INTERIOR ANALYSIS OF THE GREEN PRODUCT MIX PROBLEM

Abstract

In this paper, we present a methodology for generating and analyzing variants of the green product mix problem whose solutions assist the decision maker in framing green implementation strategy. The greening is incorporated in the modeling of optimal product mix determination in two ways: i) by restricting the emissions of production and ii) through transformation of its by-products into commodity or product. Each approach is illustrated with an example. In the search for an implementation strategy, the methodology is shown to provide useful results for assessing the impact of controlling each emission (or transformation) alone and in combination with others. The results allow the decision maker to investigate the appeal of a step-by-step implementation of emission controls as an alternative to simultaneous accommodation when aspects of the latter are prohibitive or unattractive. The methodology is adaptable to most product mix formulations.

Keywords: greening, linear programming, product mix problem, implementation strategy, sustainability

1. Introduction

Operating in an environmentally friendly manner is a widely held value and commonly practiced in one form or another among public and private, small and large, manufacturing and service, as well as new and established organizations. Many have taken steps to improve the impact of their production, procurement of materials, consumption of energy, and the after-sale consequences of their products in the environment. Frameworks for integrating green and sustainable practices in business policy are discussed in the literature review section of the paper. They include calls for incorporating the environmental consequences of production in the modeling of operations such as the determination of optimal product mix, the subject of this paper.

Let greening product mix refer to the manner in which the environmental impact (carbon footprint, gas emissions, solid wastes, effluents, etc.) of production is incorporated within the modeling of what and how much to produce. It should reflect the organization's intention to i) restrict or eliminate the impact quantities and ii) account for their economic consequences. The restrictions may take the form of voluntary or mandated caps on emissions, pollutants, and other by-products of production. In some situations, income may result from trading unused emission allowance. When emissions exceed allowances, for compliance the difference (if available) may be sourced/purchased through trading; otherwise penalties may be incurred. Greening mix may also include the transformation of the impact quantities to environmentally less harmful and possibly marketable commodities or products. The transformations may be achieved using standard as well as special purpose resources and processes.

The above approaches to greening broaden the context within which product mix is examined and modeled. They raise the perception of greening from a cost-of-compliance viewpoint to value-adding and perhaps incentivise innovation in addressing how the organization may improve its environmental impact. The value of modeling green product mix as described in this paper is the framework it provides for analyzing implementation options and strategies.

Without loss of generality, emissions will refer to gases, solid wastes, effluents, scrap, etc. that result from production and whose return to the environment in untreated state would be harmful, illegal, or perceived as poor citizenship. Transformed emissions include recyclable materials, recoverable compounds, treated effluents, composited by-products, and other forms.

Identifying optimal mix under emission restrictions and a single measure or objective of what is best is generally straightforward. However, the optimal product mix that accommodates all sought after emission controls may exclude flagship product(s); result in product volumes that may challenge maintenance of market share or be so disparate that it creates problems in moving product through the various fabrication processes; upset product/process fit; call for underutilization of expensive resources; or most importantly result in a prohibitive reduction in the optimizing objective. Clearly, the value of the objective under emission restrictions cannot be better than the value of the unrestricted product mix solution. When the difference is large, the manufacturer will seek alternatives. Although some attention is given in the literature to these aspects of greening mix, it does not include methodology and strategy for dealing with them.

In this paper, we present a methodology for generating variants of a linear programming (LP) formulation of the green product mix problem whose solutions provide the decision maker with alternatives to consider in developing a green implementation strategy. Because greening is framed within the context of an allocation problem, emission control consists of adding at least one constraint for each emission target; accommodating in the objective function charges associated with excess emissions and revenues from trading unused allowances of the same; and adding a new mix variable for each emission transformation. Successive LP problems that account explicitly or otherwise for all combinations of emission controls are solved and analyzed. In this manner, the appeal of controlling emissions (and accommodating their transformations) one, two, etc. at a time may be examined. From the results, the decision maker may compose a step-by-step implementation strategy to accommodate emission controls. We refer to the investigation as green interior analysis.

The rest of the paper is organized in the following manner. Literature relating to greening, its place in product mix determination, and strategy for its implementation are reviewed in the next section. The proposed methodology is presented in Section 3 and illustrated with examples in Section 4. The paper concludes with summary and remarks in Section 5.

2. Literature Review

Frameworks for integrating green and sustainable practices in corporate policy are found in Carter and Rogers (2008), Lee (2008), Hervani et al (2005), and Sarkis (2003). Nijkamp and van den Berg (1997) provided a typology of the underlying economic and environmental issues involved in green policy making. Among the issues, they identified the need for models that incorporate environmental impact and its economic consequences. Letmathe and Balakrishnan (2005) acknowledged the shortcoming in contemporary modeling and offered two optimization models for determining optimal product mix that included environmental considerations. Their models as formulated are examples of greening by restriction of emissions. The impact of greening optimal lot size determination and shipping/transportation is addressed in Bonney and Jaber (2011), Hua et al (2011), and Brown and Guiffrida (2011).

