Product Acquisition for Remanufacturing: A Dynamic Analysis

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We evaluate how buyback and trade-in acquisition policies affect profitability of an OEM that sells new and remanufactured product. Towards this end, we introduce a simple consumer choice model that, in aggregate form, is consistent with the classic Bass diffusion model. The choice model in combination with a product acquisition policy dictates the dynamics of new and remanufactured product demand, consumer return and repurchase decisions, and the evolution of the install base (quantity and age) over the life-cycles of new and remanufactured product.

Our analyses lead to two main findings. First, we identify a key indicator for dominance of a trade-in policy over a buyback policy. Roughly speaking, if a firm anticipates that a buyback price (on average) will be less the margin of a new product, then then a trade-in policy is likely to dominate a buyback policy. Otherwise it is possible that a buyback policy will be more profitable.

Our second main finding draws on the notion of a used product sweet spot—the sweet spot is the age of the product at which it is most economical to acquire and remanufacture. We find that profitability is highest when the time lag between new and remanufactured product introductions is at or near the sweet spot. Furthermore, when this condition holds, there is little loss in profit from using a myopic pricing instead of proactive pricing. Myopic pricing has the advantages of (i) being much simpler to implement than a proactive pricing algorithm and (ii) does not rely on accurate demand projections. A firm may wish to consider this result when making decisions on when to introduce a remanufactured version of its product to the market.

Keywords: remanufacturing, product acquisition pricing policy, product life-cycle

1. Introduction

Sustainability concerns and the rise of extended producer responsibility (EPR) legislation have generated a growing interest in product take-back and recovery practices (e.g., product reuse, remanufacturing, recycling, spare parts harvesting, incineration for energy recovery). Such practices are attractive for two reasons. First, product take-back and recovery programs increase and extend the economic value of products in the market. Second, product take-back and recovery programs positively affect the environment (e.g., landfill avoidance) and support EPR legislation that requires collection of end-of-use product. This is especially true for durable goods such as automobiles, home appliances, and computers that are costly to dispose due to size and/or the presence of hazardous materials. Indeed, durable goods producers often offer product acquisition schemes such as trade-in discounts on a new product, cash buyback, and deposit fees to encourage returns.

In this paper, we investigate the performance of buyback and trade-in policies for acquiring used product to be remanufactured. The motivation for our study comes from our discussions with management at a computer manufacturer that sells both new and remanufactured product. Consequently, we consider how buyback and trade-in product acquisition decisions impact the combined remanufactured and new product profit. A key distinguishing feature of our analysis is the consideration of time dynamics. In particular, both the quantity-condition profile of used product and the market interest in remanufactured product evolve over time, and the manner of evolution is influenced by new product sales. We introduce and analyze a series of models that reflect the dynamics of customer willingness-to-return, the size and condition of the OEM product install base, the demand for remanufactured product, and the demand for new product.

Our paper offers two broad contributions. First, we introduce a parsimonious model that captures the dynamics of demand, condition-dependent supply, and price-dependent willingness-to-return a used product over the life-cycles of a new product and the remanufactured version of the new product. The model can serve as a stepping stone for other investigations while accounting for life-cycle dynamics (e.g., timing of remanufactured product introduction and utilization of alternate acquisition strategies). We offer several suggestions for future research at the end of the paper.

Second, we identify managerial insights that are relevant for managing product acquisition programs. We find that there is little difference in profitability between a buyback program and a trade-in program when product loyalty is high. When product loyalty is not high, then a key indicator of relative performance can be found through a comparison of the (anticipated) average buyback price with the new product margin. If buyback price is less than the margin, then a trade-in program is likely to be more profitable than a buyback program.

We also identify an influential driver of profit level and offer guidance on the choice of acquisition pricing methods. Our insights along this line draw on a relationship between the time lag between new and remanufactured product introductions and the used product sweet spot. The sweet spot refers to the age of a product where the sum of acquisition and remanufacturing cost is at or near the minimum. We find that remanufactured product profit over the life-cycle is highest when the time lag is close to the sweet spot. Furthermore, we compare profits that are obtained using an optimal myopic pricing algorithm and an optimal proactive pricing algorithm. Myopic pricing does not look beyond the current period when setting acquisition prices. We find that when the time lag and sweet spot are aligned, there is little loss in profit from using myopic pricing instead of proactive pricing. Myopic pricing has the advantages of (i) being much simpler to implement than proactive pricing and (ii) does not rely on accurate demand projections. A firm may wish to consider this result when making decisions on when to introduce a remanufactured version of its product to the market.

The remainder of the paper is organized into four sections. Section 2 outlines the related literature and how our investigation differs from past research. Section 3 presents a model and analysis of a buyback program. Section 4 presents a model and analysis of a trade-in program. Section 5 investigates the relative performance of buyback and trade-in programs, and Section 6 provides a summary and suggestions for future research. A list of notation and assumptions, as well as derivations, proofs, and algorithms, can be found in the appendix.

2. Related Literature

Product trade-in and buyback policies are broadly practiced in durable goods industries (i.e., automotive, consumer electronics, computers, and industrial equipment), and are well studied in the marketing literature. The marketing literature focuses on how trade-in and buyback programs affect the profitability of new product sales, or the forward flow of new products. Earlier works in this area show that the benefits from offering trade-in programs are due largely to market segmentation and price discrimination, as the firm is able to price discriminate between owners and non-owners. Van Ackere and Reyniers (1995) develop a two-period pricing model. The first-period purchase decision segments the second-period market into owners and non-owners. The trade-in price is set to take advantage of the fact that, by virtue of a past purchase decision, the owners have a relatively high valuation of the product Okada (2001, 2006). In addition to serving as a market segmentation and price discrimination mechanism, trade-in and buyback programs can reduce cannibalization of new product sales from secondhand markets. Levinthal and Purohit (1989) examine the competition between the new and old version of durable good and show how a monopolist who introduces a new product and continues to sell the old model can improve profits by buying back some of the old products. Fudenberg and Tirole (1998) show how used textbook and software purchases by the OEM helps to increase new product sales by reducing cannibalization. Bruce et al. (2006) consider how trade-in discounts can spur demand and increase profit when consumers are otherwise reluctant to purchase a new product due to the burden of paying off an outstanding loan (e.g., as in the auto industry). Rao et al. (2009) show how a trade-in program can be used to reduce the inefficiencies associated with the "lemon problem"—the trade-in opportunity motivates more owners to purchase new goods by reducing their desire to hold on to purchased goods because of the low price the latter would fetch in a lemon market. The above marketing papers are relevant because they provide guidance on modeling return volume as a function of trade-in and buyback prices. However, these papers only consider a two-period setting where the trade-in prices are set in period two. We consider a multi-period setting. Furthermore, none of these papers incorporate the economics of product take-back for remanufacturing, which is an element that is central to our analysis.

In contrast to marketing literature, the closed-loop supply chain (CLSC) literature, focuses on how trade-in and buyback incentives affect the reverse flow of used products, e.g., for the purposes of product recovery to support remanufacturing and/or respond to take-back legislation. Product acquisition management is an important function of CLSC systems with remanufacturing. Remanufacturing depends on timely access to a reliable supply of used products that are still in relatively good condition. In the CLSC setting, trade-in and buyback prices incentivize product returns and thus act as product acquisition mechanisms. As such, they are critical because of their role in (i) shaping the quality of the returned products, and (ii) influencing the return volumes to align supply with demand (Thierry et al. 1995). Guide and Jayaraman (2000) and Flapper (2001) provide an overview of how product acquisition incentives influence the profitability of a remanufacturing firm. Klausner and Hendrickson (2000) study a German remanufacturer that once acquired used power-tools from the waste stream. They show how the firm improves profits by buying gently used tools from current holders instead of collecting used products from the waste stream, e.g., a gently used product defines the sweet spot for product acquisition.

Several recent studies model the volume of product returns as a function of acquisition price, either under a buyback program (e.g., Bakal and Akcali 2006, Guide et al. 2003, Karakayali et al. 2007) or a trade-in program (e.g., Ray et al. 2005). Guide et al. (2003) and Karakayali et al. (2007) consider models with discrete classes for the condition of used products whereas Bakal and Akcali (2006) and Ray et al. (2005) model the condition of used product as a continuous variable. All of these papers identify optimal condition-dependent acquisition prices. In addition, product selling prices—for remanufactured product in the case of Bakal and Akcali (2006), Guide et al. (2003), and Karakayali et al. (2007) and for new product in the case of Ray et al. (2005)—

are endogenous. Acquisition and selling prices are set to maximize profit in a single period. The main distinguishing features between these papers and our work are (1) we compare and contrast both buyback and trade-in programs, (2) we consider the acquisition pricing problem over the life-cycle of a product rather than a single period, and (3) we consider settings where the selling prices of new and remanufactured product are exogenous.

Both forward and reverse operations are influenced by new product diffusion and product life-cycle dynamics. The product life-cycle explains how the sale of a new product grows, matures, and declines over time (Mahajan et.al. 1990). The product life-cycle also affects the availability of used products and the demand for remanufactured products over time. Tibben-Lembke (2002) and Östlin et al. (2009) describe the relationship between new product sales, used product returns, and recovered product demand, and how these relationships vary over different stages of product life-cycle (see Figure 1). We make use of this relationship in our model of remanufactured product demand.

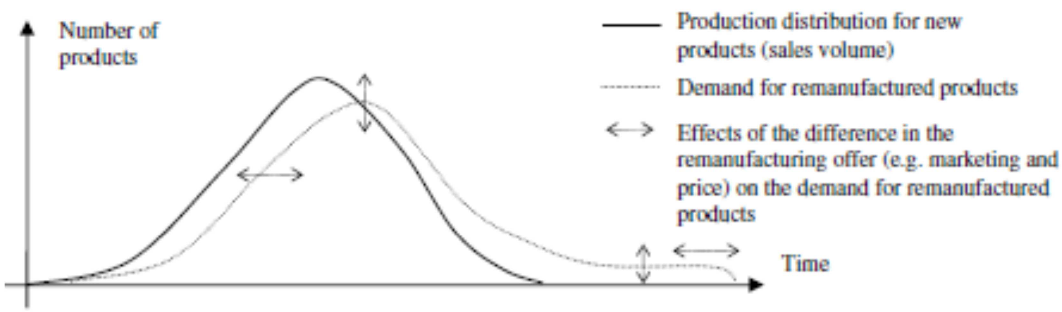

Figure 1. Illustration of the theoretical relationship between new product sales and the demand for a remanufactured version of the product (adapted from Östlin et al. 2009).

Several studies have addressed the implications of the product life-cycle on CLSC collection, recovery, and remarketing strategies. Debo et al. (2006) consider remanufacturing production strategies when the firm selects the degree of product remanufacturability in settings where the product is collected and remanufactured many times. Geyer et al. (2007) also consider a setting where the product is remanufactured many times. They focus on the economic feasibility of recovery operations and how life-cycle dynamics influence remanufacturability levels, and collection rates. Umeda et al. (2005) consider the question of how many times a device can be used over the life-time of the product. Our work is similar to these papers in that we examine the evolution of the install base of new product over time and the associated evolution of demand for remanufactured product. Our work differs from these papers in that we do not consider the number of times a product is remanufactured; however, we concentrate on the impact of life-cycle dynamics on optimal acquisition prices and on the relative merits of buyback and trade-in programs. These elements are not considered within this stream of literature.

The cost to recover used products depends on the product's condition and on the targeted return volume. Higher trade-in prices increase the flow of reusable returns (Klausner and Hendrickson 2000). Galbreth and Blackburn (2006) analyze optimal acquisition and sorting policies. They show how acquiring more used products may reduce the average cost of remanufacturing. When the difference in the cost to recover high vs. low quality products is significant, the firm can potentially benefit from acquiring more used products in order to secure better quality products. Zikopoulos and Tagaras (2007) identify optimal procurement quantities from multiple alternative sources. In contrast to these papers, we consider condition-dependent acquisition pricing. In addition, our research is distinct in that we incorporate how the condition and quantity of the install base change over time due to the linkage of these attributes with the timing and quantity of new product sales.

Finally, a few recent studies have addressed the dynamic aspects of CLSC systems. Lee et al. (2010) examine a decentralized two-echelon distribution channel where the manufacturer provides a retailer with a fixed incentive to collect used products. The retailer determines a time-variant pricing rule that includes the price of the new product and the trade-in incentive. The manufacturer chooses a constant wholesale price and offers a fixed payment for the returned products. The authors show how the retailer's behavior over time affects collection rates. Although Lee et al. (2010) develop a time-variant model, their model does not incorporate how the recovery cost changes as a function of the age or condition of the trade-in returns. This is an important feature that underlies the motivation for our study because the OEM has information on the profile of the install base (e.g., quantity and age)—information that is relevant for setting acquisition prices.

3. Buyback Policies for a Producer of Remanufactured Product

The sales rate of a product over its life-cycle follows a pattern of growth, maturity, and decline. Both the supply and the demand of a remanufactured version of the product are influenced by historical sales of the new product. In particular, the sales history exposes the quantity-age profile of the install base (e.g., number of products in the market for less than one year, less than two years, etc.) and, as a measure of market demand of the new product, can be an indicator of market interest in the remanufactured product.

