MODELING TRANSPORTATION EMISSION COSTS UNDER TWO SUPPLY CHAIN CONTRACTS

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ABSTRACT

In this paper, we study the impact of environmental costs on the supply chain performance and how revenue sharing contract under vendor managed inventory setting (VMI) affects the economic and environmental performance of the chain. In particular, we study how pollutant emission costs caused by the logistic activities may be incorporated into the decision processes of supply chain parties under both traditional wholesale price and revenue sharing contracts by assuming presence of an environmentally sensitive market. We provide the optimal solutions to the leader follower games under the two contractual settings. Numerical examples are provided to demonstrate the impact of review period and emission structure on the supply chain performance.

Keywords: Emission cost, supply chain management, vendor managed inventory, wholesale price contract, revenue sharing contract

1 Introduction

In this paper, we study how environmental costs affect supply chain decisions. We assume a supply chain of a single supplier and single retailer and two separate supply chain inventory contracts. First, is the traditional whole-price contract with the supplier as leader and the retailer as the follower. The other is revenue sharing contract with the retailer as leader and supplier as follower. Cachon (2003) provides a comprehensive and excellent review of the wide body of literature on supply chain contracts. More specifically, use of revenue sharing contract in coordinating supply chain has been studied by Cachon and Lariviere (2005). Our
work differs from the existing literature as it explicitly incorporates the environmental impact and cost into the decision making of supply chain managers. There is also a substantial body of literature on sustainable supply chains. Cornin and etal. (2011), Chabowski and Gonzales-Pardon (2011), Kleindorfer, Kaylan, and Van Wassenhove (2005), and Corbett and Kliendorfer (2001) provide surveys of sustainability related literature in strategy, management, marketing, and operations. Our work contributes to the existing literature through analytical modeling of the environmental impact and societal cost of sustainability in attempt to understand how environmental cost affects market demand and consequently, the supply chain operations. Huang and Rust (2011) models the relation between sustainability (pollution) and consumption using analytical economic models. Following a similar idea and extending it to the supply chain management area, we model the sustainability sensitive supply chain and study how companies may adjust to or even take advantage of the presence of a green sensitive market. Particularly, we want to understand how supply chain contracts may be designed for more efficient supply chain performance in a green sensitive market. Research using analytical modeling approach to study supply chain coordination is sparse. Corbett and DeCroix (2001) analyzes the impact of shared-savings contracts on alignment of profit maximization and reduction of environmental impact through lowered consumption of indirect materials. Calcott and Walls (2000) studies the extent that waste reduction policies such as a disposal fee incentivizes incorporation of environmental issues in upstream product design. Subramanian, Gupta, and Talbot (2009) examines supply chain coordination under Extended Product Responsibility legislation. Our paper falls into this category and we study how transportation pollutant emission costs affect supply chains under different contractual settings.

2 The model

The supply chain consists of one supplier with unlimited supply and one retailer facing random demand. We model the transportation activity carbon emission as a combination of fixed emission $E_f$ (per replenishment) and variable emission $E_v$ (per unit transported). If the delivered quantity is $Q$ units, the total carbon emission from each trip is expressed
as \( E = E_f + E_v Q \). If assuming that companies use taxation system for emission evaluation and the unit emission cost is \( k \), the total emission cost is \( kE \). We also assume that the carbon emission \( E \) affects the demand faced by the retailer. Let us assume that \( D = a - bE + e \), where \( a \) and \( b \) are constants, particularly, \( b \) represents the sensitivity of the market to carbon emission reduction. For single period, \( e \) is random part of the demand and i.i.d. with the single period distribution function as \( F(x) \) and the density function is \( f(x) \) on support \([-A, B]\) \((A, B > 0)\). \( e \) has the mean 0 and standard deviation \( \sigma \). For \( n \) periods, \( D_n \) is the total demand in these \( n \) periods, \( E_n \) represents the total emission; and \( e_n \) follows the \( n \)-period’s joint distribution \( F_n(x) \) with \( f_n(x) \) representing the density function. Therefore, for \( n \) periods, the total demand is \( D_n = a - bE_n + e_n \). When \( b = 0 \), the market is insensitive to emission; and when \( b > 0 \), the market is environmental impact sensitive. There has been some study on the market response to environmental impact. Empirical research (Klein (1990), Chase and Smith (1992)) and polls by polling firms (Saad 2006) and private interest groups such as National Geographic Greendex survey provide anecdotal evidence and a conflicting picture on consumer demand for products and services of environmentally sensitive companies, which provide empirical support on our demand function modeling. Therefore, our model can be used to study and explain different phenomena observed in different product markets depending on \( b \).