In the recent literature, several authors have identified the strategic benefits of green and sustainable practices. They include Porter and van der Linde (1995), who observed that the enhanced resource productivity that results from implementing green practices leads to operational innovation that in outcome strengthens the firm's competitive position in the marketplace. Klassen and McLaughlin (1996) presented a model that connected the firm's environmental scorecard with its financial performance. Empirical testing of the model supported the hypothesis that strong environmental performance positively affects the financial performance of the firm. In a study of ISO14001 certified companies, Rao and Holt (2005) demonstrated via structural equation modeling (SEM) that greening the supply chain enhanced competitiveness and economic success. Koplin et al (2007) and Seuring and Müller (2008) also addressed greening supply chains. Ambec and Lanoie (2008) identified a set of seven specific actions that could lead to potential increases in revenues or to cost reductions as a consequence of improving the organization's environmental footprint.

Although the literature is replete with discussion of the strategic value of greening, no contribution has integrated quantitative modeling of green product mix determination and implementation strategy.

3. Green interior analysis

Green interior analysis is a methodology for generating and analyzing alternate specifications of the green product mix problem that derive from an exhaustive accommodation of emissions controls taken one, two, etc. at a time. It proceeds by accommodating each subset of controls in the product mix formulation, solving the problem, and calculating important features of the resulting product mix based on the value of the objective, the achievement or otherwise of emission target(s), utilization of productive resources, and the constituency of the product mix. As such, the methodology produces implementation scenarios and their attributes that otherwise would not be examined. Some may or may not have appeal.

3.1. The proposed methodology

Interior analysis is applied to the LP form of the product mix problem in which emission controls appear as linear restrictions, terms in the objective function, or both. Let there be m number of emissions to consider. Alternate specifications of the model are generated using a scheme that exhaustively enumerates all possibilities of accommodating 1, 2, …, m of the emission restrictions. Each possibility may be framed as a variant of the product mix LP model that accommodates none of the emissions. The following is the proposed step-by-step procedure.

Interior analysis of the green product mix problem

Step 1: Identify the set of emission restrictions to be considered for inclusion/exclusion in the analysis. Go to Step 2.

Step 2: Solve the mix problem without any emission accommodation. Go to Step 4.

Step 3: Use the algorithm presented in the Appendix for streaming the next subset of emission accommodation. Edit accordingly the formulation of Step 2 or its most recently edited form and solve the resulting problem. Go to Step 4.

Step 4: Record/calculate essential features of the result. Suggestions include the following:

i. Value of the objective;
ii. Utilization of each productive resource in absolute and relative measures;
iii. Achievement of emission targets in absolute and relative measures;
iv. The constituency of the mix such as the number of mix products with zero value.

The above may be reported in comparison to the result obtained in Step 2 or some other meaningful benchmark. Go to Step 5.

Step 5: If all accommodations have been generated, go to Step 6; otherwise go to Step 3.

Step 6: Stop. Devise a strategy of stepwise implementation.

The generation scheme of Step 3 is a simpler version of backtracking schemes such as Balas (1965) and Narula and Wellington (1985).

3.2 Devising an implementation strategy

With the results for all subsets of 1, 2, …, m accommodated emission controls available, the analyst and the decision maker can identify the best single emission control to accommodate. Given this result, they can identify which accommodation is best combined with it. Continue in this manner until all emission controls are accommodated or until a partial accommodation is encountered with unappealing feature(s). Best is understood in the following sense. Given an accommodation of j (< m) number of emission controls, the next addition results in j+1 controls with superior objective function value among subsets of size j+1.

The stepwise investigation may reveal implementation strategies that are attractive and may allow time for the market value (e.g. product price and brand) of the greening to grow with emission control. If i) the emission controls can be implemented successively in a stepwise order and ii) the market value of the greening in the form of improved branding, good will, and other consequences including upward price mobility follow as suggested in the literature, the loss in objective due to greening may be softened through implementation strategies based on partial accommodation of emission targets. As such, green interior analysis provides a framework for identifying alternatives to the collective enforcement of emission targets.

4. Illustrations

Greening product mix is illustrated within the context of the LP formulation of the product mix problem for several reasons. The formulation is well known; simple to understand; solvable using a variety of commonly available software products (applications); and often used as a tool of analysis to identify production limitations due to bottlenecks (binding constraints) and product compatibility (mix of products with zero and non-zero values) and as such useful for investigating consequences of greening mix.