Section 3.1 addresses the relationship between new product sales over its life-cycle and the supply of used product for remanufacturing. Section 3.2 attends to the relationship between new product sales and the demand for remanufactured product. Section 3.3 identifies the optimal myopic age-dependent buyback pricing function and characterizes the optimal profits over the remanufactured product life-cycle. A myopic buyback price function sets prices to maximize profit in each period, with no consideration of future periods. Section 3.4 considers how the optimal buyback price function and profits are impacted when the firm is proactive in its pricing. Section 3.5 presents numerical analyses that compare and contrast the two pricing policies.

3.1. Used Product Supply, Age-Dependent Acquisition Price, and Buyback Return Volume

In this section we develop a model of return volume as a function of age-dependent buyback prices. We begin by describing the consumer-choice model that we use to define new product purchase probabilities and product return probabilities. We model the diffusion of new product sales in markets of two segments. Our model results in sales that are consistent with the diffusion model of Bass (1969), a parsimonious model that is empirically well-established for sales of consumer durables.

A generation of a product is introduced in period $t = 1$. The market is divided into two segments—innovators and imitators—that are distinguished by their valuation of the new product. The innovator segment (segment 1) valuation at the time of purchase is V_1, which is uniformly distributed. We normalize the support to [0, 1], i.e.,

$$V_1 \sim U[0, 1]. \tag{1}$$

Uniformly distributed valuation is common in the literature (e.g., Mussa and Rosen 1978, Purohit and Staelin 1994). Note that the new product price is less than the maximum innovator valuation (i.e., $p_n < 1$); otherwise there would be no sales.

The imitator segment (segment 2) valuation at the time of purchase is V_2, which is also uniformly distributed. However, the upper support of V_2 in period t is the new product purchase price plus a term that is proportional to cumulative sales at the start of the period $D_n(t-1)$, i.e.,

$$V_2 \sim U[0, p_n + \iota D_n(t-1)]. \tag{2}$$

Note that the innovator segment valuation is unaffected by historical sales whereas the maximum valuation of a consumer in the imitator segment increases with the number of users. The upper limit on total sales over the life-cycle is M. The maximum possible imitator valuation occurs at the end of the life-cycle, and its value depends upon model parameters. In the data underlying Figure 2, for example, $V_2 \leq p_n + \iota M = 0.92$.

The innovator segment makes up fraction θ of consumers who consider purchasing the product in each period, and the imitator segment makes up the balance $1 - \theta$. A consumer makes a purchase if the difference between valuation and purchase price is nonnegative. Accordingly, the sales in period t is

$$d_n(t) = \theta\left(1 - p_n\right)\left(M - D_n\left(t-1\right)\right) + \left(1-\theta\right)\iota D_n\left(t-1\right)\left(M - D_n\left(t-1\right)\right), \tag{3}$$

which matches the form of the classic Bass model:

$$d_n(t) = a\left(M - D_n\left(t-1\right)\right) + b\left(\frac{D_n\left(t-1\right)}{M}\right)\left(M - D_n\left(t-1\right)\right)$$

where $a = \theta(1 - p_n)$ is the coefficient of external influence (e.g., individual conversion ratio in the absence of adopter's influence) and $b = (1 - \theta)\iota M$ is the coefficient of internal influence (e.g., effect of each adopter on each non-adopter) (Jeuland 1981). We approach the new product sales process as a two-step flow of information between two segments. This approach is consistent with marketing literature (see Van den Bulte and Joshi 2007). Figure 2 illustrates sales over time according to (3).

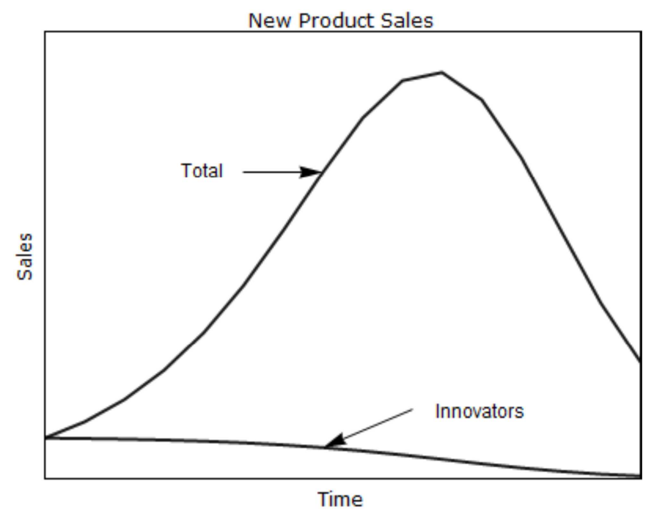

Figure 2. Illustration of data presented in Bass (1969). Sales per period with $\theta = 0.0218$, $\iota = 0.0000248$, $p_n = 0.5$, $M = 16{,}895$.These data yield the curve presented in Bass (1969) that closely matches the sales of room air conditioners between 1946 and 1961.

A product that has been used for i periods is said to be of age i. During ownership, the value of product in the eyes of the user declines with age. We let function $\upsilon(i)$ denote the valuation fraction of a product of age i, with $\upsilon(0) = 1$ and $\upsilon(i) \leq \upsilon(i - 1)$. We assume that the function $\upsilon(i)$ applies to both segments. Thus, an individual with valuation V at the time of purchase has residual valuation $V\upsilon(i)$ when the product reaches age i. The function $\upsilon(i)$ characterizes the residence index which is the ratio of the time a product is used before reaching end-of-use and the length of the product life-cycle (Georgiadis et al. 2006). The value of $V\upsilon(i)$ can also be interpreted as the disutility associated with giving up a product through a buyback program, or buyback disutility.

Assumption 1 (A1). *The valuation fraction of a product of age i,* $\upsilon(i)$, *is the same for the innovator and imitator segments*.

At time t, a customer with product of age i will return the product if the difference between the buyback price and the buyback disutility is nonnegative. The implicit assumption is that customers are not strategic in their return decision. In our setting, the firm sets the age-dependent buyback prices dynamically, and consequently there is little basis for a customer to predict how the age-dependent buyback price function will change in the future. The setting is consistent with the firm in the computer industry that motivates this work—a customer can go to a website, enter the product and condition, and receive a real-time quote of the credit for a return.

Assumption 2 (A2). *Customers are not strategic in their return decision*.

We let $c_b(t, i)$ denote the buyback price at time t for a product of age i. From (1) and (2), it follows that the probability distributions of the segment valuations in period t conditioned on a purchase transaction at time t are

$$V_1 \sim U[p_n, 1] \tag{4}$$

$$V_2 \sim U[p_n, p_n + \iota D_n(t-1)]. \tag{5}$$

Thus, the probability distributions of the residual value of a product purchased in period $t - i + 1$ that is now of age i at the end of period t are

$$V_1\upsilon(i) \sim U[p_n\upsilon(i), \upsilon(i)] \tag{6}$$

$$V_2\upsilon(i) \sim U[p_n\upsilon(i), (p_n + \iota D_n(t-i))\upsilon(i)] \tag{7}$$

and the return probabilities in response to buyback price function $c_b(t, i)$ offered at the end of period t are

$$P\left[V_1\upsilon(i) \le c_b(t,i)\right] = \min\left\{\left(\frac{c_b(t,i) - p_n\upsilon(i)}{\upsilon(i)(1-p_n)}\right)^+, 1\right\} \tag{8}$$

$$P\left[V_2\upsilon(i) \le c_b(t,i)\right] = \min\left\{\left(\frac{c_b(t,i) - p_n\upsilon(i)}{\upsilon(i)\iota D_n(t-i)}\right)^+, 1\right\} \tag{9}$$

for $i \in [1, t]$.

We next introduce expressions for the size of the install base of age i product in each segment at time t. From (3), it follows that the fraction of sales to the innovator segment in period t is

$$\theta(t) = \frac{\theta(1-p_n)}{\theta(1-p_n)+(1-\theta)\iota D_n(t-1)} \tag{10}$$

and the fraction of sales to the imitator segment in period t is $1 - \theta(t)$. The number of products of age i in the segment j install base at the end of period t is $N_{bj}(t, i)$. The number of products of age i returned in period t from segment j is $s_{bj}(t, i)$. For the purposes of defining age in functions $N_{bj}(\cdot)$ and $s_{bj}(\cdot)$, we assume that all sales occur at the beginning of the period and all returns occur at the end of the period.[1] In particular, the timing of events in each period is as follows:

<u>Start of period *t*</u>

1. Demand for new product occurs

<u>End of period *t*</u>

1. Buyback price schedule is posted and returns occur
2. Returns are remanufactured
3. Demand for remanufactured product occurs
4. Incur cost h per unit on inventory of remanufactured product

We assume that the populations of consumers who consider purchasing new products and remanufactured products are distinct. This is consistent with observations by Guide and Li (2010) who study bidding behavior of participants in eBay auctions for a consumer good (i.e., Skil jigsaw). They find that customers who bid on the new product never bid on the remanufactured version of the product, and customers who bid on the remanufactured product never bid on the new product. Feedback from each group shows awareness of both new and remanufactured versions of the product, and that the products are not perceived as substitutes.

Assumption 3 (A3). *The new product and the remanufactured product serve distinct markets—the products are not substitutes*.

A certain fraction of customers who return their product of age i in response to the buyback offer at the end of the period replace their product by purchasing a new version of the product from the firm at the beginning of the next period (i.e., customers who accept the buyback offer do not consider purchasing a remanufactured product). The repurchase fraction is a measure of customer brand loyalty that we assume is not affected by the number of periods the customer has used the product. We let γ denote this fraction. As an example, the number of products of age 1 in the install base at the end of period t is the new customer sales plus the buyback customers sales

[1] Some firms allow returns for a full refund within a specified time period after purchase for product that is in "like new" condition. These products are reintroduced to the market with minimal investment and do not enter the remanufacturing process. The volume of sales in period t, $d_n(t)$, is net of these returns.

at the beginning of period t less the buybacks at the end of period t. The buyback customer sales to segment j at the beginning of period t is the number of buybacks at the end of the previous period adjusted by the repurchase fraction, i.e.,

$$d_{bj}(t) = \gamma \sum_{k=1}^{t-1} s_{bj}(t-1,k).^{2} \tag{11}$$

for $t \geq 1$. For the computation of $d_{bj}(0, 0)$, we define $s_{bj}(0, 0) = 0$.

Assumption 4 (A4). *The repurchase rate, γ, is independent of the age of the product when returned*.

Recall that $d_n(t)$ denotes sales to new customers. Formulas for $N_{b1}(t, i)$ are illustrated below for the first few periods

$N_{b1}(1,1) = \theta(1)d_n(1) - s_{b1}(1,1)$

$N_{b1}(2,1) = \theta(2)d_n(2) + \gamma s_{b1}(1,1) - s_{b1}(2,1)$

$N_{b1}(2,2) = N_{b1}(1,1) - s_{b1}(2,2)$

$N_{b1}(3,1) = \theta(3)d_n(3) + \gamma[s_{b1}(2,1) + s_{b1}(2,2)] - s_{b1}(3,1)$

$N_{b1}(3,2) = N_{b1}(2,1) - s_{b1}(3,2)$

$N_{b1}(3,3) = N_{b1}(2,2) - s_{b1}(3,3)$,

(see Figure 3). The only differences between the $N_{b1}(t, i)$ formulas and the $N_{b2}(t, i)$ formulas are that $j = 1$ is replaced with $j = 2$ and $\theta(t)$ is replaced with $1 - \theta(t)$.

To simplify the presentation of a general expression for $N_{bj}(t, i)$, we define $N_{bj}(t, 0)$ as units that enter the segment j install base at the beginning of period $t + 1$ (equivalently, at the end of period t), which are of age 0 at the moment of entry, i.e.,

$$N_{b1}(t,0) = \theta(t+1)d_n(t+1) + d_{b1}(t+1) = \theta(t+1)d_n(t+1) + \gamma \sum_{k=1}^{t} s_{b1}(t,k)$$

$$N_{b2}(t,0) = \left[1-\theta(t+1)\right]d_n(t+1) + d_{b2}(t+1) = \left[1-\theta(t+1)\right]d_n(t+1) + \gamma \sum_{k=1}^{t} s_{b2}(t,k)$$

for $t \geq 0$. A general expression for the number of age i units in the segment j install base in period t is simply the segment j install base of age $i - 1$ product at the end of period $t - 1$ (and beginning of period t) less the number of age i units returned by segment j customers at the end of period t, i.e.,

$$N_{bj}(t, i) = N_{bj}(t-1, i-1) - s_{bj}(t, i) \text{ for } j \in \{1, 2\},\ i \in [1, t],\ t \geq 1. \tag{12}$$

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[2] The number of distinct (i.e., innovator or imitator) owners of the product does not change when a buyback credit is used for a replacement purchase. Consequently, these repeat purchases do not impact sales.

