Analysis of Ad-Supported eBook Sales Model

ABSTRACT

With the rapid and continuous increase of eBook market in recent years, retailers are exploring the ad-supported eBook model. A few companies have taken the lead in incorporating ads in their eBook such as Wiwio and Amazon. While the ad-supported eBook increases online retailers' revenue, such strategy might discourage consumers because they will experience disutility of reading ads in eBooks. Using a game theory model, we explore whether the eBook retailer will be better off to adopt the ad-supported model. We derive the equilibrium prices in both monopoly and duopoly setting and identify the conditions under which the ad-supported eBook retailer can generate more revenue. Our results show that the cross-side network effect of eBook side reduces equilibrium prices and strengthens competition. We also find that eBook consumers' disutility level plays different roles in ad-supported retailers' pricing strategies in different setting.

Keywords

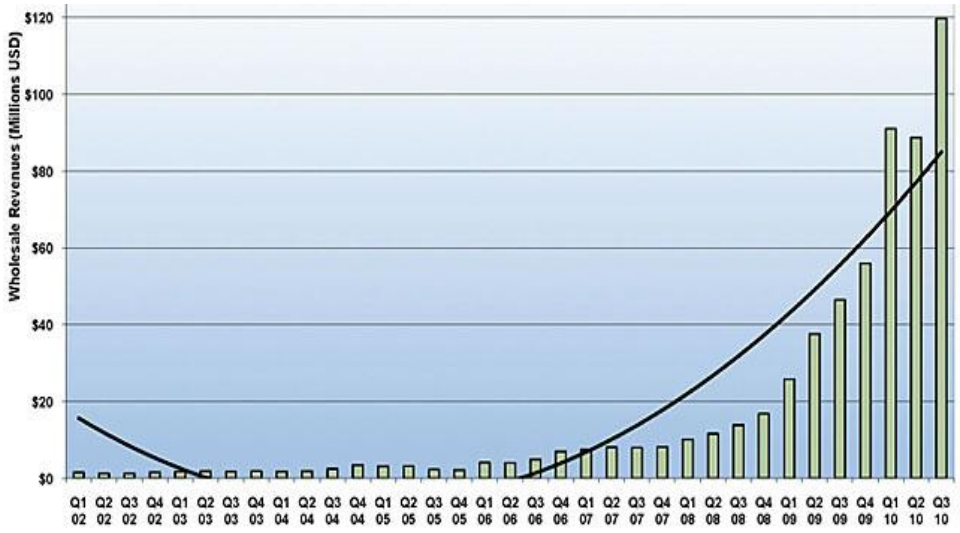
Electronic books, Ad-supported model, e-Commerce, Game theory.

1. INTRODUCTION

Recently the eBook (books in electronic/digital forms) market has seen a rapid and continuous increase in sales while the publishing industry overall has experienced a decline in sales. According to Association of American Publishers (AAP), the US book publishing industry has experienced a 5.1% decline (from $3,128.4 million in 2010 to $2,967.5 million in 2011).

However, eBook net sales in the first five months in 2011 have reached[1] $473.1 million, an increase of 160.1% compared with the first five months of 2010 (see Figure 1 for the eBook sales from 2002 till the third quarter in 2010). Meanwhile, online retailers see another trend that sales of eBooks will exceed the sales of physical books. For example, Amazon.com announced that its eBook sales had surpassed its physical books sales in May 2011 (Miller and Bosman 2011).

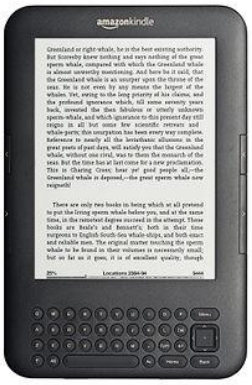
Figure 1. US Trade Wholesale E-book Sales



As the eBook market flourishes, eBook retailers and advertisers began to seek possible ways to incorporate advertising in eBook. From the retailers' aspect, the development of information technology makes it possible to incorporate ads in the eBooks such as inserting ad as a separate eBook page or putting ads as screensaver when users are not reading eBooks for a period of time. From the advertisers' point of view, eBooks serve as a perfect platform for advertising. Like other types of digital goods such as videos and music, eBooks can be spread out quickly to mass customers. The emergence of a myriad of smart reading equipment, such as Apple iPad, Apple iPhone, Amazon Kindle, Barnes & Noble Nook, etc. makes it more convenient for consumers to carry and read eBooks (please see Table 1 for a quick view).