4.1 Illustrations 1, 2 and 3

Illustration 1:

The following illustration is adapted from the product mix problem of Letmathe and Balakrishnan (2005) that identifies the optimal mix among twelve products whose quantities are denoted by $X_1, ..., X_{12}$; whose production requires use of five resources in the amounts $R_1, ..., R_5$; and whose processing results in five emissions in the amounts $E_1, ..., E_5$. The model is

Maximize (1)

$$800X_1 + 700X_2 + 500X_3 + 1{,}000X_4 + 2{,}200X_5 + 2{,}300X_6 + 1{,}600X_7 + 2{,}600X_8 + 700X_9 + 1{,}000X_{10} + 1{,}300X_{11} + 1{,}800X_{12} - 50R_1 - 100R_2 - 3R_3 - 5R_4 - 2R_5 - E_4 - 5E_5^+ + 4E_5^- \tag{1.1}$$

subject to:

$$R_1 - 3X_1 - 3X_2 - 3X_3 - 4X_4 - 10X_5 - 9X_6 - 8X_7 - 10X_8 - 3X_9 - 3X_{10} - 5X_{11} - 7X_{12} = 0 \tag{1.2}$$

$$R_2 - X_1 - X_2 - X_3 - 2X_4 - 6X_5 - 10X_6 - 4X_7 - 10X_8 - X_9 - 2X_{10} - 3X_{11} - 6X_{12} = 0 \tag{1.3}$$

$$R_3 - 20X_1 - 20X_2 - 10X_3 - 50X_4 - 80X_5 - 60X_6 - 60X_7 - 100X_8 - 30X_9 - 60X_{10} - 60X_{11} - 70X_{12} = 0 \tag{1.4}$$

$$R_4 - 30X_1 - 30X_2 - 30X_3 - 40X_4 - 100X_5 - 80X_6 - 70X_7 - 100X_8 - 30X_9 - 40X_{10} - 50X_{11} - 90X_{12} = 0 \tag{1.5}$$

$$R_5 - 50X_1 - 50X_2 - 50X_3 - 50X_4 - 50X_5 - 50X_6 - 50X_7 - 50X_8 - 50X_9 - 50X_{10} - 50X_{11} - 50X_{12} = 0 \tag{1.6}$$

$$E_1 - 4X_1 - 3X_2 - 2X_3 - 5X_4 - 3X_5 - 8X_6 - 10X_7 - 20X_8 - 4X_9 - 5X_{10} - 12X_{11} - 10X_{12} = 0 \tag{1.7}$$

$$E_2 - 10X_1 - 20X_2 - X_3 - 15X_4 - 50X_5 - 100X_6 - 50X_7 - 280X_8 - 40X_9 - 40X_{10} - 50X_{11} - 40X_{12} = 0 \tag{1.8}$$

$$E_3 - 6X_1 - 7X_2 - X_3 - 5X_4 - 4X_5 - 4X_6 - 5X_7 - 7X_8 - 5X_9 - 6X_{10} - 6X_{11} - 6X_{12} = 0 \tag{1.9}$$
$$E_4 - 4X_1 - 3X_2 - X_3 - 5X_4 - 16X_5 - 7X_6 - 7X_7 - 15X_8 - X_9 - 7X_{10} - 2X_{11} - 9X_{12} = 0 \tag{1.10}$$
$$E_5 - 2X_1 - X_2 - X_3 - X_4 - 4X_5 - 2X_6 - 3X_7 - 4X_8 - 2X_9 - 2X_{10} - 2X_{11} - 6X_{12} = 0 \tag{1.11}$$

$$X_1 \leq 1{,}600 \tag{1.12}$$
$$X_2 \leq 2{,}400 \tag{1.13}$$
$$X_3 \leq 7{,}500 \tag{1.14}$$
$$X_4 \leq 3{,}000 \tag{1.15}$$
$$X_5 \leq 2{,}000 \tag{1.16}$$
$$X_6 \leq 4{,}000 \tag{1.17}$$
$$X_7 \leq 5{,}000 \tag{1.18}$$
$$X_8 \leq 6{,}500 \tag{1.19}$$
$$X_9 \leq 1{,}000 \tag{1.20}$$
$$X_{10} \leq 2{,}200 \tag{1.21}$$
$$X_{11} \leq 1{,}000 \tag{1.22}$$
$$X_{12} \leq 1{,}800 \tag{1.23}$$

$$E_1 \leq 60{,}000 \tag{1.24}$$
$$E_2 \leq 8R_2 \tag{1.25}$$
$$E_3 \leq 6\,(X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} + X_{12}) \tag{1.26}$$
$$E_5 = 50{,}000 + E_5^+ - E_5^- \tag{1.27}$$

$$E_1, E_2, E_3, E_4, E_5, E_5^+, E_5^-, R_1, R_2, R_3, R_4, R_5 \geq 0 \tag{1.28}$$
$$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12} \geq 0. \tag{1.29}$$

The greening is addressed through restrictions (1.24) - (1.27) on the emission quantities E_1, E_2, E_3, and E_5; cost associated with emission 4 that appears in the objective function (1.1) as $-E_4$; and for emission 5 through the merit ($+4E_5^-$) and penalty ($-5E_5^+$) of under-achieving (E_5^-) or over-achieving (E_5^+) its emission allowance of 50,000 (1.27), see the treatment in the objective function (1.1). The merit ($+4E_5^-$) may be looked upon as income from trading unused allowance and the penalty ($-5E_5^+$) as the cost of sourcing over-generation of allowance. The solution to (1) with and without the five emission restrictions appears in Table 1.

