand for Product 1, positive demand for Product 2, none for Product 1.
$X01Z_{2t}$ = size of demand for Product 2, positive demand for Product 2 and none for Product 1.
X11Q = the time interval between demands when both Product 1 and 2 have positive demand.
X10Q = time interval between demands when Product 1 has positive demand & 2 has none.
X01Q = time interval between demands when Product 2 has positive demand and 1 has none.
α = smoothing parameter

If $X_{11} = 1$,
$$X11Z_{1t}'' = X11Z_{1t-1}'' + \alpha(X11Z_{1t} - X11Z_{1t-1}'')$$
$$X11Z_{2t}'' = X11Z_{2t-1}'' + \alpha(X11Z_{2t} - X11Z_{2t-1}'')$$
$$X11Q_t'' = X11Q_{t-1}'' + \alpha(X11Q - X11Q_{t-1}'')$$
$$X11Q_t = 1$$

If $X_{11} = 0$,
$$X11Z_{1t}'' = X11Z_{1t-1}''$$
$$X11Z_{2t}'' = X11Z_{2t-1}''$$
$$X11Q_t'' = X11Q_{t-1}''$$
$$X11Q_t = X11Q_{t-1} + 1$$

If $X_{10} = 1$,
$$X10Z_{1t}'' = X10Z_{1t-1}'' + \alpha(X10Z_{1t} - X10Z_{1t-1}'')$$
$$X10Q_t'' = X10Q_{t-1}'' + \alpha(X10Q - X10Q_{t-1}'')$$
$$X10Q_t = 1$$

If $X_{10} = 0$,
$$X10Z_{1t}'' = X10Z_{1t-1}''$$
$$X10Q_t'' = X10Q_{t-1}''$$
$$X10Q_t = X10Q_{t-1} + 1$$

If $X_{01} = 1$,
$$X01Z_{2t}'' = X01Z_{2t-1}'' + \alpha(X01Z_{2t} - X01Z_{2t-1}'')$$
$$X01Q_t'' = X01Q_{t-1}'' + \alpha(X01Q - X01Q_{t-1}'')$$
$$X01Q_t = 1$$

If $X_{01} = 0$,
$$X01Z_{1t}'' = X01Z_{1t-1}''$$
$$X01Q_t'' = X01Q_{t-1}''$$
$$X01Q_t = X01Q_{t-1} + 1$$

$$\text{MeanProduct1Demand}_t = (X11Z_{1t-1}'' / X11Q_{t-1}'' + X10Z_{1t-1}'' / X10Q_{t-1}'')$$

$$\text{MeanProduct2Demand}_t = (X11Z_{2t-1}'' / X11Q_{t-1}'' + X01Z_{2t-1}'' / X01Q_{t-1}'')$$

RESULTS

Comparing New Method to Croston's Method

Eighteen product pairs were identified from one "typical" store from a national retailer from a group of about 600 parts recognized as slow-moving SKU's are shown in Table 1.

TABLE 1

Product pairs, demand rates and correlation

	Demand over 2 yrs.	Demand Rate	Correlation Coefficient
Pair 1			
Part A	15	0.146	0.549
Part B	23	0.223	
Pair 2			
Part C	3	0.029	0.626
Part D	2	0.019	
Pair 3			
Part E	5	0.049	0.637
Part F	29	0.282	
Pair 4			
Part G	13	0.126	0.519
Part H	8	0.078	
Pair 5			
Part J	3	0.029	0.600
Part K	46	0.447	
Pair 6			
Part L	6	0.058	0.705
Part M	2	0.019	

	Demand over 2 yrs.	Demand Rate	Correlation Coefficient
Pair 7			
Part N	1	0.010	0.601
Part P	15	0.146	
Pair 8			
Part Q	18	0.175	0.565
Part R	56	0.544	
Pair 9			
Part S	24	0.233	0.638
Part T	30	0.291	
Pair 10			
Part U	14	0.136	0.524
Part V	9	0.087	
Pair 11			
Part W	3	0.029	0.626
Part X	2	0.019	
Pair 12			
Part Y	1	0.010	0.704
Part Z	2	0.019	

A correlation analysis was computed for the 600 slow-moving SKU's and the 12 pairs were selected from the set of SKU's with correlation coefficients over 0.5. The product pairs had correlation coefficients from 0.524 to 0.705. Table 1 shows the correlation coefficient, the average demand rate and total units of demand over a two year period using weekly data. Table 2 describes the four situations encountered for each product pair as shown in the table.

TABLE 2

Combinations of types of demand for two products.

Notation for Combinations of Positive Demand for Products 1 & 2	No Positive Demand for Product 2	Positive Demand for Product 2
No Positive Demand for Product 1	P(00)	P(01)
Positive Demand for Product 1	P(10)	P(11)

We empirically validate a modification of Croston's (1972) method that utilizes the demand rates of complementary products to improve the ability to forecast the demand rates for each item. We computed the improvement in the MSE for our modified Croston's method for each of two complementary products with correlated demand and individually for each item using Croston's method compared to SES. The initial forecast uses the actual demand rate and forecasts are then made ahead one period. Only 100 periods of data were available, so the known rate was used for the initial. We believed this made the best use of the limited data and it is not unreasonable to that managers would have fairly accurate estimates of product demand averages. We forecasted each period from the start of the series, using the initial estimate of the actual mean demand and performed a rolling forecast for the 100 periods. The probability of both products having demand ranged from 1% to 7% of the time, of neither products having demand ranged from 73% to 98% of the time and only one of the products having a demand ranged from 1% to 19% of the time. While the demand rates are from actual products a common product sales price was used for each product in the investigation. This was to make it easier to compare the MSE for each forecast and to eliminate the effect of different revenue levels on the model. It is assumed that when both products experienced demands, the first product would generate $500 in revenue and the second would generate $200 in revenue. When only one product experienced demand in a period we assumed the revenue for product 1 would be $200 and product 2 would be $50.