[1] http://www.publishers.org/press/41/

Table 1. Popular eBook reading Equipment

Apple iPad	Apple iPhone	Amazon Kindle
		
55.28 million[2]	37.04 million[3]	Estimated around 6 million[4]

Several online retailers have already taken the lead of Ad-supported eBook sales. For example, Wowio.com started selling ads in the e-books consumers purchased from its website and download to their personal computers or e-Book readers like Apple's iPad and Amazon's Kindle. Wowio's ad-supported e-Book model directly incorporates the ads as eBook pages, usually including an introduction page, a closing page and another full-page ad (Steel 2010). Wowio charges advertisers certain amount for each book downloaded. Recently, Amazon announced Kindle with Special Offers which allows consumers to purchase the eBook reading equipment Amazon Kindle at a cheaper price $79.00 when they agree to receive sponsored screensaver directly on their Kindle[5] (Kindle without special offers is sold at $109.00). Examples of special offers include (1) $30 for one month of unlimited yoga classes in your city ($119 value), (2) $1 for a Kindle books, and (3) $15 for $30 worth of groceries at your local store, etc. Amazon claimed that special offers displayed as Kindle's screensaver will not interrupt reading.

[2] Source: http://en.wikipedia.org/wiki/IPad
[3] Source: http://en.wikipedia.org/wiki/IPhone
[4] Source: http://en.wikipedia.org/wiki/Amazon_Kindle
[5] http://www.amazon.com/Kindle-eReader-eBook-Reader-e-Reader-Special-Offers/dp/B0051QVESA/ref=sr_1_1?ie=UTF8&qid=1333918030&sr=8-1

While the Ad-supported eBook models help retailers to generate more revenue, they might decrease ad-supported eBooks evaluation because eBook consumers often experience disutility in reading ads in eBooks. In addition, unlike the ad-supported video such as Hulu.com and Youtube.com or the sponsored ad in the movie, books are not closely related to ads. In this paper, we explore the question whether it is strategically advantageous for retailers to adopt the ad-supported eBook model. Although we focus on eBooks, our research question is also relevant to other information goods such as digital magazine, video and music. Specifically, our study addresses the following research questions: (1) Under what conditions can retailers generate higher revenue when adopting ad-supported eBook model? (2) What are the effect, and responding strategies of adopting ad-supported eBook in both monopoly and competition?

2. LITERATURE REVIEW

Our study is closely related to the stream of research on ad-supported business models, especially in the domain of e-commerce. Fan et al. (2008) studied the optimal pricing and advertising strategies from an aspect of a media monopoly. They claimed that the media monopoly can optimize its overall profits by distributing over both traditional and online channels and showed how the channel difference factors, such as online channel access cost, advertising level affect the monopoly's optimal price and advertising levels. Kumar and Sethi (2009) claimed that pure revenue model (free-access or pure subscription fee-based models) cannot support the online information sellers. They investigated how a company can use a hybrid model and dynamic pricing policy to optimize the advertising revenue. A recent study, Lin et al. (2011) studied vertical differentiation of online service providers in both monopoly and duopoly settings. They found that the optimal strategy in monopoly setting is to offer both ad-free and ad-

supported services. In a duopoly setting, exactly one service provider will offer both ad-free and ad-supported services when the ad revenue rate is sufficiently high.

The second stream our study is related is the literature of Internet intermediary. The eBook retailers serve as the intermediary of both advertisers and eBook consumers. They charge the advertisers for the service of adding their ads and charges eBook consumers for the eBooks. Bhargava and Choudhary (2004) studied the aggregation benefits of an information intermediary. They suggest the intermediary offer two levels of service quality: high-quality and low-quality and they found that the low-quality level aids in increasing aggregation benefits. Yoo et al. (2007) explored welfare implications of different ownership structures of the intermediary. They found that different ownership will influence both sides' participation and welfare. Following Yoo et al. (2007), Bakos and Katsamakas (2008) developed a generalized model that discussed the intermediary's pricing and surplus in this type of two-sided network. Our study adds to the literature by examining under what conditions digital goods retailers' benefits from one side (revenue of advertising attached to digital goods such as eBooks) will compensate for the loss of the other side (eBook sales decline due to the ad disutility).