{Table 1}

From the linear programming properties of (1) under emission control, we expect the value of the objective to be no greater than \$5,386,000. However, the consequences of simultaneously accommodating the five emission controls include significant reduction in the optimizing objective, from \$5,386,000 to \$1,781,188; significant changes in the constituency of the optimal mix with elimination of five products (6-8, 10, 11); and dramatic changes in resource use, see Table 2. The results of Tables 1 and 2 clearly indicate the inability of the processes as modeled to accommodate the five emission controls with reasonable operating consequences.

{Table 2}

The results of interior analysis of (1) under various scenarios of accommodation among the five emissions appear in Tables 3, 4, and 5. There are $2^5 = 32$ scenarios (column 2) with emissions 1 though 5 included alone or in combination with two, three, four, or all five. Each scenario was framed as a variant of (1) in the following way. For each i appearing in a scenario of column 1, $E_i \geq 0$ in (1) and for each i omitted $E_i = 0$ in the same. The constraint set and objective function of (1) were accordingly edited and the resulting form of the problem was solved. The step-by-step procedure of Section 3 including the algorithm (Step 3) for sequencing the accommodated emissions produced the results that populate Tables 3-5. The results were reorganized in the order shown in column 2 of the Tables. See column 5 of Table A1 for the sequencing of emission of the accommodated emissions produced by Step 3.

{Table 3}

Among the thirty-two scenarios of Table 3, the quantities X_1, X_2, X_4, and X_5 do not change; X_8, X_9, X_{10}, and X_{12} each have two distinct values; X_{11} has three distinct values; X_6 and X_7, four values; and X_3, five values. Over one-third of the product mix values are zero. Further, the inclusion of emission control 3 in any scenario does not change the value of the objective. It is not a binding constraint in any scenario. The addition of emission 4 to any scenario does not improve the objective. However, in twelve of the fifteen scenarios in which emission 5 is combined with other emissions in pairs, triples, etc., the values of the objective are greater with its inclusion. Row 32 displays the results for no emission controls that served as a benchmark for comparison purposes.

Measures of operational impact associated with each scenario are given in Table 4. For each scenario, ratio (in percent) of the sum of its optimal product mix quantities $X_1, \ldots, X_{12}$ to the sum (= 30,500) for the benchmark mix (row 32, with no emission controls) is recorded in column 3 and the number of zeros among the values of $X_1, \ldots, X_{12}$ is noted in column 4. Note that the benchmark mix has one decision variable ($=X_3$) with zero value. The entries of column 5 indicate the percent reduction in the use of each resource relative to its use under the benchmark scenario. The entries of column 6 indicate similarly for percent reductions in emissions generated under each scenario.

{Table 4}

From inspection of Table 4 using the scenario of no emission controls for comparative purposes, it is clear that in some scenarios the imposition of controls is achieved with significant changes in the mix quantities, loss in objective, and diminished resource use. Except for the scenarios of rows 3-5, 13-15, and 25, the emission accommodations of column 2 are achieved through i) elimination (=0) of five quantities among the $X_1, \ldots, X_{12}$ and ii) reductions between 33.83% and 78.23% in resource use where the smallest changes occurred with resource 5 in the interval [33.83%, 55.74%], see column 5-R5 of Table 4, and the largest with resource 2 in the range [71.22%, 78.235%], see column 5-R2 of the same. Among the percent reductions in emissions, the smallest changes are found with emission 5 in column 6-E5 and the largest with emission 2 in column 6-E2. Note that the accommodation of emissions 1 and 2 alone or in combinations with the others results in at least a 55.62% reduction in use of resource 1, 71.22% reduction in use of resource 2, 63.49% with resource 3, 54.12% with resource 4, and 33.83% with resource 5. Given the impact on resources, accommodation of emission 1 and/or 2 controls would likely upset the fit of product and process.