Twelve product pairs were investigated. The reduction in MSE using Croston's method over SES for Product 1, the reduction in MSE using Croston's method over SES for Product 2, the reduction in MSE using the new modified Croston's method for related products over SES for Product 1 and the reduction in MSE using the new modified Croston's method for related products over SES for Product 2 are reported below. We then performed at Directional One Tail t-test for paired samples using Croston's method to the new method for the first product in each pair and then for the second product in each pair. The t-tests showed that the differences in the two forecasting methods were statistically significant. For product 1 the T-test statistic was 2.51 with a p-value of 0.014. For product 2 the T-test statistic was 2.32 with a p-value of 0.020. For the 12 product pairs used in the investigation, the average improvement using the new method over SES showed a statistical improvement over Croston's (1972) method. While modest, it suggests that improved forecasts can be achieved when the correlated demand is used.

TABLE 3

Reduction in MSE compared to SES

	PRODUCT 1		PRODUCT 2	
Product Pair	Croston MSE Reduction	New Model MSE Reduction	Croston MSE Reduction	New Model MSE Reduction
1	6.6	7.1	5.8	6.7
2	5.2	5.6	8.6	8.7
3	3.0	7.5	10.2	9.9
4	8.2	8.6	8.3	8.3
5	5.7	5.7	3.4	3.8
6	4.4	4.4	3.4	3.8
7	1.3	1.2	8.3	8.3
8	7.5	8.3	7.4	8.2
9	6.3	8.4	7.1	7.6
10	7.8	8.6	8.3	8.1
11	7.6	8.9	9.4	9.5
12	4.0	4.5	8.2	8.5
Average	5.6	6.6	7.4	7.6

CONCLUSIONS

While the improvement in the forecast using the new method is not always much higher than using Croston's method (and for some pairs is actually less) overall many products, on average, showed improvement compared to the benchmark of SES. This analysis using real world data from one store over a two year period shows the proposed method can provide a better forecast using data from related products. While this improvement may be small for each pair, the potential savings can be considerable when it is applied to many items across thousands of stores. Managers may benefit from improved forecasts when they are managing large inventories of slow-moving products with correlated demand.

Although more extreme improvements would be desired, the decrease in the MSE for the actual data is in line with the lower end of the range of improvement experienced in previous simulations on the model by the authors. Empirical validation was the next step in exploring the proposed two-product modification of Croston's (1972) method. This investigation uses real data. However, it is limited in that only 100 periods of weekly data was available. It appears that the demand pattern may affect the quality of the forecast as well. Demand that was equally dispersed over the period seemed to have better forecasts than periods when the demand was skewed early or late in the period. It would also be interesting to investigate more product pairs with longer periods of demand and different product costs.

REFERENCES

Altay, N., Rudisill, F., and Litteral, L. A., 2008. Adapting Wright's modification of Holt's method to forecasting intermittent demand. *IJPE*, *111*(2), 389-408.

Boylan, J. E. and Syntetos, A. A., 2007. The accuracy of a modified Croston procedure. *International Journal of Production Economics*, 107(2), 511-517.

Croston, J. D., 1972. Forecasting & stock control for int. demands. *ORQ*, 23(3), 289-303.

Johnston, F. R., Boylan, J. E., & Shale, E. A. 2003. An exam. of the size of orders from cust., their characterization, … control of slow moving items. *JORS, 54*(8), 833-837.

Lindsey, M. D. & Pavur R. 2008. A Comparison of Methods for Forecasting Intermittent Demand with Increasing or Decreasing Probability of Demand Occurrences. *Advances in Business and Management Forecasting, 5*, 115-132.

Syntetos, A. A., & Boylan, J. E. 2001. On the bias of intermittent demand estimates. *International Journal of Production Economics, 71*(1/3), 457-466.

Syntetos, A. A., & Boylan, J. E. 2005. The accuracy of intermittent demand estimates. *International Journal of Forecasting, 21*(2), 303-314.

Syntetos, A. A., & Boylan, J. E. 2006. On the stock-control performance of intermittent demand estimators. *International Journal of Production Economics, 103*(1), 36-47.

Shale, E. A., Boylan, J. E., & Johnston, F. R. 2006. Forecasting for intermittent demand: The estimation of an unbiased average. *JORS, 57*(5), 588-592.

Teunter, R., Syntetos, A.A., Babai, M.Z., 2010. Determining Order-Up-To Levels under Periodic Review for Compound Binomial (Intermittent) Demand. *EJOR, 203* (3), 619-624.

Willemain, T. R., Smart, C. N. and Schwarz, H. F., 2004. A new approach to forecasting intermittent demand for service parts inventories. *IJ F*, 20(3), 375-387.

Willemain, T. R., Smart, C. N., Shockor, J. H. and DeSautels, P. A., 1994. Forecasting int. demand in mfg: A comparative evaluation of Croston's method. *IJF*, 10(4), 529-538.