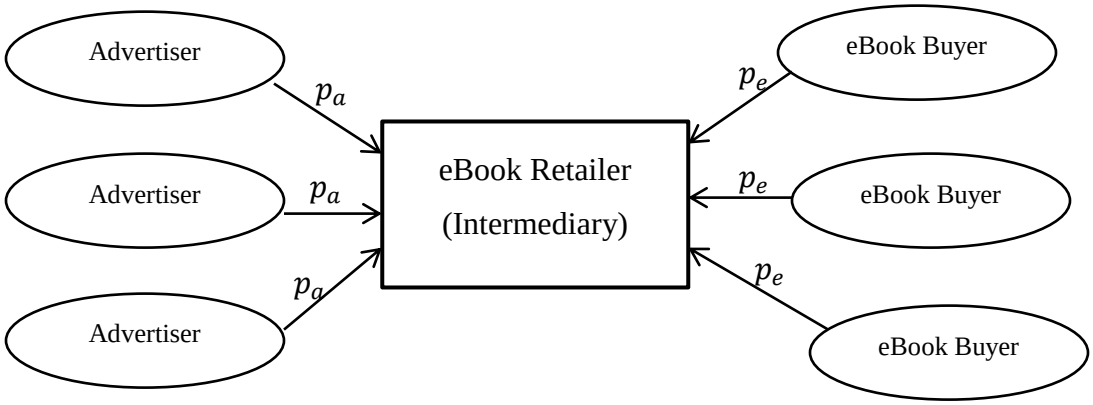
Additionally, our study adds to the current online books sales research. The current research has focused more on the issue of price differentiation of online book sales (Clay et al. 2002), and the entry of eBook seller under the condition of market asymmetries of print book and eBook markets (Jiang and Katsamakas 2010). However, our paper takes on a different aspect to address the effect of ad-supported eBook model and consumers' utility.

3. MODEL

We started with the market in which a monopoly retailer sells ad-supported eBooks and then discuss the competition in a duopoly setting. An eBook retailer might have a monopoly

position when the retailer has its unique channel presence such as Amazon Kindle. Following the general model framework of Bakos and Katsamakas (2008), we assume that the eBook retailers will generate revenue from both the advertisers and the eBook buyers. For the advertisers, they need to pay participation fee for putting their ads in the eBooks. For the consumers, they need to pay for the eBooks in which ads are incorporated. Figure 2 shows the conceptual model.

Figure 2. Ad-Supported eBook Model



Suppose the retailer charges p_a to advertisers and advertisers' utility is $U_A = \theta_a \alpha n_e - p_a$, where θ_a refers to the sensitivity of the eBook market size (the number of eBook consumers) and θ_a follows a uniform distribution [0,1]. The parameter α captures the cross-side network effect and n_{e} refers to the size eBook readers. This phenomenon is similar to the network effect in the software free trial literature (Liebowitz and Margolis 1994) and is commonly used in model the intermediary in e-commerce (Barkos and Katsamakas 2008) . The retailer charges p_e as the book price. The eBook buyers' utility is $U_e = v - \theta_e k - p_e$, where $\theta_e \epsilon\, U[0,1]$ refers to the consumers' sensitivity level towards the advertisement. The parameter v refers to consumers' reserved value for eBooks, and k refers to the consumers' disutility of reading ads. In our model, both advertisers and eBook buyers will choose to whether accept the price based on their respective utilities. Table 2 summarizes the variables used in our model.

Table 2. Summary of Variables in the Model

Notation	Definition
v	Consumers' reserved value for eBooks
θ_e	Consumers' sensitivity level towards the advertisement $\theta_e \epsilon\ U[0,1]$
θ_a	Advertisers' sensitivity level of eBook consumer size $\theta_a \epsilon\ U[0,1]$
k	Consumers' disutility of reading ads in eBooks
p_a	Participation fee for advertisers
p_e	Price of eBooks
n_e	Size of eBooks readers
α	Network effect parameter of the eBook reader side
β	Ratio of reading-break ad disutility over reading-time ad disutility

3.1 Monopoly Setting

Case One: Ad-free eBooks model.