{Table 5}

The results of interior analysis displayed in Table 5 can assist in devising an implementation strategy. If the scenarios are looked upon as partial accommodations of the five emissions controls and if the emission control technologies are amenable to stepwise implementation, consider the following. From inspection of Table 5, enforcement of emission 3's target is the best single implementation; emission 3 is best paired with emission 5; emissions 3 and 5 are best in triple with emission 4. Best refers to the largest value of the objective for a scenario of $n < m$ controlled emissions. For the scenario 3,4,5, note that the values of $X_1, \ldots, X_{12}$ at each step/stage (scenario 3; scenario 3,5; scenario 3, 4, 5) of implementation are identical to the $X_1, \ldots, X_{12}$

values for the benchmark mix without emission controls. Consequently, the implementation 3,4,5 can be made without alterations to the mix quantities. If the implementation occurs in a stepwise manner (3 → 5 → 4), the values of the objective would be respectively $5,386,000, $5,212,000, $4,956,300 that are reductions of 0%, 3.23%, and 7.97%. The 3,4,5 implementation could also occur as 3,5 followed by 4 or as 3,4,5 simultaneously. After the 3,4,5 implementation, observe that the addition of emissions 1 and 2 results in dramatic changes in the mix quantities X_1, …, X_{12} and the values of the objective function, see scenarios of rows 29-31 in Table 4.

Interior analysis helps in revealing features of the solution set that are not otherwise evident and as shown above they can become the basis for framing implementation strategy. As noted above, the timing and manner of the implementations may allow the market value of the greening to grow with the accommodation of controls. They may also allow the parallel development and implementation of offsets to the reductions in objective that accompany emission control. The results of interior analysis may move the decision maker to consider 'satisficing' the emission targets not enforced in a scenario of partial implementation of controls. For the partial implementation scenario 3,4,5 the unrestricted quantities E_1 (=291,600) and E_2 (=2,929,00) are respectively 4.86 (=E_1/60,000) and 2.20 (=E_2/(8*166,200)) times their target values, see (1.24) and (1.25). Although satisficing does not appear attractive for this partial implementation, in other applications the multiples for the unrestricted emissions may be acceptable i.e. satisficing. If so, a partial implementation scenario may serve as a first strategy of accommodation.

The partial implementation 3,4,5 may also be attractive for reason of process fit. The accommodation of emissions 1 and 2 alone or in combinations with the other emissions would likely upset the fit of product, process, and locus of production i.e. the what, where, how, and other aspects of process choice, Katariina et al (2008). However, the mix values X_1, …, X_{12} for the scenario 3,4,5 as well as every subset thereof are identical to the values for the scenario of no emission controls. Under the assumption that the latter has good product/process fit, 3,4,5 may be a good first implementation of controls to maintain the alignment. Its staging may permit development of a strategy for subsequently accommodating controls 1 and 2. When the green product mix solution is perceived as poor fit, it will be incumbent upon the analyst to turn to methodology such as interior analysis to produce alternatives with better fit.

Generally, the accommodation of emission controls results in elimination of product(s) from the mix and in turn reductions in resource use. Offsets to the reduction in objective such as the gain/penalty accompanying generation of emission 5 soften the impact of emission enforcement and invite envisioning other offsets. They may include transforming emissions to commodity/product that has market value. The following illustration is a demonstration of this point.

Illustration 2:

Let recycling emissions for internal use or external sale/disposal; separating and bundling scrap for salvage; compositing scrap into briquettes or other forms to be used as fuel that drives equipment or provides heat; and recovering and treating water for reuse serve as examples of transforming emissions. The resulting commodities and products may/not have market value. However, their transformations consume resources and consequently call for resource allocations. As such, the conversions do properly belong within the modeling framework of product mix determination and its implementation strategy.

Because emission transformations have varied impact on objective and other features of good product mix and some are challenging to operationalize, it may be useful to investigate their implementation one, two, etc. at a time using interior analysis. The approach is illustrated with modifications to model (1) and application of the proposed scheme for streaming the accommodation of emission transformations one, two, etc. at a time in the model. The underlying technology of producing products 1-12 in the Lemathe and Balakrishan (2005) model was not available. Consequently a simple scheme for converting emissions 1-5 to four transformed products was assumed and appears in Tables 6 and 7.

{Table 6}

In Table 6, note that emissions 1, 2, and 5 convert respectively to transformed products 1, 2, and 4 in quantities denoted by TP_1, TP_2, and TP_4. Emissions 3 and 4 convert to transform product 3 in the amount TP_3. Standard resources in the amounts $RS_1, \ldots, RS_5$ are required for fabricating mix products 1-12. Quantities of standard resources utilized for the transformation of emissions 1-5 to new products 1-4 are denoted by $RT_1, \ldots, RT_5$. Quantities of standard resources used for both purposes are represented by $R_i = RS_i + RT_i$, $i = 1, \ldots, 5$.

{Table 7}

Table 7 is the display of the per unit standard and transformation specific (dedicated) resource requirements for each unit of transformed products 1-4. The per unit standard resource requirements for mix products 1-12 are unchanged from those specified in (1.2) – (1.6) of formulation (1). Quantities of special resources dedicated to transforming emissions 1-5 to new products 1-4 are denoted by $ST_1, \ldots, ST_4$.