2.1 Whole-sale price contract

In the traditional whole-sale price contract, the supplier offers the wholesale price \( w \) as leader, then the retailer acts as follower by setting up the order quantity \( Q \). In order to study the emission impact from logistic activity, we first assume the retailer replenishment interval as \( R \) periods, i.e., the retailer reviews inventory and orders every \( R \) period. The order decision is \( Q \). The total emission from transportation activities during these \( R \) periods is \( E_R = E_f + E_v Q \). The demand during these \( R \) periods is \( D_R \). According to the assumptions, \( D_R = a - bE_R + e_R = a - bE_f - bE_v Q + e_R \). We also assume that there is a penalty cost \( p \) per unit unsatisfied demand and the salvage value per unit is \( v \). The selling price is \( s \) per
unit. Therefore, the retailer’s average profit per period ($\Pi_1$) can be written as,

$$\Pi_1 = E_{D_R} \left[ \frac{1}{R} \left[ s \min(D_R, Q) - wQ - p(D_R - Q)^+ + v(Q - D_R)^+ - kE_R \right] \right] \quad (1)$$

The following proposition describes the follower’s response function and the optimal order quantity $Q$ for given whole-sale price $w$.

**Proposition 1** $\Pi_1$ is concave function of $Q$. If denoting $\Delta = (1 + bE_v)Q + bE_f - a$, the optimal $Q$ for given wholesale price $w$ can be found from,

$$F_R(\Delta) = \frac{s - w + p + (pb - k)E_v}{(1 + bE_v)(s + p - v)}. \quad (2)$$

The proofs are included in the Appendix. To further analyze the impact of parameters on $Q$, the following corollary describes some sensitivity analysis results.

**Corollary 1** For given $w$ and $R$, $Q$ increases as the wholesale price $w$ decreases, or the penalty cost $p$ increases, or the salvage value $v$ decreases, or the emission cost $k$ decreases. As for selling price $s$, when $E_v \geq \frac{w - v}{pb - k}$, or the variable emission $E_v$ is above certain threshold, $\frac{dQ}{ds} \leq 0$; otherwise, $\frac{dQ}{ds} > 0$.

The equation in Proposition 1 can also be expressed as $F_R(\Delta) = \frac{sbE_v + w + kE_v - v(1 + bE_v)}{(1 + bE_v)(s + p - v)}$. From Proposition 1, we have one to one mapping between $w$ and $Q$ for given review period $R$ and we can find out $Q(w)$ for any given $w$. At the same time, we can write the response function as, $w(Q) = s + p + (pb - k)E_v - (1 + bE_v)(s + p - v)F_R(\Delta)$. Based on this response function and assuming the supplier’s production/purchasing cost is $c$ per unit product, the supplier’s average profit per period $\Pi_2$ can be expressed as,

$$\Pi_2 = \frac{1}{R} [w(Q) - c]Q \quad (3)$$

The following proposition demonstrates both the leader and the follower’s optimal decisions on $Q$ and $w$.

**Proposition 2** If the distribution of $D_R$ satisfies increasing failure rate (IFR) property, $\Pi_2$ is a unimodal function for given optimal response function $Q(w)$. The optimal $Q^*$ can be found from solving the following equation,

$$(s + p - v)F_R(\Delta) = -v + \frac{sbE_v + kE_v + c}{1 + bE_v} + (1 + bE_v)(s + p - v)Qf_R(\Delta) \quad (4)$$
And the optimal wholesale price $w$ can be found from the response function $w = s + p + (pb - k)E_v - (1 + bE_v)(s + p - v)FR(\Delta)$ after derivation of $Q^*$. 

Under fixed review period $R$, Proposition 2 describes the solution structure of this leader follower game. This provides us with the structural results and tools to further analyze how cost structure, demand function and emission structure impact the optimal decisions in this supply chain under wholesale price contract. Further, we will discuss the impact of review periods on supply chain decisions.