Case one serves as our benchmark scenario in which the retailer sells ad-free eBooks. In this case, the retailers' goal is to maximize its revenue π: $\text{Max}_{p_e}\, \pi = (v - p_e)p_e$. Solving this optimization problem yields the optimal eBook prices: $p_e^* = \frac{v}{2}$ and the optimal revenue $\pi^* = \frac{v^2}{4}$.

Case Two: Ad-Supported eBook model

In this case, the retailer adopts ad-supported eBook model which collects revenue from both the advertisers' side and the eBook buyers' side. Since the advertisers' utility function is $U_A = \theta_a \alpha n_e - p_a$, the indifference level is $\widehat{\theta_a} = \frac{p_a}{\alpha n_e}$. Thus, the revenue from advertisers is $\pi_a = \left(1 - \widehat{\theta_a}\right)p_a = \left(1 - \frac{p_a k}{\alpha(v - p_e)}\right)p_a$. Similarly, the indifference level for consumers is$\widehat{\theta_e} = \frac{v - p_e}{k}$, which is n_e. Thus, the revenue from eBook consumers is $\pi_e = \left(\widehat{\theta_e} - 0\right)p_e = (\frac{v - p_e}{d})p_e$.

The retailer seeks to maximize its revenue by setting prices for both ads and eBooks:

$$\underset{p_a,p_e}{\text{Max}}\,\pi = \left(1 - \frac{p_a k}{\alpha(v - p_e)}\right)p_a + (\frac{v - p_e}{k})p_e$$

Lemma 1. When the retailer adopts ad-supported eBook model, the equilibrium price of the eBooks and ads and the retailer's optimal revenue at equilibrium are as follows:

(1) For $0 < \alpha < 4v$, $p_a^* = \frac{\alpha(\alpha+4v)}{16k}$, $p_e^* = \frac{(-\alpha+4v)}{8}$, and $\pi^* = \frac{(\alpha+4v)^2}{64k}$;

(2) For $\alpha > 4v, p_a^* = \frac{\alpha v}{2k}$, $p_e^* = 0$, and $\pi^* = \frac{av}{4k}$.

Proof. Please see the Appendix.

Now that we have calculated the retailer's optimal revenue in both case one (ad-free eBook model) and case two (ad-supported eBook model), we are able to compute the revenue difference and discuss under what conditions the retailer can generate more revenue by adopting the ad-supported eBook model.

Proposition 1. The retailer prefers to adopt ad-supported eBook model for a higher level of revenue when $Max(4(\sqrt{k} - 1)v, 0) < \alpha < 4v$ or $\alpha > Max(k, 4) * v$.

Proof. Please see the Appendix.

Proposition 1 illustrates the two regions that the retailer will realize more revenue when introducing ad-supported eBook model: the network effect α is greater than the lower threshold $Max(4(\sqrt{k} - 1)v, 0)$, and α is less than the upper threshold $4v$ in region one or α is greater than the lower threshold $Max(k, 4) * v$ (see Table 3 below).

Table 3. Region and Value of α in favor of Ad-supported Model

Region	**Value of α in favor of Ad-supported Model**	
Region One ($\alpha < 4v$)	$k \leq 1$	$0 < \alpha < 4v$
	$1 < k < 4$	$4(\sqrt{k} - 1)v < \alpha < 4v$
Region Two ($\alpha > 4v$)	$0 < k \leq 4$	$\alpha > 4v$
	$k > 4$	$\alpha > kv$

From the retailer's perspective, our results suggest that there is ample opportunity for retailers to adopt ad-supported eBook model. In region one with a relatively low network effect α, if $k \leq 1$ then the entire region is good for the retailer; if $1 < k < 4$, retailers can adopt ad-supported model when $4\left(\sqrt{k}-1\right)v < \alpha < 4v$. In region two with a relatively high network effect α, if $0 < k \leq 4$, the whole region is good for the retailer; if $k > 4$ retailers can adopt ad-supported model when $\alpha > kv$.