The determination of optimal product mix under accommodation of the five emissions as specified in (1) and the four emission transformations as described above appears in (2) where (2.7) - (2.11) identify the quantities $RS_1, \ldots RS_5$ of standard resources required for producing the mix quantities $X_1, \ldots, X_{12}$; (2.12) – (2.16), the quantities $RT_1, \ldots RT_5$ of standard resources required for producing transformed emission quantities $TP_1, \ldots, TP_4$; (2.17) – (2.21), the amounts $R_1, \ldots R_5$ of standard resources required for both purposes; (2.22) - (2.26), the quantities of emissions 1-5 generated with the production of mix products 1-12 in the amounts $X_1, \ldots, X_{12}$; (2.27) - (2.30), the assumed form of the transformation of emissions 1-5 to products 1-4 in the amounts $TP_1, \ldots TP_4$; (2.31) - (2.34), quantities $ST_1, \ldots ST_4$ of resources dedicated exclusively to the transformations; and (2.35) - (2.54) are adaptations of (1.19) - (1.29) of model (1). In the objective function, the revenue contribution for each unit of transformed emissions 1-4 is \$250, see (2.3). The per unit costs of the $ST_1, \ldots, ST_4$ were respectively assigned values of \$5, \$3, \$2, and \$4, see (2.6). The remaining elements of the objective function appear as given in (1.1). The manner in which emissions 1-5 are generated in the production of mix products 1-12 is unchanged from that given in (1.7) – (1.11) of formulation (1). In (2), it is assumed that the transformations of emissions 1-5 to products 1-4 do not generate new emission types or additional amounts of $E_1, \ldots, E_5$.

The outcomes of accommodating transformed emission products 1-4 alone, in pairs, etc. are given in the rows of Table 8. There are 2^4 scenarios (column 2) for accommodating the four hypothetical transformations. Each scenario was framed as a variant of (2) in the following way. For each i appearing in a scenario of column 1, $TP_i \geq 0$ in formulation (2) and for each i omitted $TP_i = 0$ in the same. The constraint set and objective function of model (2) were accordingly edited and the resulting form of the problem was solved. No edits were made to terms with $E_1, \ldots, E_5$. The step-by-step procedure of Section 3 including the algorithm (Step 3) for sequencing the inclusion of emission transformations produced the results that populate Table 8.

{Table 8}

Let the results of row 1 of Table 8 serve as reference or benchmark (all five emissions accommodated) for assessing the appeal of the scenarios of rows 2 - 16. The latter may be looked upon as scenarios that offer offsets to the reduction in objective that accompany accommodation of the five emissions. In Table 8, except for the scenario of row 2, each situation offers an improvement in the benchmark value (=\$1,781,188) of the objective. Among the scenarios of accommodation in Table 8, the ratio of the sum of the mix quantities $X_1, \ldots, X_{12}$ to the sum for the benchmark scenario of row 1 varies within the range [-0.10%, 1.95%]. Hence, some consistency is generally seen among the mix values $X_1, \ldots, X_{12}$. The results indicate the best that can be expected from simultaneous accommodation of emissions controls 1-5 and emission transformations as modeled.

The scenario of row 5 in Table 8 offers the largest objective function value (=\$3,712,643). Its utility is seen in comparison to i) the benchmark scenario for accommodating simultaneously emissions 1-5 with objective function value of \$1,781,188 and ii) the partial implementation of emission controls 3,4,5 identified in Illustration 1 with objective of \$4,956,300. As a strategy, the emergence of emission transformations in whole or part could be synchronized to coincide with or precede the addition of emissions controls 1 and 2 to implementation 3,4,5 that otherwise would result in reduction of objective to \$1,781,188. Results of interior analysis assist the analyst in assessing the timing, need for, and capacity of the emission transformations to offset the losses in objective that accompany the additions of controls 1 and 2. Table 9 is a display of the solutions for this illustration.

{Table 9}

The transformations of emissions to products given in (2) were modest in resource requirements, cost of resources, and per unit contributions to objective. The result is a simple non-general demonstration of the manner in which the results of interior analysis may move the decision maker to consider a variety on implementation strategies based on stepwise accommodation of emission controls with or without emission transformations.