### 2.2 Revenue sharing contract under VMI

The second contract is revenue sharing contract under vendor managed inventory setting. We want to compare this contract with the wholesale price contract and study how revenue sharing contract may be applied to improve supply chain performance and further reduce the environmental impact. In this setting, the retailer (the leader) determines the revenue sharing contract parameter $\alpha$ which represents the percentage of revenue received by the retailer for every unit sale. The supplier (the follower) decides on $Q$, the shipment quantity each delivery. The supplier follows a review interval $R$, i.e. every $R$ periods, the supplier performs inventory review and replenishment. For each product sale, the supplier shares $1 - \alpha$ of the unit profit $s$ and the retailer receives $\alpha s$. Therefore, the firms average profit functions per period can be expressed as,

\[
\Pi_2 = E_{DR} \left( \frac{1}{R} \left[ (1 - \alpha) \min(D_R, Q)s - kE_R - p(D_R - Q)^+ + v(Q - D_R)^+ - cQ \right] \right) \quad (5)
\]

\[
\Pi_1 = E_{DR} \left[ \frac{1}{R} \alpha \min(D_R, Q)s \right] \quad (6)
\]

To find the optimal solution to this leader follower game and similar to the former section, we first determine the optimal response function of the follower, which is the supplier here. Then substitute the optimal response function back into the leader’s profit function to solve the optimal leader decision. The following proposition describes the supplier’s response function.

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Proposition 3  The supplier expected profit is concave on his quantity decision $Q$ and the response function for given $\alpha$ is,

$$F_R(\Delta) = \frac{(1 - \alpha)s - c - kE_v + p(1 + bE_v)}{((1 - \alpha)s + p - v)(1 + bE_v)}$$  (7)

Therefore, for a fixed replenishment period length, the optimal order size is then given by:

$$Q^* = \frac{F_R^{-1}[(1 - \alpha)s - c - kE_v + p(1 + bE_v)] + a - bE_f}{1 + bE_v}$$  (8)

From Proposition 3, we can rewrite the response function as,

$$\alpha(Q) = 1 - \frac{kE_v + c - p - pbE_v - (1 + bE_v)(p - v)F_R(\Delta)}{s[1 - F_R(\Delta)(1 + bE_v)]}$$  (9)

Substitute this equation into the retailer’s profit function $\Pi_1(\alpha) = \frac{1}{R}\alpha \min(D_R, Q)s$, and we have $\Pi_1$ as a function of $Q$ only.

$$\Pi_1(Q) = \frac{1}{R}[1 - \frac{kE_v + c - p - pbE_v - (1 + bE_v)(p - v)F_R(\Delta)}{s[1 - F_R(\Delta)(1 + bE_v)]}]sE_{D_R}[\min(D_R, Q)]$$  (10)

The following proposition describes the optimal revenue sharing contract.

Proposition 4  For given review policy $R$, the retailer’s profit function is unimodal on $Q$ if assuming the demand distribution function $F_R(\cdot)$ is increasing failure rate function (IFR). Therefore, the optimal supply quantity $Q^*$ can be found from $\frac{d\Pi_1}{dQ} = 0$ and the optimal revenue sharing contract share $\alpha$ can be found from the response function above.

3 Impact of review period $R$ and emission structure

The length of the review period $R$ not only affects the demand distribution’s mean and standard deviation, but also the emission costs. The analytical solution of the optimal review period is extremely difficult to find for both contracts. Hence, we provide some numerical examples and analysis to gain a few insights on setting up the optimal $R$. From the numerical examples conducted, the supply chain profit behaves as a unimodal function of $R$. However, we are not able to show this property analytically. A search algorithm is provided below for finding the optimal review interval. For example, for the wholesale price contract, to find the optimal combination of $(Q, w, R)$, we perform a searching procedure.
• Step 1: set \( R = 1 \);

• Step 2: for given \( R \), compute the optimal \( Q \) and \( w \) according to Proposition 2.

• Step 3: compute the supply chain profit (or the retailer’s profit) function \( \Pi_1 \) by using \((R, Q, w)\) defined;

• Step 4: update the maximum profit if there is any; set \( R = R + 1 \) and repeat step 2,3 and 4.

Similar procedure is used for revenue sharing contract. Numerical analysis is performed based on both contracts. We are interested in understanding how emission cost structure \((k, E_f, E_v)\) impacts the supply chain. Detailed discussion on numerical examples is included in the full version of the paper.

4 Further research and conclusion

This paper provides an analytical model for incorporating the impact of transportation pollutant emission cost into the traditional inventory replenishment problem under a two echelon supply chain of a supplier and retailer. We used a simple linear emission structure to model the transportation emission and assumed a linear emission cost structure. For both the wholesale price contract and the revenue sharing contract, we provided the analytical optimal solution under given review period duration. There are a few questions that we are interested to further explore and expand upon. First, the demand function is assumed to be a linear decreasing additive function, which may be further expanded to multiplicative model and even more general demand function. We conjecture that similar solutions may be found, which need to be further proven. Second, the profit functions are based on average profit per period for either party, which is an approximation to the multiple period model. This could be well fit in a more general setting using dynamic programming formulation by using more general assumptions on salvaging and inventory holding.
References