3.2 Duopoly Setting

Next, we look at the duopoly in which two eBook retailers compete in the eBook market. We assume that the two eBook retailers provide perfectly substitutable eBooks, for which eBook buyers have the valuation v. We also assume that the firms have zero marginal cost and the market is covered. Firms adopt different strategies. We denote the ad-free eBook model as "AF". For ad-supported model, the retailer can choose from two strategies: (1) "Ad as content", which is to insert the ad as part of the eBook content such as Wowio Inc., and (2) "Ad as screensaver", which is to incorporate the ad as screensaver that does not disturb reading, the practice used by Amazon. We denote the strategy of "Ad as content" as "AC" and denote "Ad as screensaver" as "AS".

We assume that one retailer will choose "AF" while another will choose either "AC" or "AS" strategy. If both retailers choose "AF", then the competition turns out to be a simple Bertrand competition game. Without loss of generality, let retailer 1 adopt "AF" strategy (ad-free retailer) and retailer 2 choose either "AC" or "AS" strategy (ad-supported retailer). Consumers' preference between ad-free retailer's eBooks and ad-supported retailer's eBooks with ads depends on the prices and their valuation for ads. The split of consumers will also influence the revenue of the advertisers' side.

Lemma 2. The equilibrium price for the strategy set (AF, AC) is as follows:

(1) For $0 < \alpha < 2k$, $p_e^{1*} = \frac{(-\alpha+8k)}{12}$, $p_e^2 = \frac{(-\alpha+2k)}{6}$ and $p_a^2 = \frac{\alpha(\alpha+4k)}{24}$;

(2) For $2k < \alpha < 8k$, $p_e^{1*} = \frac{k}{2}$, $p_e^2 = 0$, and $p_a^2 = \frac{\alpha}{4}$, $\pi_2^* > \pi_1^*$.

(3) For $\alpha > 8k$, no equilibrium prices exist.

Proof. Please see the Appendix.

Next, we focus on the case when ad-supported retailer undertakes the "AS" strategy. In this case, consumers who will buy eBooks from ad-supported retailer can get a lower price on reading equipment or in a more general term a bonus b. Note that consumers' disutility of ad is mainly from the ad screensaver during the reading-break. Thus, we denote β as the ratio of reading-break disutility over reading-time disutility and we assume that $\beta \in (0, 1)$. Therefore, the utility function of eBook consumers of retailer two is revised as $U = v - \beta\theta_e k - p_e^2 + b$.

Lemma 3. The equilibrium price for the strategy set (AF, AS) is as follows:

(1) For $0 < \alpha < 2\beta(k-b)$, $p_e^{1*} = \frac{(-\alpha+4b\beta+8\beta k)}{12\beta}$, $p_e^{2*} = \frac{(-\alpha-2b\beta+2\beta k)}{6\beta}$, and $p_a^{2*} = \frac{\alpha(\alpha+4\beta k-4\beta b)}{24\beta^2 k}$.

(2) For $2\beta(k-b) < \alpha < 4\beta(b+2k)$, $p_e^{1*} = \frac{k+b}{2}$, $p_e^{2*} = 0$, and $p_a^2 = \frac{\alpha(k-b)}{4\beta k}$;

(3) For $\alpha > 4\beta(b+2k)$, no equilibrium prices exist.

Proof. Please see the Appendix.

Based on the result of both Lemma 2 and Lemma 3, we study the equilibrium prices of both ad-free retailer and ad-supported retailer in the following Proposition.

Proposition 2. For the duopoly competition under the strategy set (AF, AC)

(1) When $p_e^{2*} > 0$, both retailers' equilibrium eBooks prices p_e^{1*} and p_e^{2*} are decreasing in the network effect α and are increasing in the ad disutility k. The ad price p_a^{2*} increases in both ad network effect α and disutility k;

(2) When $p_e^{2*} = 0$, ad-free retailer' s eBook price only depends on disutility of ad k and the ad price p_a^{2*} only depends on network effect α.

For the duopoly competition under the strategy set (AF, AS)

(1) When $p_e^{2*} > 0$, both retailers' equilibrium eBook prices p_e^{1*} and p_e^{2*} are decreasing in the network effect α and are increasing in the disutility of ad k and the reading-break disutility ratio β. The ad price p_a^{2*} will increase in both network effect α but the influence of disutility of ad k is uncertain;

(2) When $p_e^{2*} = 0$, ad-free retailer's eBook price only depends on disutility of ad k and bonus b.

Proof. Please see the Appendix.