Maximize (2)

$$800X_1 + 700X_2 + 500X_3 + 1{,}000X_4 + 2{,}200X_5 + 2{,}300X_6 + 1{,}600X_7 + 2{,}600X_8 + 700X_9 + 1{,}000X_{10} \tag{2.1}$$
$$+ 1{,}300X_{11} + 1{,}800X_{12} \tag{2.2}$$
$$+ 250TP_1 + 250TP_2 + 250TP_3 + 250TP_4 \tag{2.3}$$
$$- 50R_1 - 100R_2 - 3R_3 - 5R_4 - 2R_5 \tag{2.4}$$
$$- E_4 - 5E_5^{+} + 4E_5^{-} \tag{2.5}$$
$$- 5ST_1 - 3ST_2 - 2ST_3 - 4ST_4 \tag{2.6}$$

subject to:

$$RS_1 - 3X_1 - 3X_2 - 3X_3 - 4X_4 - 10X_5 - 9X_6 - 8X_7 - 10X_8 - 3X_9 - 3X_{10} - 5X_{11} - 7X_{12} = 0 \tag{2.7}$$
$$RS_2 - X_1 - X_2 - X_3 - 2X_4 - 6X_5 - 10X_6 - 4X_7 - 10X_8 - X_9 - 2X_{10} - 3X_{11} - 6X_{12} = 0 \tag{2.8}$$
$$RS_3 - 20X_1 - 20X_2 - 10X_3 - 50X_4 - 80X_5 - 60X_6 - 60X_7 - 100X_8 - 30X_9 - 60X_{10} - 60X_{11} - 70X_{12} = 0 \tag{2.9}$$
$$RS_4 - 30X_1 - 30X_2 - 30X_3 - 40X_4 - 100X_5 - 80X_6 - 70X_7 - 100X_8 - 30X_9 - 40X_{10} - 50X_{11} - 90X_{12} = 0 \tag{2.10}$$
$$RS_5 - 50X_1 - 50X_2 - 50X_3 - 50X_4 - 50X_5 - 50X_6 - 50X_7 - 50X_8 - 50X_9 - 50X_{10} - 50X_{11} - 50X_{12} = 0 \tag{2.11}$$

$$RT_1 - 1TP1 - 0TP2 - 0.10TP3 - 0.2TP4 = 0 \tag{2.12}$$
$$RT_2 - 1TP1 - 0.5TP2 - 0.10TP3 - 0.2TP4 = 0 \tag{2.13}$$
$$RT_3 - 0.6TP1 - 0.1TP2 - 0.75TP3 - 0.65TP4 = 0 \tag{2.14}$$
$$RT_4 - 1TP1 - 0TP2 - 0TP3 - 1.40TP4 = 0 \tag{2.15}$$
$$RT_5 - 0TP1 - 0.5TP2 - 0.50TP3 - 0.50TP4 = 0 \tag{2.16}$$

$$R_1 - RS_1 - RT_1 = 0 \tag{2.17}$$
$$R_2 - RS_2 - RT_2 = 0 \tag{2.18}$$
$$R_3 - RS_3 - RT_3 = 0 \tag{2.19}$$
$$R_4 - RS_4 - RT_4 = 0 \tag{2.20}$$
$$R_5 - RS_5 - RT_5 = 0 \tag{2.21}$$

$$E_1 - 4X_1 - 3X_2 - 2X_3 - 5X_4 - 3X_5 - 8X_6 - 10X_7 - 20X_8 - 4X_9 - 5X_{10} - 12X_{11} - 10X_{12} = 0 \tag{2.22}$$
$$E_2 - 10X_1 - 20X_2 - X_3 - 15X_4 - 50X_5 - 100X_6 - 50X_7 - 280X_8 - 40X_9 - 40X_{10} - 50X_{11} - 40X_{12} = 0 \tag{2.23}$$
$$E_3 - 6X_1 - 7X_2 - X_3 - 5X_4 - 4X_5 - 4X_6 - 5X_7 - 7X_8 - 5X_9 - 6X_{10} - 6X_{11} - 6X_{12} = 0 \tag{2.24}$$
$$E_4 - 4X_1 - 3X_2 - X_3 - 5X_4 - 16X_5 - 7X_6 - 7X_7 - 15X_8 - X_9 - 7X_{10} - 2X_{11} - 9X_{12} = 0 \tag{2.25}$$
$$E_5 - 2X_1 - X_2 - X_3 - X_4 - 4X_5 - 2X_6 - 3X_7 - 4X_8 - 2X_9 - 2X_{10} - 2X_{11} - 6X_{12} = 0 \tag{2.26}$$

$$TP_1 – 0.05E_1 = 0 \tag{2.27}$$
$$TP_2 – 0.025E_2 = 0 \tag{2.28}$$
$$TP_3 – 0.05E_3 – 0.048E_4 = 0 \tag{2.29}$$
$$TP_4 – 0.05E_1 = 0 \tag{2.30}$$

$$ST_1 - 10TP_1 - 7TP_2 - 9TP_3 - 11TP_4 = 0 \tag{2.31}$$
$$ST_2 - 11TP_1 - 11TP_2 - 7TP_3 - 8TP_4 = 0 \tag{2.32}$$
$$ST_3 - 6TP_1 - 9TP_2 - 6TP_3 - 6TP_4 = 0 \tag{2.33}$$
$$ST_4 - 12TP_1 - 12TP_2 - 10TP_3 - 13TP_4 = 0 \tag{2.34}$$