Based on the consumer choice model, the volume of age i product that is returned by each segment at the end of period t is given by

$$s_{bj}(t, i) = P\left[V_j \upsilon(i) \leq c_b(t,i)\right] N_{bj}(t-1, i-1) \text{ for } j \in \{1, 2\},\ i \in [1, t],\ t \geq 1.^{3} \tag{13}$$

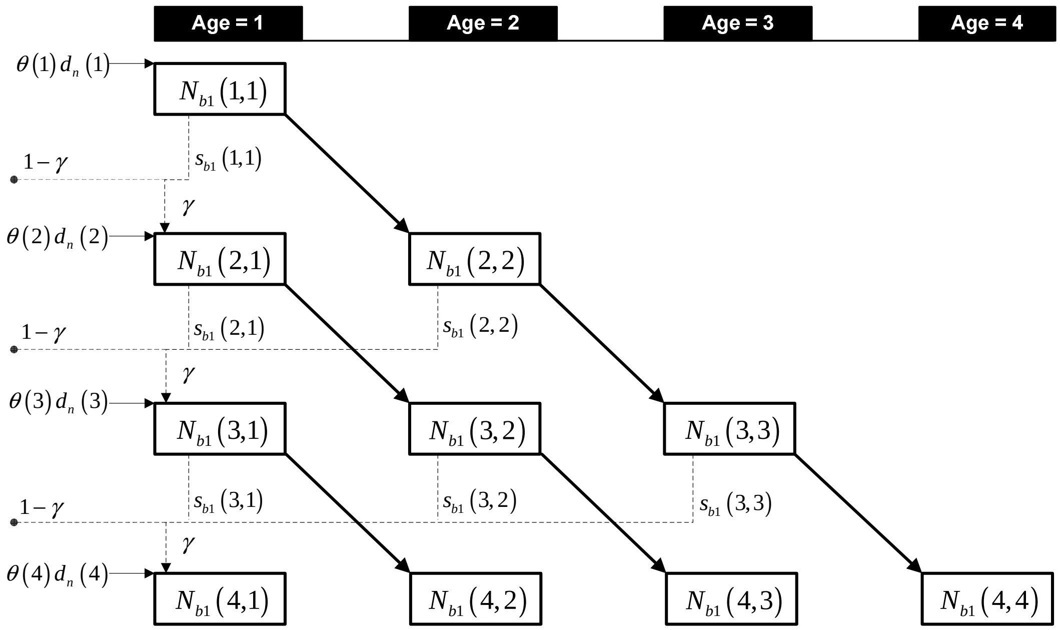

Figure 3. Illustration of the evolution of the innovator segment install base.

3.2. Demand for Remanufactured Product

Section 3.1 describes a model for the relationship between new product sales over its life-cycle and the supply of used products for remanufacturing. In this section we turn our attention to the relationship between new product sales and the demand for remanufactured products. Figure 4 shows monthly new product sales and remanufactured product demand (actual and projected) of a model of a photocopy machine between 2005 and 2012. The sales patterns in the figure are consistent with the linkage between new product sales and remanufactured product demand illustrated in Figure 1.

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3 In practice, there may be some fraction of products in the install base that cannot be returned (e.g., lost or discarded products, customers who would never consider returning product due to lack of awareness, interest, etc.). The right-hand side of (13) could be proportionally reduced to account for this effect and the following results carry through. In the interest of parsimony, we do not include this parameter in the expressions.

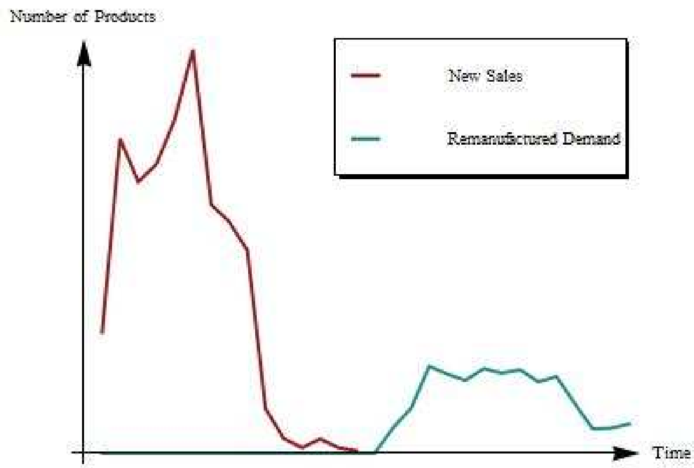

Figure 4. Relationship between new product sales of a photocopy machine and the demand for a remanufactured version of the photocopier. The time frame is 2005 –2012.

We use two parameters to specify the remanufactured product demand in terms of new product sales—a relative-size parameter α and a time-lag parameter τ. The value of α defines the size of the remanufactured product market relative to the new product market. The value of τ is the number of periods that the remanufactured product demand is behind the new product demand. In Figure 4, for example, the value of τ is about four years and the value of α is approximately 25%.

As noted above, the populations of consumers who consider purchasing new products and remanufactured products are distinct (Guide and Li 2010). The remanufactured product demand in period t is

$$d_r(t) = \alpha d_n(t - \tau) \text{ for } \tau < t \tag{14}$$

and the total sales in periods 1 through t is

$$D_r(t) = \alpha D_n(t - \tau) = \sum_{j=1}^{t} d_r(j) \text{ for } \tau < t. \tag{15}$$

Recall that in our model, demand for new product occurs at the beginning of the period, whereas returns occur at the end of the period (i.e., so that the install base at the time that buybacks take place is net of any "like new" returns to the retailer that take place shortly after purchase). We assume that demand for remanufactured product occurs at the end of the period, and thus can be satisfied using the buyback returns in the period. If the duration of a period is long, then it is possible that $\tau = 0$. However, there is typically a time lag for products to be returned (Umeda et al. 2005), which can lead to a multi-period time lag between the patterns of new and remanufactured product demand. In practice, the magnitude of the time lag may evolve over time, though in the interest of parsimony, we assume a fixed value over the duration of the life-cycle.

3.3 Remanufacturing Cost and the Myopic Buyback Problem

The net out-of-pocket costs of transforming a returned unit into a remanufactured product of age i is $c_m(i)$, is which is non-decreasing in age (i.e., age is a proxy for the condition of a returned product). This cost is comprised of the pure remanufacturing cost less a constant that reflects the profit benefit of advancing the purchase by a loyal consumer who would have purchased the product in the future if the buyback program did not exist. In other words, the value of $c_m(i)$ includes the discounted benefit associated with advancing a replacement purchase. Guide et al. (2003), for example, report that ReCellular's remanufacturing cost is convex increasing in i, where the larger the value of i, the worse the condition of the returned product. According to our notation, the total cost to acquire and remanufacture a product of age i in period t is

$$c_b(t, i) + c_m(i). \tag{16}$$

We assume that each unit acquired through the buyback program is remanufactured (e.g., some portion of each returned product is reused). Of course, once product is beyond a certain age, the value of $c_m(i)$ may reach a point where it is not profitable to remanufacture the product and the firm will not offer to buyback these products.

The total return volume in period t of product of age i is $s_b(t, i) = s_{b1}(t, i) + s_{b2}(t, i)$ or in expanded form (see (13)),

$$s_b(t, i) = \min\left\{\left(\frac{c_b(t,i) - p_n\upsilon(i)}{\upsilon(i)(1-p_n)}\right)^+, 1\right\} N_{b1}(t-1, i-1) + \min\left\{\left(\frac{c_b(t,i) - p_n\upsilon(i)}{\upsilon(i)\iota D_n(t-i)}\right)^+, 1\right\} N_{b2}(t-1, i-1). \tag{17}$$

We denote the total number of units of age i in the install base just prior to returns at the end of period t as $N_b(t, i)$, i.e.,

$$N_b(t, i) = N_{b1}(t, i) + N_{b2}(t, i). \tag{18}$$

Note that if $N_b(t-1, i-1) = 0$, then there can be no returns of age i products in period t (i.e., because there are no such products in the install base). We let $\Omega(t)$ denote the set of product ages for which returns are possible in our optimization problem, i.e., $\Omega(t) = \{i : N_b(t-1, i-1) > 0\}$.

The return volume function, $s_b(t, i)$, is constant for any $c_b(t, i) \geq \max\{\upsilon(i), \upsilon(i)[p_n + \iota D(t-i)]\}$, e.g., 100% of the age i product owned by the innovator segment is returned if $c_b(t, i) \geq \upsilon(i)$, and 100% of the age i product owned by the imitator segment is returned if $c_b(t, i) \geq \upsilon(i)[p_n + \iota D(t - i)]$. For any age $i \in \Omega(t)$, we see from (17) that $s_b(t, i)$ is strictly increasing over the range of viable buyback prices

$$c_b(t, i) \in \Omega_{c_b}(t,i) \equiv \left[p_n \upsilon(i), \upsilon(i) \max\{1, p_n + \iota D_n(t-i)\} \right], \; i \in \Omega(t). \tag{19}$$

Thus, we can invert (17) to express buyback price as a function of volume over the corresponding range of viable volumes, denoted $\Omega_{s_b}(t,i)$ for product of age i, i.e.,

$$\Omega_{s_b}(t,i) \equiv \left[0, N_b(t-1, i-1)\right] \tag{20}$$

If $1 \geq p_n + \iota D(t - i)$, we say that the age i cost structure conforms to Regime 1; otherwise the age i cost structure conforms to Regime 2. If Regime 1 applies, then at $c_b(t, i) = \upsilon(i)[p_n + \iota D(t - i)]$, 100% of age i product owned by the imitator segment is returned and the total return volume is

$$A_{b1}(t, i) = \left(\frac{\iota D_n(t-i)}{1 - p_n} \right) N_{b1}(t-1, i-1) + N_{b2}(t-1, i-1) \tag{21}$$

(obtained by substituting $c_b(t, i) = \upsilon(i)[p_n + \iota D(t - i)]$ into (17)). At $c_b(t, i) = \upsilon(i)$, the entire age i install base is returned.

If Regime 2 applies, then at $c_b(t, i) = \upsilon(i)$, 100% of age i product owned by the innovator segment is returned and the total return volume is

$$A_{b2}(t, i) = N_{b1}(t-1, i-1) + \left(\frac{1 - p_n}{\iota D_n(t-i)} \right) N_{b2}(t-1, i-1) \tag{22}$$

(obtained by substituting $c_b(t, i) = \upsilon(i)$ into (17)). At $c_b(t, i) = \upsilon(i)[p_n + \iota D(t - i)]$, the entire age i install base is returned. Inverting (17), the buyback price under Regime $k \in \{1, 2\}$ is

$$c_b(t, i) = \begin{cases} a_b(t,i) s_b(t,i) + b_b(t,i), \; s_b(t,i) \in \left[0, A_{bk}(t,i)\right] \\ a_{bk}(t,i) s_b(t,i) + b_{bk}(t,i), \; s_b(t,i) \in \left[A_{bk}(t,i), N_b(t-1, i-1)\right] \end{cases} \tag{23}$$

where

$$a_b(t, i) = \frac{\upsilon(i)(1 - p_n)\iota D_n(t-i)}{(1 - p_n) N_{b2}(t-1, i-1) + \iota D_n(t-i) N_{b1}(t-1, i-1)}$$

$$b_b(t, i) = p_n \upsilon(i)$$

$$a_{b1}(t, i) = \frac{\upsilon(i)(1 - p_n)}{N_{b1}(t-1, i-1)} \qquad b_{b1}(t, i) = \upsilon(i)\left(p_n - \frac{(1 - p_n) N_{b2}(t-1, i-1)}{N_{b1}(t-1, i-1)} \right)$$

$$a_{b2}(t,i) = \frac{\upsilon(i)\iota D_n(t-i)}{N_{b2}(t-1,i-1)} \qquad b_{b2}(t,i) = \upsilon(i)\left(p_n - \frac{\iota D_n(t-i)N_{b1}(t-1,i-1)}{N_{b2}(t-1,i-1)}\right).$$

The value of $b_b(t, i)$ is the upper limit of the buyback price $c_b(t, i)$ for which there is no return volume (i.e., $c_b(t, i)$ must be above $b_b(t, i)$ in order for some customers to accept the buyback offer). The value of $a_b(t, i)$ is the increase in the buyback price required to generate an additional unit in return volume once the buyback price passes the threshold value $b_b(t, i)$. Depending on the regime, the slope of the buyback price function shifts to either $a_{b1}(t, i)$ or $a_{b2}(t, i)$ once all customers in a segment with age i product have accepted the buyback offer. The profit due to buyback decisions in period t is

$$\Pi_b(t) = \sum_{i\in\Omega(t)} \left(p_r - c_b(t,i) - c_m(i)\right)s_b(t,i). \tag{24}$$

The value of $p_r - c_b(t, i) - c_m(i)$ is the profit on each unit that is returned and sold as a remanufactured product.

The myopic pricing/acquisition problem treats each period independently. We assume that unmet demand in a period is not backordered. The optimal remanufactured product profit in period t is

$$\Pi_b^m(t) = \max_{s_b(t,i)\in\Omega_{s_b}(t,i), i\in\Omega(t)} \left\{\Pi_b(t) \mid \sum_{i\in\Omega(t)} s_b(t,i) \le d_r(t)\right\}. \tag{25}$$

By substituting (23) into (24), we see that $\Pi_b(t)$ is a piecewise concave function that is separable in the decision vector $\mathbf{s_b}(t) = (s_b(t, 1), \ldots, s_b(t, t))$. The marginal profit associated with age i product under Regime $k\in \{1, 2\}$ is

$$\frac{\partial \Pi_b(t)}{\partial s_b(t,i)} = \begin{cases} p_r - c_m(i) - b_b(t,i) - 2a_b(t,i)s_b(t,i), & s_b(t,i)\in\left[0, A_{bk}(t,i)\right) \\ p_r - c_m(i) - b_{bk}(t,i) - 2a_{bk}(t,i)s_b(t,i), & s_b(t,i)\in\left(A_{bk}(t,i), N_b(t-1,i-1)\right] \end{cases}. \tag{26}$$

Assumption 5 (A5). *Unsatisfied demand in a period results in a lost sale* (*i.e., no backorders*).