Proposition 2 shows that in the duopoly setting, whether ad-supported retailer provides eBook for free will lead to different equilibrium price results. One common finding in both strategy sets (AF, AC) and (AF, AS) is that when ad-supported retailer does not provide free eBooks, a higher network effect α causes both retailers to reduce eBook prices. Ad-supported retailer might want to expand the eBook consumer market size to increase his ad-supported revenue (together by increase ad price), which forces ad-free retailer to reduce the price to stay in the competition.

The finding that the equilibrium price will increase as ad disutility k increases seems counter-intuitive. However, a higher k will discourage ad-supported retailer to incorporate more ads which forces him to rely more on generating eBook revenues and increase the equilibrium prices. This might also explain another interesting finding: when ad-supported retailer provides free eBooks, ad-free retailer's eBook price depends only on disutility k under the strategy set (AF, AC) and on disutility k and bonus b under the strategy set (AF, AS). This implies that when

ad-support retailer can rely completely on ad revenue, ad-free retailer has to take advantage of consumers' ad disutility and charge higher price.

Next we compare the revenue of both ad-free retailer and ad-supported retailer in the following Proposition.

Proposition 3. For the duopoly competition under the strategy set (AF, AC)

(1) For $0 < \alpha < 2k$, ad-free retailer has a higher level of revenue $\pi_1^* > \pi_2^*$;

(2) For $2k < \alpha < 8k$, ad-supported retailer has a higher level of revenue $\pi_1^* < \pi_2^*$.

For the duopoly competition under both the strategy set (AF, AS)

(1) For $0 < \alpha < 2\beta(k-b)$, ad-free retailer has a higher level of revenue $\pi_1^* > \pi_2^*$;

(2) For $2\beta(k-b) < \alpha < 4\beta(b+2k)$, if $\beta > \frac{\alpha(k-b)}{2(k+b)^2}$, ad-free retailer has a higher level of revenue $\pi_1^* > \pi_2^*$; if $\beta < \frac{\alpha(k-b)}{2(k+b)^2}$, ad-supported retailer has a higher level of revenue $\pi_2^* > \pi_1^*$.

Proof. Please see the Appendix.

Proposition 3 suggests that in equilibrium under strategy set (AF, AC), the Ad-free retailer generates a higher level of revenue than the ad-supported retailer when the network effect α is lower than the threshold $2k$. When the network effect is beyond that threshold, the ad-supported retailer gains advantage and generates more revenue. In the region where the ad revenue is extremely high, the ad-supported retailer can force the eBook price to zero and dominate the whole market. Under the strategy set (AF, AS), we have a similar finding in the region when the network effect is lower than a threshold $\underline{\alpha} = 2\beta(k-b)$ and in the region when the ad revenue is higher than the threshold $\bar{\alpha} = 4\beta(b+2k)$. However, in the middle region, whether ad-supported retailer or ad-free retailer generates more revenue depends on the reading-break disutility ration. When eBook consumers experience a high disutility level when they see

the ad in the screensaver (the disutility of reading ad in eBook pages and in screensaver are close), the ad-free retailer will generate a higher level of revenue.

4. CONCLUSION

Motivated by the rapid development of eBook industry and increasing awareness of ad-supported model in e-commerce, we explore the ad-supported models in the eBook industry. While retailers are able to generate more revenue from incorporating ads in their eBooks, they also take the risk of losing consumers because of the disutility of reading eBook ads. In this paper, we propose a game-theory model to analyze ad-supported eBook models which were understudied in the previous literature of eBooks. We first derive the equilibrium prices for both monopoly and duopoly setting and identify conditions under which retailers might be better off by selling ad-supported eBooks. Our results suggest that there are ample opportunities for retailers to adopt ad-supported eBooks model in both monopoly and duopoly setting. Our study also found that the network effect of eBooks consumer side will intensify the competition in duopoly and reduces the equilibrium eBook prices in both monopoly and duopoly setting. In addition, we found that eBooks consumers' disutility will increase the equilibrium prices in duopoly competition. One possible extension to this paper is to study the impact of network effect on the advertisers' side. Another interesting topic for future research is to examining different ad revenue model such as pay-per-click (CPC) and pay-per-thousand impressions (CPM).

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APPENDIX

Due to the limits of the paper length, only brief proof is presented.