The profit function is non-differentiable at $s_b(t, i) = A_{bk}(t, i)$. However, as we show below, the marginal profit at $s_b(t, i) = A_{bk}(t, i)^-$ is not less than the marginal profit at $s_b(t, i) = A_{bk}(t, i)^+$. We can use this fact to specify a simple greedy algorithm that solves (25).

Observe that

$$a_b(t, i) \le a_{bk}(t, i) \text{ for } k\in \{1, 2\} \text{ and } i\in \{1, \ldots, t\}. \tag{27}$$

The profit contribution of age i product under Regime $k\in \{1, 2\}$, denoted $\Pi_b(t, t, s_b(t, i))$, is piecewise continuous, and at $s_b(t, i) = A_{bk}(t, i)$ we have

$$\Pi_b(t, t, A_{bk}(t, i)) = A_{bk}(t, i)[p_r - c_m(i) - b_b(t, i) - a_b(t, i)A_{bk}(t, i)]$$

$$= A_{bk}(t, i)[p_r - c_m(i) - b_{bk}(t, i) - a_{bk}(t, i)A_{bk}(t, i)]$$

which implies

$$p_r - c_m(i) - b_b(t,i) - a_b(t,i)A_{bk}(t,i) = p_r - c_m(i) - b_{bk}(t,i) - a_{bk}(t,i)A_{bk}(t,i). \quad (28)$$

Expression (27) says that the slope of the buyback price function is lower when there are age i products owned by both segments than when one segment has returned all age i product (e.g., each additional unit of return volume requires a lower increase in buyback price when the install base includes both segments). As noted above, the profit expression is a piecewise continuous function comprised of two curves that intersect at return volume $s_b(t, i) = A_{bk}(t, i)$, yielding (28). Thus, from (27) and (28), it follows that

$$p_r - c_m(i) - b_b(t,i) - 2a_b(t,i)A_{bk}(t,i) \geq p_r - c_m(i) - b_{bk}(t,i) - 2a_{bk}(t,i)A_{bk}(t,i) \quad (29)$$

An optimization algorithm for solving (25) begins by ranking the values of $\left.\frac{\partial \Pi_b(t)}{\partial s_b(t,i)}\right|_{s_b(t,i)=0}$ from largest-to-smallest for $i \in \Omega(t)$. Of course, the value of $\frac{\partial \Pi_b(t)}{\partial s_b(t,i)}$ changes as return volume is allocated to $s_b(t, i)$. The algorithm tracks marginal profit and allocates volume in a manner that maximizes the increase in profit per unit increase in return volume. Volume is allocated as long as marginal profit is positive and the constraint $\sum_{i \in \Omega(t)} s_b(t,i) \leq d_r(t)$ is satisfied. The algorithm steps are described below (see the appendix for details on the implementation of Step 3). We suppress the parameter t in our description.

<u>Optimal Algorithm for the Myopic Buyback Problem</u>

1. Initialize the decision vector, $\mathbf{s_b} = (0, \ldots, 0)$
2. From among the viable ages (contained in Ω), identify the age(s) with the maximum marginal profit, say age J.
3. Add volume to $s_b(J)$ in an amount that is the minimum of 5 values: (1) quantity that results in marginal profit of age J to equal the second-highest marginal profit, (2) quantity at which $s_b(J) = A_{bk}(J)$, (3) quantity at which $s_b(J) = N_b(J - 1)$, (4) quantity at which the marginal profit of age J is zero (or negative), (5) quantity at which total return volume is equal to demand d_r.
4. If the quantity added to $s_b(J)$ is equal to the value given in either (4) or (5), then exit.
5. If the quantity added to $s_b(J)$ is equal to the value given in (3), then remove J from the viable age set Ω.
6. Go to step 2.

3.4 Proactive Buyback Problem

The previous section solves a myopic optimization problem. In this section we consider the problem of setting a buyback price schedule in the current period so as to maximize profit over the duration of the product life-cycle. In effect, this requires determining the buyback price schedule in each future period, though in practice, such future period price schedules would be finalized once the period is reached based on the sales history and demand projections available at that time.

Myopic buyback pricing has the advantage of being relatively simple. One objective of our analysis of myopic and proactive pricing problems is to identify conditions under which myopic buyback pricing is nearly optimal and far from optimal. Building on the comment at the end of the preceding paragraph, it is worth emphasizing that we assume known and deterministic behavior throughout the life-cycle. In practice, there are market uncertainties that work against the power of a proactive pricing policy, and thus our comparative results present a conservative picture of the attractiveness of a myopic pricing policy.

We let T denote the last period in the remanufactured product life-cycle. The particular value of T is a management decision that we assume to be exogenous to our problem, though period T would typically be in the decline stage of the life-cycle, i.e., $T > t^*$ where t^* denotes the period of peak sales for the remanufactured product. For example, in the continuous-time analog of our Bass diffusion model of demand, remanufactured product sales reaches its peak in period

$$t^* = \left(\frac{1}{\theta(1-p_n)+(1-\theta)\iota M}\right)\ln\left(\frac{(1-\theta)\iota M}{\theta(1-p_n)}\right)+\tau \tag{30}$$

(obtained from the first-order condition). The first term in (30) is the period of peak sales for new product, which is advanced by τ periods to yield the period of peak demand for the remanufactured version of the product.

The problem is to maximize total discounted profit over the planning horizon of T periods. We define the problem by presenting a math programming formulation below. The formulation conveys the problem in a relatively simple and clear manner, but introduces a non-smooth objective function. In the appendix, we present an alternative equivalent formulation that yields a smooth objective function, but is more complex.

We assume that the firm remanufactures the unit in the period in which it is returned. We make this assumption in order to simplify the problem while still capturing the essence of the anticipatory buyback pricing problem. Without this assumption, our formulation would need to track inventory by condition and we would need to assure that units in inventory are remanufactured in order of best-to-worst condition (i.e., from smallest-to-largest $c_m(i)$).

Assumption 6 (A6). *The firm remanufactures the unit in the period in which it is returned*.

Math Programming Formulation

We require the following additional notation:

h = inventory holding cost per unit-period for remanufactured product

r = net discount rate, e.g., cost of capital less inflation

$x(t)$ = sales of remanufactured product in period t

$I(t)$ = inventory of remanufactured product at the end of period t, with $I(0) = 0$

The problem is

$$\Pi_b^p = \max \sum_{t=1}^{T} (1-r)^t \, \Pi_b(t)$$

where

$$\Pi_b(t) = p_r x(t) - hI(t) - \sum_{i=1}^{t} \left(c_b(t,i) + c_m(i)\right) s_b(t,i) \tag{31}$$

subject to

$$N_{bj}(t,i) + s_{bj}(t,i) - N_{bj}(t-1,i-1) = 0,\ i \in \{1,\ldots,t\},\ j \in \{1,2\}, t \in \{1,\ldots,T\} \tag{32}$$

$$I(t) + x(t) - \sum_{i=1}^{t} s_b(t,i) - I(t-1) = 0,\ t \in \{1,\ldots,T\} \tag{33}$$

$$x(t) \le d_r(t),\ t \in \{1,\ldots,T\} \tag{34}$$

$$s_{b1}(t,i) - \min\left\{ \left(\frac{c_b(t,i) - p_n \upsilon(i)}{\upsilon(i)(1-p_n)} \right)^{+}, 1 \right\} N_{b1}(t-1,i-1) = 0,\ i \in \{1,\ldots,t\}, t \in \{1,\ldots,T\} \tag{35}$$

$$s_{b2}(t,i) - \min\left\{ \left(\frac{c_b(t,i) - p_n \upsilon(i)}{\upsilon(i) \iota D_n(t-i)} \right)^{+}, 1 \right\} N_{b2}(t-1,i-1) = 0,\ i \in \{1,\ldots,t\}, t \in \{1,\ldots,T\} \tag{36}$$

$$s_b(t,i), N_{bj}(t,i), x(t), I(t) \ge 0,\ i \in \{1,\ldots,t\},\ j \in \{1,2\}, t \in \{1,\ldots,T\} \tag{37}$$

Constraints defined in (32) are the flow-balance constraints for the install base of each age and segment over time. Constraints defined in (33) are flow-balance constraints for remanufactured product inventory. Constraints defined in (34) state that sales can be no more than demand. Constraints defined in (35) and (36) specify returns by segment given the buyback price $c_b(t, i)$, as determined by total return volume $s_b(t, i)$. Constraints defined in (37) enforce nonnegativity of the decision variables. The expressions for $c_b(t, i)$ depend on the regime, and are given below (see (23)):

if $p_n + \iota D_n(t-i) \le 1$, then

$$c_b(t,i) = \begin{cases} \upsilon(i)\left(\dfrac{(1-p_n)\iota D_n(t-i)s_b(t,i)}{(1-p_n)N_{b2}(t-1,i-1)+\iota D_n(t-i)N_{b1}(t-1,i-1)}+p_n\right), \\ \qquad \text{if } s_b(t,i) \in \left[0, \left(\dfrac{\iota D_n(t-i)}{1-p_n}\right)N_{b1}(t-1,i-1)+N_{b2}(t-1,i-1)\right] \\ \upsilon(i)\left(\dfrac{(1-p_n)s_b(t,i)}{N_{b1}(t-1,i-1)}+p_n-\dfrac{(1-p_n)N_{b2}(t-1,i-1)}{N_{b1}(t-1,i-1)}\right), \\ \qquad \text{if } s_b(t,i) \in \left[\begin{array}{l}\left(\dfrac{\iota D_n(t-i)}{1-p_n}\right)N_{b1}(t-1,i-1)+N_{b2}(t-1,i-1), \\ N_{b1}(t-1,i-1)+N_{b2}(t-1,i-1)\end{array}\right] \end{cases}$$

if $p_n + \iota D_n(t-i) \ge 1$, then

$$c_b(t,i) = \begin{cases} \upsilon(i)\left(\dfrac{(1-p_n)\iota D_n(t-i)s_b(t,i)}{(1-p_n)N_{b2}(t-1,i-1)+\iota D_n(t-i)N_{b1}(t-1,i-1)}+p_n\right), \\ \qquad \text{if } s_b(t,i) \in \left[0, N_{b1}(t-1,i-1)+\left(\dfrac{1-p_n}{\iota D_n(t-1)}\right)N_{b2}(t-1,i-1)\right] \\ \upsilon(i)\left(\dfrac{\iota D_n(t-i)s_b(t,i)}{N_{b2}(t-1,i-1)}+p_n-\dfrac{\iota D_n(t-i)N_{b1}(t-1,i-1)}{N_{b2}(t-1,i-1)}\right), \\ \qquad \text{if } s_b(t,i) \in \left[\begin{array}{l}N_{b1}(t-1,i-1)+\left(\dfrac{1-p_n}{\iota D_n(t-i)}\right)N_{b2}(t-1,i-1), \\ N_{b1}(t-1,i-1)+N_{b2}(t-1,i-1)\end{array}\right] \end{cases}$$

3.5 Numerical Illustrations

This section compares the performance of myopic and proactive solutions through a few numerical illustrations. Note that the cost to acquire and remanufacture a unit of age *i* product from a segment 1 customer is in the following range:

$$\upsilon(i)p_n + c_m(i) < c_b(t, i) + c_m(i) \le \upsilon(i) + c_m(i). \tag{38}$$

The range follows from the fact that the range of valuations at the time of purchase is in the interval $[p_n, 1]$ (see (4)). Thus, the buyback price for age *i* must be at least $\upsilon(i)p_n$ before a segment 1 customer will accept the offer, and all segment 1 customers will accept the offer at buyback price $c_b(t, i) = \upsilon(i)$. Similarly, the cost to acquire and remanufacture a unit of age *i* from a segment 2 customer is in the following range:

$$\upsilon(i)p_n + c_m(i) < c_b(t, i) + c_m(i) \le \upsilon(i)[p_n + \iota D(t-1)] + c_m(i). \tag{39}$$

For given functions $\upsilon(i)$ and $c_m(i)$, we can examine how the lower and upper limits of the ranges given in (38) and (39) vary as a function of i. And for certain functional forms, there may exist a relatively narrow range of ages associated with lowest acquisition and remanufacturing cost, e.g., a sweet spot defining a band of ages that are most cost effective. Guide et al. (2003), for example, report ReCellular's cost and acquisition percentage data by condition (see Table 3 in Guide et al. 2003). The ReCellular cost data show a distinctive sweet spot. In settings with a pronounced sweet spot, say i^*, we find that the difference between i^* and the time lag τ is an indicator of the relative performance between a myopic solution and a proactive solution. In particular, when i^* is close to τ, then the profit of the myopic solution is close to the profit of the proactive solution. As these parameter values get farther apart, the use of a proactive solution method in place of the myopic algorithm is more likely to add value. This effect is illustrated in Table 1 below.

i^*	τ	Π_M/Π_P	$\%_M$	$\%_P$	Π_P
6	0	0.875	42%	35%	583
6	6	0.968	86%	85%	1,205
6	10	0.763	36%	50%	544

Table 1. The ratio of optimal myopic to proactive buyback profits over the remanufactured product life-cycle, the percent of remanufactured product demand satisfied by myopic and proactive buyback policies, and proactive buyback profit.

The results in Table 1 correspond to a 12 year new product diffusion process. The diffusion parameters are based on sales data from Xerox DocuTech copiers sold from 1991 to 2000 where the market size, coefficient of innovation, and coefficient of imitation are M = 38,833 units, p = 0.015 and q = 0.346 respectively (Van de Capelle 2004). Given a value $p_n = 0.5$, the values of θ = 0.3 and ι = 0.00001273 correspond with the values of p and q; however, we adjust these values and let θ = 0.15 and ι = 0.00001329. These adjustments preserve the fundamental structure of the diffusion process, and yet allow us to generate a product life-cycle of approximately 12 years, ensuring that the maximum valuation for imitator type is not more than 1. We assume $\upsilon(i)$ is a decreasing linear function that reaches zero at $i = 12$ (i.e., $\upsilon(i) = 1 - i/12$). This assumption suggests a residence index = 1, which implies the typical consumer's usage cycle matches the product life-cycle. We assume $c_m(i) = (i/12)^4$, a convex increasing function that reflects an increasing rate of deterioration over time. Finally we let $\gamma = 0.6$, $p_r = 0.4$ and $\alpha = 0.8$. These parameters and functions yield a sweet spot of about six periods. The sweet spot is evident in the lower bound curve in Figure 5, which is the cost to acquire and remanufacture the first unit at different ages. We see that the curve reaches a low point at about six periods. We used the algorithm outlined in Section 3.3 to generate the optimal myopic profits. We used *Large Scale SQP* solver to search for optimal (near optimal) solutions to the math programming formulation.

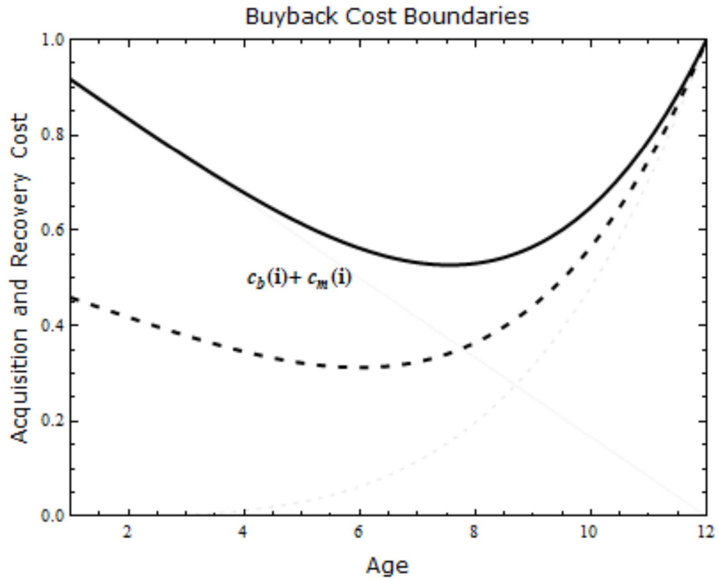

Figure 5. Upper and lower bounds on buyback acquisition and recovery cost given in (38).

The results in Table 1 show that when the time lag is aligned with the sweet spot (i.e., $\tau = i^*$), there is little difference between optimal profits under the myopic and the proactive buyback pricing policies (i.e., 97.3%). This is because when $\tau = i^*$, the availability of used products that are both affordable and suitable for cost effective remanufacturing is high. Columns 4 and 5 of Table 1 contain the percent of remanufactured demand satisfied by the myopic and proactive solutions respectively. The return volumes under the optimal myopic and proactive policies show that when the time lag matches the sweet spot, both polices satisfy roughly 83% of the demand for remanufactured product. However, the proactive solution has a slight advantage when $\tau < i^*$ and when $\tau > i^*$, but for different reasons. When $\tau < i^*$, the proactive decision maker acquires fewer products in the early stages only to acquire them later at a lower price. The myopic decision maker will acquire end-of-use returns as long as marginal cost is less than marginal profits. When $\tau = 0$, the myopic decision maker satisfies 41% of the remanufactured product demand while the proactive active decision maker only satisfies 34% of the remanufactured product demand, yet earns higher profits. When $\tau > i^*$, the opposite is true. The myopic decision maker is less able to accommodate the demand for the remanufactured product in later periods due to insufficient supply of products that are affordable to recover. On the other hand, the proactive decision maker accumulates an inventory of used products in earlier periods (e.g., where acquisition and recovery cost are low), in order to satisfy demand in later periods (e.g., where recovery costs are high). For instance, when $\tau = 10$, we see that in Table 1 the optimal myopic policy satisfies 33% of the remanufactured product demand while the proactive policy satisfies 47% of total remanufactured product demand and earns larger profits.

Importantly, Table 1 also shows that profit is highest when the sweet spot is well-aligned with the time lag. We see a manifestation of this result in the higher fraction of remanufactured product demand satisfied at $i^* = \tau$ (83%) than at $i^* \neq \tau$ (34% and 47%). The table shows the benefits of alignment between peak demand and peak supply of product with low acquisition and remanufacturing cost.

4. Trade-in Policies for a Producer of New and Remanufactured Products

In this section, we consider the relative performance of myopic and proactive trade-in acquisition policies. A firm that sells both new and remanufactured product has the option to offer a trade-in price for a return.[4] A customer who accepts the trade-in offer receives a price discount on the purchase of the new product. In the next section, we will compare the profitability of buyback and trade-in policies for an OEM/remanufacturer.

4.1. Used Product Supply, Age-Dependent Acquisition Price, and Trade-in Return Volume

Recall from Section 3.1 that the buyback disutility for a customer with age i product and valuation V at the time of purchase is $V\upsilon(i)$. This means that a customer who is offered a cash amount of $c_b(t, i)$ satisfying $c_b(t, i) \geq V\upsilon(i)$ will return his or her product. A trade-in transaction, on the other hand, has strings attached. A customer who is offered a trade-in price of $c_t(t, i)$ must purchase a new product from the firm to receive the credit, e.g., the new product purchase price on a trade-in transaction is $p_n - c_t(t, i)$. We let $\phi \in [1, \infty)$ be a measure of the consumer's perceived cost of reduced flexibility associated with a trade-in transaction relative to a buyback transaction. More precisely, ϕ is the ratio of trade-in- to-buyback disutility, which we assume to be independent of product's age. Thus, at time t, a customer with product of age i will accept the trade-in offer and return the product if $c_t(t, i) \geq \phi V\upsilon(i)$.

Assumption 7 (A7). *The ratio of trade-in- to-buyback disutility,* ϕ*, is independent of product age*.

Recall from Section 3.1 that γ is the fraction of buyback customers with product of age i who prefer to replace their returned product with a new product from the firm. The parameters ϕ and γ are related. For example, if $\gamma = 1$, then a trade-in transaction offers no disadvantage relative to a

[4]The more common terms are trade-in *discount* or trade-in *credit*. We use *price* to be consistent with buyback terminology, which simplifies our wording later on when we discuss these policies together.

buyback transaction, and $\phi = 1$; customers prefer to use the cash from a buyback transaction to purchase a new product from the firm, i.e.,

$$\gamma = 1 \Rightarrow \phi = 1 \tag{40}$$

In general, the values of ϕ and γ are inversely related, e.g., a product with a large value of $1 - \gamma$ is likely to have a large value of $\phi - 1$. A small value of γ (or large value of $1 - \gamma$) means that few customers prefer to replace their old product with a new product from the firm, which implies a high value ϕ, i.e., a high cost of the reduced flexibility associated with a trade-in transaction relative to a buyback transaction.

As one may suspect (and as will be shown below), the buyback and trade-in profit expressions are equivalent when customers are very loyal to the firm (i.e., $\gamma = \phi = 1$). In essence, when $\gamma = 1$, customers perceive no difference between a buyback credit and a trade-in credit. Differences in the relative performance of buyback and trade-in programs arise in settings where $\gamma < 1$.

From the valuation probability distributions associated with customers who purchased the product (see (4) and (5)), it follows that

$$\phi V_1 \upsilon(i) \sim U[\phi p_n \upsilon(i),\ \phi\upsilon(i)] \tag{41}$$

$$\phi V_2 \upsilon(i) \sim U[\phi p_n \upsilon(i),\ \phi(p_n + \iota D_n(t-i))\upsilon(i)] \tag{42}$$

and the return probabilities in response to trade-in price function $c_t(t, i)$ offered at the end of period t are

$$P\left[\phi V_1 \upsilon(i) \le c_t(t,i)\right] = \min\left\{\left(\frac{c_t(t,i) - \phi p_n \upsilon(i)}{\phi\upsilon(i)(1-p_n)}\right)^+, 1\right\} \tag{43}$$

$$P\left[\phi V_2 \upsilon(i) \le c_t(t,i)\right] = \min\left\{\left(\frac{c_t(t,i) - \phi p_n \upsilon(i)}{\phi\upsilon(i)\iota D_n(t-i)}\right)^+, 1\right\} \tag{44}$$

for $i \in [1, t]$.

The number of products of age i in the segment j install base at the end of period t is $N_{tj}(t, i)$. The number of products of age i returned in period t from segment j is $s_{tj}(t, i)$. The expressions for $N_{tj}(t, i)$ are similar to the expressions for $N_{bj}(t, i)$. The only difference stems from the fact that each trade-in transaction is associated with the purchase of a new product from the firm, whereas only fraction γ of buybacks result in the purchase of a new product from the firm. Replacing γ with 1 in (11) yields the trade-in customer sales to segment j at the beginning of period t, i.e.,

$$d_{tj}(t) = \sum_{k=1}^{t-1} s_{tj}\left(t-1,k\right). \tag{45}$$

for $t \geq 1$. For the computation of $d_{tj}(0, 0)$, we define $s_{tj}(0, 0) = 0$.

Recall that $d_n(t)$ denotes sales to new customers in period t. Following Section 3.1, we define

$$N_{t1}(t, 0) = \theta\left(t+1\right)d_n\left(t+1\right)+d_{t1}\left(t+1\right)=\theta\left(t+1\right)d_n\left(t+1\right)+\sum_{k=1}^{t}s_{t1}\left(t,k\right)$$

$$N_{t2}(t, 0) = \left[1-\theta\left(t+1\right)\right]d_n\left(t+1\right)+d_{t2}\left(t+1\right)=\left[1-\theta\left(t+1\right)\right]d_n\left(t+1\right)+\sum_{k=1}^{t}s_{t2}\left(t,k\right)$$

for $t \geq 0$, and the general expression for the install base is

$$N_{tj}(t, i) = N_{tj}(t-1, i-1) - s_{tj}(t, i) \text{ for } j \in \{1, 2\} \text{ and } i \in [1, t],\ t \geq 1. \tag{46}$$

The volume of age i product that is returned by each segment at the end of period t is given by

$$s_{tj}(t, i) = P\left[\phi V_j \upsilon\left(i\right) \leq c_t\left(t,i\right)\right]N_{tj}\left(t-1,i-1\right) \text{ for } j \in \{1, 2\},\ i \in [1, t],\ t \geq 1. \tag{47}$$

4.2 Myopic Trade-in Problem

The total return volume in period t of product of age i is $s_t(t, i) = s_{t1}(t, i) + s_{t2}(t, i)$, or in expanded form (see (47)),

$$s_t(t, i) = \min\left\{\left(\frac{c_t\left(t,i\right)-p_n\phi\upsilon\left(i\right)}{\phi\upsilon\left(i\right)\left(1-p_n\right)}\right)^{+},1\right\}N_{t1}\left(t-1,i-1\right)+$$

$$\min\left\{\left(\frac{c_t\left(t,i\right)-p_n\phi\upsilon\left(i\right)}{\phi\upsilon\left(i\right)\iota D_n\left(t-i\right)}\right)^{+},1\right\}N_{t2}\left(t-1,i-1\right). \tag{48}$$

We denote the total number of units of age i in the install base just prior to returns at the end of period t as $N_t(t, i)$, i.e.,

$$N_t(t, i) = N_{t1}(t, i) + N_{t2}(t, i). \tag{49}$$

Recall from Section 3.3 that $\Omega(t)$ is the set of ages at the end of period t for which returns are viable, i.e., $\Omega(t) = \{i : N_t(t, i) > 0\}$. The return volume function, $s_t(t, i)$, is constant for any $c_t(t, i) \geq \max\{\phi\upsilon(i), \phi\upsilon(i)[p_n + \iota D(t-i)]\}$, e.g., 100% of the age i product owned by the innovator segment is returned if $c_t(t, i) \geq \phi\upsilon(i)$, and 100% of the age i product owned by the imitator segment is returned if $c_t(t, i) \geq \phi\upsilon(i)[p_n + \iota D(t-i)]$. For any age $i \in \Omega(t)$, we see from (48) that $s_t(t, i)$ is strictly increasing over the range of viable trade-in prices

$$c_t(t, i) \in \Omega_{c_t}\left(t,i\right) \equiv \left[p_n\phi\upsilon\left(i\right), \phi\upsilon\left(i\right)\max\left\{1, p_n+\iota D_n\left(t-i\right)\right\}\right],\ i \in \Omega(t). \tag{50}$$

Thus, we can invert (48) to express trade-in price as a function of volume over the corresponding range of viable volumes, denoted $\Omega_{s_t}(t,i)$ for product of age i, i.e.,

$$\Omega_{s_t}(t,i) \equiv \left[0, N_t(t-1,i-1)\right]. \tag{51}$$

If $1 \geq p_n + \iota D(t-i)$, we say that the age i cost structure conforms to Regime 1; otherwise the age i cost structure conforms to Regime 2. If Regime 1 applies, then at $c_t(t, i) = \phi\upsilon(i)[p_n + \iota D(t-i)]$, 100% of age i product owned by the imitator segment is returned and the total return volume is

$$A_{t1}(t, i) = \left(\frac{\iota D_n(t-i)}{1-p_n}\right) N_{t1}(t-1,i-1) + N_{t2}(t-1,i-1) \tag{52}$$

(obtained by substituting $c_t(t, i) = \phi\upsilon(i)[p_n + \iota D(t-i)]$ into (48); see (21)). At $c_t(t, i) = \phi\upsilon(i)$, the entire age i install base is returned.