Proof of Lemma 1

Case Two: Ad-supported eBook

Retailer's revenue function is as follows:

$$\underset{p_a,p_e}{\text{Max}}\,\pi = \left(1 - \frac{p_a k}{\alpha(v - p_e)}\right)p_a + (\frac{v - p_e}{k})p_e$$

We first take the derivative on both p_a and p_e: $\frac{\partial \pi}{\partial p_a} = 1 - \frac{2p_a k}{\alpha(v-p_e)} = 0$ and $\frac{\partial \pi}{\partial p_e} = \frac{v-2p_e}{k} - \frac{2kp_a^2}{\alpha(v-p_e)^2} = 0$

Solving these two equations together, we get the following cases:

(1) For $0 < \alpha < 4v$, $p_a^* = \frac{\alpha(\alpha+4v)}{16k}$, and $p_e^* = \frac{(-\alpha+4v)}{8}$. The retailer's optimal revenue is $\pi^* = \frac{(a+4v)^2}{64(k+v)}$.

(2) For $\alpha > 4v$, $p_a^* = \frac{\alpha(\alpha+4v)}{16k}$ and $p_e^* = 0$. The retailer's optimal revenue is $\pi^* = \frac{(a+4v)^2}{64(k+v)}$.

Proof of Proposition 1

For $0 < \alpha < 4v$, $\Delta\pi = \frac{(a+4v)^2}{64k} - \frac{v^2}{4}$. To have $\Delta\pi > 0$, we need to satisfy $-\frac{a+4v}{4\sqrt{k}} < v < \frac{a+4v}{4\sqrt{k}}$.

Since $v > 0$, $-\frac{a+4v}{4\sqrt{k}} < v$ is satisfied. The inequality $v < \frac{a+4v}{4\sqrt{k}}$ equals to $Max(4(\sqrt{k}-1)v, 0) < \alpha < 4v$.

For $\alpha > 4v$, $\Delta\pi = \frac{av}{4k} - \frac{v^2}{4}$. To have $\Delta\pi > 0$, we need to satisfy $\frac{a}{k} > v$. Thus, $\alpha > Max(k, 4) * v$

Proof of Lemma 2.

Utility of buying from retailer 1 is:$v - p_e^1$, and utility of buying from retailer 2 is: $v - \theta_e k - p_e^2$

The utility difference $\Delta U = p_e^2 - p_e^1 + \theta_e k$, and the indifference level $\widehat{\theta_e} = \frac{p_e^1 - p_e^2}{k}$.

Thus, if $\widehat{\theta_e} > \frac{p_e^1 - p_e^2}{k}$, consumers buy from retailer 1 and if $\widehat{\theta_e} < \frac{p_e^1 - p_e^2}{k}$, buy from retailer 2.

$\pi_1 = (1 - \frac{p_e^1 - p_e^2}{k})p_e^1$ and $\pi_2 = \left(1 - \frac{p_a^2 k}{\alpha(p_e^1 - p_e^2)}\right)p_a + \frac{p_e^1 - p_e^2}{k}p_e^2$

Taking derivative of p_e^1 , p_e^2 and p_a^2, we get the following:

$\frac{\partial\pi}{\partial p_e^1} = 1 + \frac{p_e^2 - 2p_e^1}{k} = 0$; $\frac{\partial\pi}{\partial p_e^2} = \frac{p_e^1 - 2p_e^2}{k} - \frac{k(p_a^2)^2}{\alpha(p_e^1 - p_e^2)^2} = 0$; and $\frac{\partial\pi}{\partial p_a^2} = 1 - \frac{2kp_a^2}{\alpha(p_e^1 - p_e^2)} = 0$

Solving these three equations together, we have the following cases:

(1) For $0 < \alpha < 2k$, $-\alpha + 2k > 0$, $p_e^{1*} = \frac{(-\alpha+8k)}{12}$, $p_e^2 = \frac{(-\alpha+2k)}{6}$ and $p_a^2 = \frac{\alpha(\alpha+4k)}{24k}$.

(2) For $2k < \alpha < 8k$, p_e^{2*} is driven to zero. $p_e^{1*} = \frac{k}{2}$, $p_e^{2*} = 0$ and $p_a^2 = \frac{\alpha}{4}$,

If $\alpha > 8k$, both p_e^{1*} and p_e^{2*} are driven to zero. In this case, no equilibrium prices.