If Regime 2 applies, then at $c_t(t, i) = \phi\upsilon(i)$, 100% of age i product owned by the innovator segment is returned and the total return volume is

$$A_{t2}(t, i) = N_{t1}(t-1,i-1) + \left(\frac{1-p_n}{\iota D_n(t-i)}\right) N_{t2}(t-1,i-1) \tag{53}$$

(obtained by substituting $c_t(t, i) = \phi\upsilon(i)$ into (48); see (22)). At $c_t(t, i) = \phi\upsilon(i)[p_n + \iota D(t-i)]$, the entire age i install base is returned. Inverting (48), the trade-in price under Regime $k \in \{1, 2\}$ is

$$c_t(t, i) = \begin{cases} a_t(t,i)s_t(t,i) + b_t(t,i), & s_t(t,i) \in \left[0, A_{tk}(t,i)\right] \\ a_{tk}(t,i)s_t(t,i) + b_{tk}(t,i), & s_t(t,i) \in \left[A_{tk}(t,i), N_t(t-1,i-1)\right] \end{cases} \tag{54}$$

where

$$a_t(t, i) = \frac{\phi\upsilon(i)(1-p_n)\iota D_n(t-i)}{(1-p_n)N_{t2}(t-1,i-1) + \iota D_n(t-i)N_{t1}(t-1,i-1)}$$

$$b_t(t, i) = p_n\phi\upsilon(i)$$

$$a_{t1}(t, i) = \frac{\phi\upsilon(i)(1-p_n)}{N_{t1}(t-1,i-1)} \qquad b_{t1}(t, i) = \phi\upsilon(i)\left(p_n - \frac{(1-p_n)N_{t2}(t-1,i-1)}{N_{t1}(t-1,i-1)}\right)$$

$$a_{t2}(t, i) = \frac{\phi\upsilon(i)\iota D_n(t-i)}{N_{t2}(t-1,i-1)} \qquad b_{t2}(t, i) = \phi\upsilon(i)\left(p_n - \frac{\iota D_n(t-i)N_{t1}(t-1,i-1)}{N_{t2}(t-1,i-1)}\right).$$

Recall that $c_m(i)$ is the remanufacturing cost less γm, which is discounted profit associated with advancing a replacement purchase by loyal customers; the value of $\gamma \in [0, 1]$ represents the repurchase rate and the value of m represents the margin on a new product sale. A trade-in program requires that a consumer replace his returned product with a new product from the from, and thus relative to a buyback transaction, generates an additional $(1-\gamma)m$ in profit contribution. Accordingly, the profit in period t is

$$\Pi_t(t)=\sum_{i\in\Omega(t)}\left(p_r+(1-\gamma)m-c_t(t,i)-c_m(i)\right)s_t(t,i). \tag{55}$$

The optimal remanufactured product profit in period t is

$$\Pi_t^m(t)=\max_{s_t(t,i)\in\Omega_{s_t}(t,i),i\in\Omega(t)}\left\{\Pi_t(t)\,|\sum_{i\in\Omega(t)}s_t(t,i)\le d_r(t)\right\}. \tag{56}$$

$\Pi_t(t)$ is a piecewise concave function that is separable in the decision vector $\mathbf{s_t}(t)=(s_t(t,1),\ldots,s_t(t,t))$. The marginal profit associated with age i product under Regime $k\in\{1,2\}$ is

$$\frac{\partial\Pi_t(t)}{\partial s_t(t,i)}=\begin{cases}p_r+(1-\gamma)m-c_m(i)-b_t(t,i)-2a_t(t,i)s_t(t,i), & s_t(t,i)\in\left[0,A_{tk}(t,i)\right)\\ p_r+(1-\gamma)m-c_m(i)-b_{tk}(t,i)-2a_{tk}(t,i)s_t(t,i), & s_t(t,i)\in\left(\begin{matrix}A_{tk}(t,i),\\ N_t(t-1,i-1)\end{matrix}\right]\end{cases}. \tag{57}$$

The problem structure follows the structure of the myopic buyback problem. The steps of an algorithm to solve problem (56) steps are essentially identical to the buyback algorithm, except that subscript b is replaced with t.

4.3 Numerical Illustrations

The proactive trade-in problem can be formulated in a manner similar to the proactive buyback trade-in problem (see the appendix for details). This section compares the performance of myopic and proactive solutions through a few numerical illustrations. The cost to acquire and remanufacture a unit of age i from a segment 1 customer is in the following range:

$$\phi\upsilon(i)p_n+c_m(i)<c_t(t,i)+c_m(i)\le\phi\upsilon(i)+c_m(i). \tag{58}$$

Similarly, the cost to acquire and remanufacture a unit of age i from a segment 2 customer is in the following range:

$$\phi\upsilon(i)p_n+c_m(i)<c_t(t,i)+c_m(i)\le\phi\upsilon(i)[p_n+\iota D_n(t-1)]\quad+c_m(i). \tag{59}$$

The structures of (58) and (59) follow the structures observed in (38) and (39) under analysis of a buyback policy (i.e., the only difference is the inclusion of factor ϕ). Figure 6 illustrates the upper and lower bounds on the trade-in credits and product recovery cost. We see that the sweet spot is approximately six years. The figure is based on the same parameter values and functions used to create Figure 5, except the required additional term ϕ is set to 1.4.

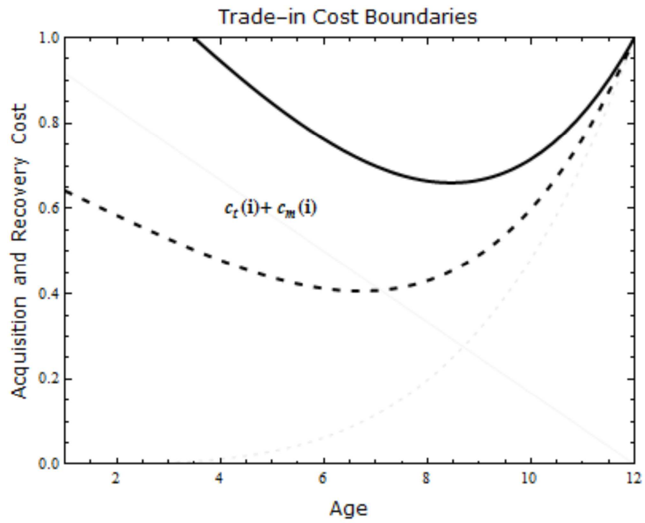

Figure 6. Upper and lower bounds on trade-in acquisition and recovery cost given in (58).

Table 2 shows how the differences between i^* and the time lag τ affects the relative performance of myopic and proactive policies.

i^*	τ	Π_M/Π_P	$\%_M$	$\%_P$	Π_P
6	0	0.866	43%	35%	1,035
6	6	0.966	100%	100%	2,845
6	10	0.624	53%	91%	2,093

Table 2. The ratio of optimal myopic to proactive trade-in profits over the remanufactured product life-cycle, the percent of remanufactured product demand satisfied by myopic and proactive trade-in policies, and proactive trade-in profit.

The results in Table 2 are based on $\phi = 1.4$ and $m = 0.4$ with the remaining parameter values set to match the parameter values that are used to generate Table 1. The main conclusions are consistent with the conclusions in Section 3.5. When τ is aligned with the sweet spot i^*, there is little difference between optimal myopic trade-in profits and optimal (or near optimal) proactive trade-in profits, and total profit is highest. The proactive policy is more advantageous when the values of τ and i^* are quite different. When $\tau = 0$, for example, the proactive decision maker acquires fewer end-of-use products and satisfies less demand for the remanufactured product. In the earlier stages of the product life-cycle, demand for the remanufactured product is low and used products are costly to acquire (i.e., the residual valuations are high). As the new product matures, demand for the remanufactured product begins to pick up. When $\tau < i^*$, the proactive decision maker acquires fewer products in the earlier stages, only to acquirer them later at a lower prices. When $\tau = 10$, the opposite is true; that is, the proactive decision maker satisfies a larger portion of the remanufactured product demand. Note that when $\tau = 10$, the proactive trade-in policy is considerably more profitable than the myopic counterpart. The proactive decision maker launches take-back and recovery activities well before observing demand for the remanufactured product.

5. Comparison of Buyback and Trade-in Policies

We begin with two propositions that help characterize the relative performance among optimal (proactive) buyback and trade-in policies. For given return volumes, we let $\Pi_{b|s_b(t,i)}$ and $\Pi_{t|s_t(t,i)}$ denote the total discounted profit for buyback and trade-in programs, respectively, and we let $\bar{c}_{b|s_b(t,i)}$ denote the average discounted buyback cost per unit, i.e.,

$$\bar{c}_{b|s_b(t,i)} = \left(\sum_t (1-r)^t \sum_i c_b(t,i) s_b(t,i) \right) \Big/ \left(\sum_t (1-r)^t \sum_i s_b(t,i) \right).$$

We use superscript * to denote optimal values.

Proposition 1. *a) If* $\bar{c}_{b|s_b(t,i)} \le \left(\frac{1-\gamma}{\phi - 1} \right) m$ *, then* $\Pi_{t|s_t(t,i)=s_b(t,i)} \ge \Pi_{b|s_b(t,i)}$. *b) If* $\bar{c}_b^* \le \left(\frac{1-\gamma}{\phi - 1} \right) m$ *, then* $\Pi_t^* \ge \Pi_b^*$.

Proposition 2. *a) If* $\phi = 1$*, then* $\Pi_t^* \ge \Pi_b^*$. *b) If* $\gamma = \phi = 1$*, then* $\Pi_t^* = \Pi_b^*$.

Recall that the value of $1 - \gamma$ is the fraction of buyback customers who choose not to use their cash from the return toward the purchase of a replacement product from the firm, and that the value of $\phi - 1$ is the required percentage increase in the buyback credit to yield the same fraction of trade-in returns as buyback returns. In essence, $1 - \gamma$ is a measure of customer resistance to repurchasing from the firm and $\phi - 1$ is a measure of the cost to overcome this resistance. In settings where these measures are equal (i.e., $1 - \gamma = \phi - 1$), Proposition 1 indicates that if management has good reason to believe that the average buyback price (in current dollars) will be less than the new product margin, then a trade-in program is likely to be more profitable than a buyback program.

The simple interpretation that applies to the case of $1 - \gamma = \phi - 1$ can be generalized by using the notion of an *adjusted margin*, which is the product of margin and the resistance-to-cost ratio $\frac{1-\gamma}{\phi - 1}$. We see that a decrease in the resistance-to-cost ratio increases the relative attractiveness of a buyback program. Similarly, an increase in the resistance-to-cost ratio increases the relative attractiveness of a trade-in program. From Proposition 1a, we see that a requirement for a buyback program to dominate a trade-in problem is that the average buyback price is greater than the adjusted margin. The underlying intuition for this result is that, for the same return volume, the percentage increase in buyback price required to offset the resistance to a trade-in offer (i.e., percentage increase is $\phi - 1$) more than offsets the gain from locking in the margin from fraction $1 - \gamma$ of customers who would not purchase a new product if not for the trade-in requirement (i.e., gain is $(1 - \gamma)m$).

We emphasize that the conditions identified in Proposition 1 are sufficient, but not necessary, for the dominance of a trade-in program. A trade-in program can result in higher profit than a buyback program when the conditions do not hold. This is because the trade-in program offers the additional benefit of install base expansion due to 100% repurchase rate instead of repurchase rate $\gamma < 1$. A larger install base translates into a lower acquisition cost to acquire a given number of units (e.g., see (17)). Indeed, if we compare a buyback program to a modified trade-in model that only allows fraction γ to enter the install base, then the condition of Proposition 1a becomes a necessary and sufficient condition for dominance of a trade-in program under this modified model (e.g., see proof of Proposition 1). The additional fraction $1 - \gamma$ of returned units entering the install base increases the profitability of a trade-in program relative to the modified trade-in model.

The effects of changing values of $1 - \gamma$ and $\phi - 1$ on profits under buyback and trade-in programs are illustrated in Table 3 and Table 4. To generate the results, we used the same data used to generate tables 1 and 2, but with the time lag τ set to match the sweet spot, i.e., $\tau = i^* = 6$. We used the myopic algorithm to generate the profits.

Π_b/Π_t c_b, $\left(\frac{1-\gamma}{\phi-1}\right)m$		$\phi - 1$			
		0.2	0.4	0.6	0.8
$1 - \gamma$	0.2	0.624 0.287, 0.40	1.784 0.287, 0.20	14.014 0.287, 0.13	∞ 0.287, 0.10
	0.4	0.266 0.285, 0.80	0.425 0.285, 0.40	0.866 0.285, 0.267	2.357 0.285, 0.20
	0.6	0.163 0.284, 1.20	0.214 0.284, 0.60	0.308 0.284, 0.40	0.513 0.284, 0.30
	0.8	0.113 0.282, 1.60	0.138 0.282, 0.80	0.174 0.282, 0.533	0.231 0.282, 0.40

Table 3a. Ratios of buyback profit to trade-in profit when the remanufactured product market is large ($\alpha = 0.8$).

Π_b/Π_t c_b, $\left(\frac{1-\gamma}{\phi-1}\right)m$		$\phi - 1$			
		0.2	0.4	0.6	0.8
$1-\gamma$	0.2	0.624 0.287, 0.40	1.784 0.287, 0.20	14.014 0.287, 0.13	∞ 0.287, 0.10
	0.4	0.266 0.285, 0.80	0.425 0.285, 0.40	0.866 0.285, 0.267	2.357 0.285, 0.20
	0.6	0.163 0.284, 1.20	0.214 0.284, 0.60	0.308 0.284, 0.40	0.513 0.284, 0.30
	0.8	0.113 0.282, 1.60	0.138 0.282, 0.80	0.174 0.282, 0.533	0.231 0.282, 0.40

Table 3b. Ratios of buyback profit to trade-in profit when the remanufactured product market is small ($\alpha = 0.2$).

The analysis provides the following insights. For a given margin (m) and trade-in disutility (ϕ) the relative profitability of the buyback program would decline when $1 - \gamma$ increases (refer to Table 3a and 3b). Higher $1 - \gamma$ implies smaller replacement purchase (i.e., lesser loyalty) by the buyback customer. In contrast, for a given margin (m) and replacement purchase fraction (γ) the relative profitability of the buyback program would increase if the trade-in disutility (i.e., ϕ) becomes higher (refer to Table 3a and 3b). Finally, consistent with Proposition 1, we see that the trade-in program dominates the buyback program when the buyback price is less than $\left(\frac{1-\gamma}{\phi-1}\right)m$, and that the buyback program tends to dominate the trade-in program when the inequality doesn't hold (an exception being the combination of $1 - \gamma = 0.4$ and $1 - \phi = 0.6$; refer to Table 3a and 4a). As discussed, this exception is because of the relatively larger install base in case of trade-in program as a result of 100% repurchase rate instead of a 60% repurchase rate under a buyback program.

$\%_B$ $\%_T$		$\phi - 1$			
		0.2	0.4	0.6	0.8
$1-\gamma$	0.2	88.8 97.8	88.8 68.0	88.8 23.6	88.8 0
	0.4	86.5 100	86.5 100	86.5 86.9	86.5 56.1
	0.6	83.8 100	83.8 100	83.8 100	83.8 94.9
	0.8	79.7 100	79.7 100	79.7 100	79.7 100

Table 4a. Percent of remanufactured product demand satisfied under buyback and trade-in product acquisition policies when the remanufactured product market is large ($\alpha = 0.8$).

$\%_B$ $\%_T$		$\phi - 1$			
		0.2	0.4	0.6	0.8
$1 - \gamma$	0.2	100 100	100 100	100 85.6	100 0
	0.4	100 100	100 100	100 100	100 100
	0.6	100 100	100 100	100 100	100 100
	0.8	100 100	100 100	100 100	100 100

Table 4b. Percent of remanufactured product demand satisfied under buyback and trade-in product acquisition policies when the remanufactured product market is small ($\alpha = 0.2$).

Table 4a shows that the percent of remanufactured demand satisfied through a buyback program is larger when the replacement purchase fraction (γ) in buyback program is larger. Replacement purchases replenish the install base and a higher replacement purchase fraction means a higher supply (install base), lower buyback prices, higher buyback volume, and greater sales of remanufactured product. Table 4a also shows that the percent of remanufactured demand satisfied through a trade-in program is larger when the trade-in disutility (i.e., ϕ) is smaller (e.g., compared to high trade-in disutility, the firm can generate higher return volumes at a given price when trade-in disutility is low). Note that for fixed trade-in disutility (i.e., ϕ), the percent of remanufactured demand satisfied in trade-in program increases when $1 - \gamma$ increases. The higher incremental profit contribution relative to buyback program, $(1 - \gamma)m$, induces higher acquisition rates and a greater percentage of remanufactured demand satisfied.

6. Summary and Conclusions

This study was motivated by discussions with supply chain managers who oversee aspects of product take-back and recovery activities at several companies. These companies use take-back and recovery programs as a means to meet a demand for remanufactured products. There is a genuine interest in understanding the marketing and operational merits of various product acquisition policies. We respond to these managerial concerns by studying buyback and trade-in programs. Our analysis focuses on the time dynamics of the used product acquisition pricing problem. Our models incorporate elements of consumer willingness-to-return and replacement purchase behavior. The main results of our analyses are summarized below.

Product life-cycle dynamics influence the profitability of remanufacturing activities. The diffusion of new product sales, the consumers' depreciating valuations of the owned product, and age-dependent remanufacturing cost shape product acquisition and recovery cost. We refer to the age of a product where acquisition and recovery cost are most economical as the sweet spot. The number of used products in the install base that are at or near the sweet spot is affected by the product life-cycle dynamics. We notice that any optimal acquisition policy will target used products that are at or near the sweet spot. A firm will want to identify the sweet spot when designing buyback or trade-in policies, and use this information when specifying the model and model year of products that qualify for the take-back deal.

One of our findings relates to the relationship between the sweet spot and the time lag between new and remanufactured product introductions. We find that the firm's profit over the remanufactured product life-cycle is sensitive to the degree of alignment between the sweet spot and time lag and, most importantly, that profit is highest when the time lag is at or near the sweet spot. Furthermore, if the time lag is close to the sweet spot, then there is relatively little loss in profit from using a myopic pricing algorithm instead of a proactive pricing algorithm. Myopic pricing is much simpler to implement and is not affected by inaccurate projections of future demand. However, if there is a large mismatch between the time lag and sweet spot, then proactive pricing may generate significantly higher profits, particularly when the time lag is long relative to the sweet spot. In these settings, proactive pricing exploits a surplus of used product near the sweet spot through heavy acquisitions that are held in inventory and sold in future periods when remanufactured product demand is high. One important managerial implication from these findings arises when a firm has some flexibility on when to introduce a remanufactured version of its product to the market. Timing the introduction to align with the sweet spot affords the dual benefits of high profit potential and the opportunity to use a simple and robust pricing algorithm.

We also study the relative performance of buyback and trade-in acquisition policies. Our analysis accounts for replacement purchase rates and consumer preferences for buyback cash and trade-in credits. We find that a rough rule of thumb for selecting between buyback and trade-in programs is to compare the (anticipated) average buyback price with the new product margin. If the price is not more than the margin, then a trade-in policy dominates a buyback policy. This simple rule of thumb can be refined by using an adjusted margin—the margin multiplied by a resistance-to-cost ratio. The resistance-to-cost ratio is the consumer resistance to buying a new product from the firm divided by the percentage increase in a buyback price required for a consumer to be indifferent between buyback and trade-in alternatives. A sufficient condition for the dominance of trade-in program is an average buyback price that is less than or equal to the adjusted margin. We note that a trade-in program has an important secondary advantage over a buyback program that can cause the trade-in program to be more profitable than a buyback program even when the inequality noted above does not hold—this is due to the benefit of increased supply of used product, and consequent lower acquisition costs, by requiring trade-in customers to repurchase the firm's product.

Regarding the generality and the applicability of our results, we assume the residual valuation function for innovator and imitator market segments is the same. The residual valuation function reflects how a consumer's perception of the value of the owned good changes over time. This function is a proxy for voluntary replacement decision which could depend on several factors, such as the owner's usage rate (Raymond et. al. 1993), desires for newness, and situational factors such as storage space and alternative uses (Jacoby et. al. 1977). These studies suggest that owners are, to some degree, heterogeneous with respect to their residual valuation of the used product. A model that captures heterogeneity in the residual valuation could be of potential use. We also assume the replacement fraction is constant, where in practice the replacement fraction may depend on the age of the product. The math programming formulations can be extended to examine how age-dependent replacement rates influence policy preferences.

We consider the company that offers either a trade-in program or a buyback program. Some companies allow the consumer to choose between a trade-in credit and buyback cash (i.e., BestBuy). The trade-in credit is generally larger than the corresponding buyback rebate. An interesting extension to this work would be a model that examines the optimal trade-in credit and buyback price when both programs are offered. Second, while our analysis focused on age-dependent policies, some companies will offer a flat credit regardless of the age or condition of the returned product. For example, in 2010 Snap On Equipment offered a fixed fee trade-in credit with only one condition—the dealer returns a wheel balancing systems – regardless of brand and working condition. A dynamic study of a family of fixed fee trade-in or buyback credits could provide new and interesting insights. Another contribution would be a rigorous examination of trade-in policies that allow consumers to return competitor equipment. A model that accounts for uncertain remanufactured product value could also provide interesting insights about the effect of uncertainty on product acquisition policy choice.

Our models and analyses do not consider the possibility of competing buyback and trade-in programs. The number of companies offering trade-in and buyback programs are growing. An analysis of how competition effects policy choice could add valuable insight to the growing body of literature related to competition in product recovery markets.

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8. Appendix

8.1 Notation

L	=	time until all warranties expire if there is no trade-in program, i.e., length of the warranty horizon
M	=	total number of purchases over the life of the product
θ	=	fraction of new purchases influenced by external source, i.e., innovator segment
α	=	size of the remanufactured product market relative to the new product market
τ	=	number of periods remanufactured product demand lags behind the new product demand
p	=	coefficient of innovation denotes adoption due to external influences
q	=	coefficient of imitation denotes adoptions due to internal market influences.
$d_n(t)$	=	new product demand at time t.
$d_r(t)$	=	remanufactured product demand in period t
$d_{bj}(t)$, $d_{tj}(t)$	=	new product sales to segment j resulting from buyback / trade-in transactions
$\theta(t)$	=	the fraction of sales to the innovator segment in period t
$D_n(t-1)$	=	cumulative new product sales at the start of period t
$D_r(t)$	=	total remanufactured product sales in periods 1 through t
$N_{bj}(t, i)$, $N_{tj}(t, i)$	=	The number of products of age i in the segment j install base at the end

		of period t
$s_{bj}(t, i)$, $s_t(t, i)$	=	The number of products of age i returned in period t from segment j
$c_b(t, i)$	=	denote the buyback price at time t for a product of age i.
$c_t(t, i)$	=	trade-in price reduction on the new model of the product at time t according to policy i, i.e., trade-in discount
$c_m(i)$	=	cost to remanufacture a returned product of age i less the profit benefit of advancing the purchase by a loyal consumer who would have purchased the product in the future if the buyback program did not exist
h	=	inventory holding cost per unit-period excluding the cost of capital (e.g., storage fees)
M	=	margin on the new model of the product
p_n	=	price of the new model of the product, i.e., $p_n = c_n + m$
p_r	=	remanufactured product selling price
r	=	net discount rate, e.g., cost of capital less inflation
γ	=	fraction of customers who repurchase the product when returning the old product for a buyback price
ϕ	=	the ratio of trade-in- to-buyback disutility
V	=	consumer's valuation of the new product
i	=	age of a product
$\upsilon(i)$	=	consumer's valuation fraction of a product of age i,

8.2 Assumptions

A1. The valuation fraction of a product of age i, $\upsilon(i)$, is the same for the innovator and imitator segments.

A2. Customers are not strategic in their return decision.

A3. The new product and the remanufactured product serve distinct markets—the products are not substitutes.

A4. The repurchase rate, γ, is independent of the age of the product when returned.

A5. Unsatisfied demand in a period results in a lost sale (i.e., no backorders).

A6. The firm remanufactures the unit in the period in which it is returned.

A7. The ratio of trade-in- to-buyback disutility, ϕ, is independent of product age.

8.3 Step 3 of the Myopic Algorithm

The logic of Step 3 of the myopic algorithm is presented at a high level in the body of the paper. We provide additional detail on implementation in this section. We use the myopic buyback algorithm for purposes of illustration; the implementation of the myopic trade-in algorithm is identical except for changes in notation.

Recall that the profit associated with age i product under Regime $k \in \{1, 2\}$ is

$$\frac{\partial \Pi_b(t)}{\partial s_b(t,i)} = \begin{cases} p_r - c_m(i) - b_b(t,i) - 2a_b(t,i)s_b(t,i), \; s_b(t,i) \in \left[0, A_{bk}(t,i)\right) \\ p_r - c_m(i) - b_{bk}(t,i) - 2a_{bk}(t,i)s_b(t,i), \; s_b(t,i) \in \begin{pmatrix} A_{bk}(t,i), \\ N_b(t-1,i-1) \end{pmatrix}\!\!\Bigg] \end{cases}.$$

For a given volume, Δ, to allocate to ages in set J, the fraction of volume allocated to age $i \in J$ in order to maintain identical marginal profits is

$$\rho_i = \left[\sum_{j \in J} \frac{a_i}{a_j}\right]^{-1}$$

where

$$a_i = \begin{cases} a_b(t,i), \; s_b(t,i) \in \left[0, A_{bk}(t,i)\right) \\ a_{bk}(t,i), \; s_b(t,i) \in \begin{pmatrix} A_{bk}(t,i), \\ N_b(t-1,i-1) \end{pmatrix}\!\!\Bigg] \end{cases}$$

(obtained by setting marginal profits equal after allocating $\rho_i\Delta$ to each age $i \in J$ and solving for ρ_i). To simplify the notation, let

$$b_i = \begin{cases} p_r - c_m(i) - b_b(t,i), \; s_b(t,i) \in \left[0, A_{bk}(t,i)\right) \\ p_r - c_m(i) - b_{bk}(t,i), \; s_b(t,i) \in \begin{pmatrix} A_{bk}(t,i), \\ N_b(t-1,i-1) \end{pmatrix}\!\!\Bigg] \end{cases}.$$

The value of Δ is the minimum of five values, $\Delta_1, \ldots, \Delta_5$, that are explained below.