Proof of Lemma 3.

Utility of buying from retailer 1 is: $v - p_e^1$, and utility of buying from retailer 2 is: $v - \beta\theta_e k - p_e^2 + b$.

The utility difference $\Delta U = p_e^2 - p_e^1 + b + \beta\theta_e k$, and the indifference level $\widehat{\theta_e} = \frac{p_e^1 - p_e^2 - b}{k\beta}$.

Similar to the previous strategy set (AF vs. AC), if $\widehat{\theta_e} > \frac{p_e^1 - p_e^2 - b}{k\beta}$, consumers will buy eBooks from retailer 1 and if $\widehat{\theta_e} < \frac{p_e^1 - p_e^2 - b}{k\beta}$., consumers will buy eBooks from retailer 2.

$\pi_1 = (1 - \frac{p_e^1 - p_e^2 - b}{k\beta})p_e^1$ and $\pi_2 = \left(1 - \frac{p_a^2\beta k}{\alpha(p_e^1 - p_e^2 - b)}\right)p_a + \frac{p_e^1 - p_e^2 - b}{k\beta}p_e^2$. Taking derivative of p_e^1 and p_e^2,

$\frac{\partial\pi}{\partial p_e^1} = 1 + \frac{p_e^2 + b - 2p_e^1}{k} = 0$; $\frac{\partial\pi}{\partial p_e^2} = \frac{p_e^1 - 2p_e^2 - b}{k} - \frac{\beta k(p_a^2)^2}{\alpha(p_e^1 - p_e^2)^2} = 0$; and $\frac{\partial\pi}{\partial p_a^2} = 1 - \frac{2\beta kp_a^2}{\alpha(p_e^1 - p_e^2 - b)} = 0$

Solving these three equations together, we have the following cases

(1) For $0 < \alpha < 2\beta(k - b)$, $(-\alpha - 2b\beta + 2\beta k) > 0$

$p_e^{1*} = \frac{(-\alpha+4b\beta+8\beta k)}{12\beta}$, $p_e^2 = \frac{(-\alpha-2b\beta+2\beta k)}{6\beta}$ and $p_a^2 = \frac{\alpha(\alpha+4\beta k-4\beta b)}{24\beta^2 k}$.

(2) For $2\beta(k-b) < \alpha < 4\beta(b+2k)$, p_e^{2*} is driven to zero.

$p_e^{1*} = \frac{k+b}{2}$, $p_e^{2*} = 0$ and $p_a^2 = \frac{\alpha(k-b)}{4\beta k}$,

(3) For $\alpha > 4\beta(b+2k)$, both p_e^{1*} and p_e^{2*} are driven to zero. In this case, no equilibrium prices.

Proof of Proposition 2

Based on Lemma 2, we have

(1) For $0 < \alpha < 2k$, $\pi_1^* = \frac{(\alpha-8k)^2}{144k}$ and $\pi_2^* = \frac{(\alpha+4k)^2}{144k}$. Since $\alpha < 2k$, $\pi_1^* > \pi_2^* > 0$.

(2) For $2k < \alpha < 8k$, $\pi_1^* = \frac{k}{4}$ and $\pi_2^* = \frac{\alpha}{8}$. Since $\alpha > 2k$, $\pi_2^* > \pi_1^* > 0$.

Based on Lemma 3, we have

(1) For $0 < \alpha < 2\beta(k-b)$, $\pi_1^* = \frac{(\alpha-4\beta(b+2k))^2}{144\beta^2 k}$ and $\pi_2^* = \frac{(\alpha-4\beta(-b+k))^2}{144\beta^2 k}$.

Since $\alpha < 2\beta(k-b) < 2\beta k < 6\beta k$, $\pi_1^* > \pi_2^* > 0$.

(2) For $2\beta(k-b) < \alpha < 4\beta(b+2k)$, $\pi_1^* = \frac{1}{4k}(k+b)^2$ and $\pi_2^* = \frac{\alpha(k-b)}{8\beta k}$.

For $\beta > \frac{\alpha(k-b)}{2(k+b)^2}$, $\pi_1^* > \pi_2^* > 0$; For $\beta < \frac{\alpha(k-b)}{2(k+b)^2}$, $\pi_2^* > \pi_1^* > 0$.