1. Quantity that results in marginal profit of age J to equal the second-highest marginal profit:

   Let j denote the age of the second-highest marginal profit. If there is no positive second-highest marginal profit, then $\Delta_1 = \infty$; otherwise the quantity is

   $$\Delta_1 = \min_{i \in J} \left\{ \frac{b_i - a_i s_b(i) - b_j}{a_i \rho_i} \right\}$$

2. Quantity at which $s_b(J) = A_{bk}(J)$:

$$\Delta_2 = \min_{i\in J}\begin{Bmatrix} \dfrac{A_{bk}(i)-s_b(i)}{\rho_i}, & \text{if } s_b(i) < A_{bk}(i) \\ \infty, & \text{if } s_b(i) \geq A_{bk}(i) \end{Bmatrix}$$

3. Quantity at which $s_b(J) = N_b(J-1)$:

$$\Delta_3 = \min_{i\in J}\begin{Bmatrix} \infty, & \text{if } s_b(i) < A_{bk}(i) \\ \dfrac{N_b(t-1,i-1)-s_b(i)}{\rho_i}, & \text{if } s_b(i) \geq A_{bk}(i) \end{Bmatrix}$$

4. Quantity at which the marginal profit of age J is zero:

$$\Delta_4 = \min_{i\in J}\left\{\frac{b_i - a_i s_b(i)}{a_i\rho_i}\right\}$$

5. Quantity at which total return volume is equal to demand d_r:

$$\Delta_5 = d_r - \sum_{i\in J} s_b(i)$$

The total quantity allocated to the ages in J is

$$\Delta = \min\{\Delta_1, \ldots, \Delta_5\}$$

and $s_b(i) = s_b(i) + \rho_i\Delta \ \forall i \in J$.

8.4 Math Programming Formulations

Alternative Math Programming Formulation of the Buyback Problem

We use $y_-(t)$ to denote unsatisfied demand in period t and we use $y(t)$ to denote the difference between supply and demand in period t. Thus, the inventory at the end of period t is $y(t) + y_-(t)$, which is initialized at 0, i.e., $y(0) + y_-(0) = 0$. A number of other intermediate variables are computed in the constraints. The problem is

$$\Pi_b^p = \max \sum_{t=1}^{T} (1-r)^t \, \Pi_b(t) \tag{60}$$

where

$$\Pi_b(t) = p_r\left[d_r(t) - y_-(t)\right] - h\left[y(t) + y_-(t)\right] - \sum_{i=1}^{t}\left(c_b(t,i) + c_m(i)\right)\left[s_{b1}(t,i) + s_{b2}(t,i)\right]$$

subject to

$$z1(t,i) \leq \frac{c_b(t,i) - p_n\upsilon(i)}{\upsilon(i)(1-p_n)}, \ i\in\{1,\ldots,t\}, t\in\{1,\ldots,T\} \tag{61}$$

$$z2(t,i) \leq \frac{c_b(t,i) - p_n\upsilon(i)}{\upsilon(i)\imath D_n(t-i)}, \ i\in\{1,\ldots,t\}, t\in\{1,\ldots,T\} \tag{62}$$

$$zj(t,i) \geq 0, \ i\in\{1,\ldots,t\}, \ j\in\{1,2\}, t\in\{1,\ldots,T\} \tag{63}$$

$$zj(t,i) \le 1,\ i \in \{1,...,t\},\ j \in \{1,2\}, t \in \{1,...,T\} \tag{64}$$

$$s_{bj}(t,i) = zj(t,i) N_{bj}(t-1,i-1),\ i \in \{1,...,t\},\ j \in \{1,2\}, t \in \{1,...,T\} \tag{65}$$

$$N_{bj}(t,i) = N_{bj}(t-1,i-1) - s_{bj}(t,i),\ i \in \{2,...,t\},\ j \in \{1,2\}, t \in \{1,...,T\} \tag{66}$$

$$y(1) = s_{b1}(1,1) + s_{b2}(1,1) - d_r(1) \tag{67}$$

$$y(t) = y(t-1) + y_-(t-1) + \sum_{i=1}^{t} \left(s_{b1}(t,i) + s_{b2}(t,i) \right) - d_r(t)\ \ , t \in \{2,...,T\} \tag{68}$$

$$y_-(1) \ge d_r(1) - \left(s_{b1}(1,1) + s_{b2}(1,1) \right) \tag{69}$$

$$y_-(t) \ge d_r(t) - \sum_{i=1}^{t} \left(s_{b1}(t,i) + s_{b2}(t,i) \right) - y(t-1) - y_-(t-1)\ \ , t \in \{2,...,T\} \tag{70}$$

$$y_-(t) \ge 0,\ t \in \{1,...,T\} \tag{71}$$

Constraints (61) – (64) ensure that the min{(·)+, 1} terms in (17) take on the proper values. Constraints (65) – (66) are implementations of (17) and (12). Constraints (67) and (68) define the difference between supply and demand at the end of periods 1 though *T*. Constraints (69) and (70) define unsatisfied demand in periods 1 through *T*; the inequality is tight when demand is more than supply because of the cost.

Math Programming Formulation of the Trade-in Problem

The following formulation parallels the formulation of the buyback problem presented in Section 3.4. It is relatively straightforward to write an alternative equivalent formulation that parallels the above, and thus we omit the details.

The problem is

$$\Pi_t^p = \max \sum_{t=1}^{T} (1-r)^t \, \Pi_t(t)$$

where

$$\Pi_t(t) = p_r x(t) - hI(t) - \sum_{i=1}^{t} \left(c_t(t,i) + c_m(i) - (1-\gamma) m \right) s_t(t,i) \tag{72}$$

subject to

$$N_{tj}(t,i) + s_{tj}(t,i) - N_{tj}(t-1,i-1) = 0,\ i \in \{1,...,t\},\ j \in \{1,2\}, t \in \{1,...,T\} \tag{73}$$

$$I(t) + x(t) - \sum_{i=1}^{t} s_t(t,i) - I(t-1) = 0,\ t \in \{1,...,T\} \tag{74}$$

$$x(t) \le d_r(t),\ t \in \{1,...,T\} \tag{75}$$

$$s_{t1}(t,i)-\min\left\{\left(\frac{c_t(t,i)-p_n\phi\upsilon(i)}{\phi\upsilon(i)(1-p_n)}\right)^+,1\right\}N_{t1}(t-1,i-1)=0,\ i\in\{1,\dots,t\},t\in\{1,\dots,T\} \tag{76}$$

$$s_{t2}(t,i)-\min\left\{\left(\frac{c_t(t,i)-p_n\phi\upsilon(i)}{\phi\upsilon(i)\iota D_n(t-i)}\right)^+,1\right\}N_{t2}(t-1,i-1)=0,\ i\in\{1,\dots,t\},t\in\{1,\dots,T\} \tag{77}$$

$$s_t(t,i),N_{tj}(t,i),x(t),I(t)\geq 0,\ i\in\{1,\dots,t\},\ j\in\{1,2\},t\in\{1,\dots,T\} \tag{78}$$

Constraints defined in (73) are the flow-balance constraints for the install base of each age and segment over time. Constraints defined in (74) are flow-balance constraints for remanufactured product inventory. Constraints defined in (75) state that sales can be no more than demand. Constraints defined in (76) and (77) specify returns by segment given the trade-in price $c_t(t, i)$ as determined by total return volume $s_t(t, i)$. Constraints defined in (78) enforce nonnegativity of the decision variables. The expressions for $c_t(t, i)$ depend on the regime, and are given below (see (54)):

if $p_n + \iota D_n(t-i) \leq 1$, then

$$c_t(t,i)=\begin{cases}\phi\upsilon(i)\left(\dfrac{(1-p_n)\iota D_n(t-i)s_t(t,i)}{(1-p_n)N_{t2}(t-1,i-1)+\iota D_n(t-i)N_{t1}(t-1,i-1)}+p_n\right), \\ \qquad \text{if } s_t(t,i)\in\left[0,\left(\dfrac{\iota D_n(t-i)}{1-p_n}\right)N_{t1}(t-1,i-1)+N_{t2}(t-1,i-1)\right] \\ \phi\upsilon(i)\left(\dfrac{(1-p_n)s_t(t,i)}{N_{t1}(t-1,i-1)}+p_n-\dfrac{(1-p_n)N_{t2}(t-1,i-1)}{N_{t1}(t-1,i-1)}\right), \\ \qquad \text{if } s_t(t,i)\in\left[\begin{array}{l}\left(\dfrac{\iota D_n(t-i)}{1-p_n}\right)N_{t1}(t-1,i-1)+N_{t2}(t-1,i-1), \\ N_{t1}(t-1,i-1)+N_{t2}(t-1,i-1)\end{array}\right]\end{cases}$$

if $p_n + \iota D_n(t-i) \geq 1$, then

$$c_t(t,i)=\begin{cases}\phi\upsilon(i)\left(\dfrac{(1-p_n)\iota D_n(t-i)s_t(t,i)}{(1-p_n)N_{t2}(t-1,i-1)+\iota D_n(t-i)N_{t1}(t-1,i-1)}+p_n\right), \\ \qquad \text{if } s_t(t,i)\in\left[0, N_{t1}(t-1,i-1)+\left(\dfrac{1-p_n}{\iota D_n(t-1)}\right)N_{t2}(t-1,i-1)\right] \\ \phi\upsilon(i)\left(\dfrac{\iota D_n(t-i)s_t(t,i)}{N_{t2}(t-1,i-1)}+p_n-\dfrac{\iota D_n(t-i)N_{b1}(t-1,i-1)}{N_{t2}(t-1,i-1)}\right), \\ \qquad \text{if } s_t(t,i)\in\left[\begin{array}{l}N_{t1}(t-1,i-1)+\left(\dfrac{1-p_n}{\iota D_n(t-1)}\right)N_{t2}(t-1,i-1), \\ N_{t1}(t-1,i-1)+N_{t2}(t-1,i-1)\end{array}\right]\end{cases}$$

8.5 Derivations and Proofs

Proof of Proposition 1. For a given install base and return volume in period t of age i product, the relationship between the associated trade-in price and buyback price is

$$c_t(t, i) = \phi c_b(t, i) \tag{79}$$

(see (23) and (54)). Consider an alternative to the trade-in model where the repurchase fraction is the same as the buyback model (i.e., repurchase fraction is γ instead of 1) yet the trade-in-to-buyback disutility of ϕ still applies. We let the subscript bt denote this modified trade-in model. Due to (79) and the fact that, for a given set of return volumes $s(t, i)$ for all t and i, the evolution of the install base for the buyback model and the bt-model is identical, i.e.,

$$N_{bj}\left(t,i\,|\,s_b(t,i)=s(t,i)\right)=N_{btj}\left(t,i\,|\,s_{bt}(t,i)=s(t,i)\right) \text{ for } j = 1, 2 \text{ and all } t \text{ and } i, \tag{80}$$

we have

$$\begin{aligned}\Pi_{bt|s_{bt}(t,i)=s_b(t,i)}-\Pi_{b|s_b(t,i)} &= \sum_t (1-r)^t \sum_i \left[(1-\gamma)m-(\phi-1)c_b(t,i)\right]s_b(t,i) \\ &= \left(\sum_t (1-r)^t \sum_i s_b(t,i)\right)\left[(1-\gamma)m-(\phi-1)\overline{c}_{b|s_b(t,i)}\right]\end{aligned} \tag{81}$$

$$\begin{aligned}\Pi_{bt|s_{bt}(t,i)=s_b^*(t,i)}-\Pi_b^* &= \sum_t (1-r)^t \sum_i \left[(1-\gamma)m-(\phi-1)c_b^*(t,i)\right]s_b^*(t,i) \\ &= \left(\sum_t (1-r)^t \sum_i s_b^*(t,i)\right)\left[(1-\gamma)m-(\phi-1)\overline{c}_b^*\right].\end{aligned} \tag{82}$$

Note that

$$N_{tj}\left(t,i\,|\,s_t(t,i)=s(t,i)\right)\geq N_{btj}\left(t,i\,|\,s_{bt}(t,i)=s(t,i)\right) \text{ for } j = 1, 2 \text{ and all } t \text{ and } i,$$

and thus,

$$\Pi_{t|s_t(t,i)=s(t,i)} \geq \Pi_{bt|s_{bt}(t,i)=s(t,i)}. \tag{83}$$

Setting (81) and (82) greater than or equal to zero and noting (83) yields Proposition 1a and 1b.

Proof of Proposition 2. Part a follows directly from Proposition 1. For part b, we compare (11) and (23) with (45) and (54) to see that buyback and trade-in models are identical when $\gamma = \phi = 1$.