$$X_1 \leq 1,600 \tag{2.35}$$
$$X_2 \leq 2,400 \tag{2.36}$$
$$X_3 \leq 7,500 \tag{2.37}$$
$$X_4 \leq 3,000 \tag{2.38}$$
$$X_5 \leq 2,000 \tag{2.39}$$
$$X_6 \leq 4,000 \tag{2.40}$$
$$X_7 \leq 5,000 \tag{2.41}$$
$$X_8 \leq 6,500 \tag{2.42}$$
$$X_9 \leq 1,000 \tag{2.43}$$
$$X_{10} \leq 2,200 \tag{2.44}$$
$$X_{11} \leq 1,000 \tag{2.45}$$
$$X_{12} \leq 1,800 \tag{2.46}$$

$$E_1 \leq 60,000 \tag{2.47}$$
$$E_2 \leq 8R_2 \tag{2.48}$$
$$E_3 \leq 6 (X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} + X_{12}) \tag{2.49}$$
$$E_5 = 50,000 + E_5^+ - E_5^- \tag{2.50}$$

$$E_1, E_2, E_3, E_4, E_5, E_5^+, E_5^- \geq 0 \tag{2.51}$$
$$R_1, R_2, R_3, R_4, R_5, RS_1, RS_2, RS_3, RS_4, RS_5, RT_1, RT_2, RT_3, RT_4, RT_5 \geq 0 \tag{2.52}$$
$$ST_1, ST_2, ST_3, ST_4, ST_5, TP_1, TP_2, TP_3, TP_4 \geq 0 \tag{2.53}$$
$$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12} \geq 0 \tag{2.54}$$

Illustration 3:

The search for alternatives to the product mix solution that accommodates all emission controls may also include investigation of subsets of the mix products. The following represents the search for the best subsets of p (< 12) mix products for the Lemathe and Balakrishnan (2005) mix problem under accommodation of all emission controls.

Maximize (3)

$$800X_1 + 700X_2 + 500X_3 + 1,000X_4 + 2,200X_5 + 2,300X_6 + 1,600X_7 + 2,600X_8 + 700X_9 + 1,000X_{10} + 1,300X_{11} + 1,800X_{12} - 50R_1 - 100R_2 - 3R_3 - 5R_4 - 2R_5 - E_4 - 5E_5^+ + 4E_5^- \tag{3.1}$$

subject to:

$$R_1 - 3X_1 - 3X_2 - 3X_3 - 4X_4 - 10X_5 - 9X_6 - 8X_7 - 10X_8 - 3X_9 - 3X_{10} - 5X_{11} - 7X_{12} = 0 \tag{3.2}$$
$$R_2 - X_1 - X_2 - X_3 - 2X_4 - 6X_5 - 10X_6 - 4X_7 - 10X_8 - X_9 - 2X_{10} - 3X_{11} - 6X_{12} = 0 \tag{3.3}$$
$$R_3 - 20X_1 - 20X_2 - 10X_3 - 50X_4 - 80X_5 - 60X_6 - 60X_7 - 100X_8 - 30X_9 - 60X_{10} - 60X_{11} - 70X_{12} = 0 \tag{3.4}$$
$$R_4 - 30X_1 - 30X_2 - 30X_3 - 40X_4 - 100X_5 - 80X_6 - 70X_7 - 100X_8 - 30X_9 - 40X_{10} - 50X_{11} - 90X_{12} = 0 \tag{3.5}$$

$$\mathrm{R}_5 - 50X_1 - 50X_2 - 50X_3 - 50X_4 - 50X_5 - 50X_6 - 50X_7 - 50X_8 - 50X_9 - 50X_{10} - 50X_{11} - 50X_{12} = 0 \tag{3.6}$$

$$\mathrm{E}_1 - 4X_1 - 3X_2 - 2X_3 - 5X_4 - 3X_5 - 8X_6 - 10X_7 - 20X_8 - 4X_9 - 5X_{10} - 12X_{11} - 10X_{12} = 0 \tag{3.7}$$
$$\mathrm{E}_2 - 10X_1 - 20X_2 - X_3 - 15X_4 - 50X_5 - 100X_6 - 50X_7 - 280X_8 - 40X_9 - 40X_{10} - 50X_{11} - 40X_{12} = 0 \tag{3.8}$$
$$\mathrm{E}_3 - 6X_1 - 7X_2 - X_3 - 5X_4 - 4X_5 - 4X_6 - 5X_7 - 7X_8 - 5X_9 - 6X_{10} - 6X_{11} - 6X_{12} = 0 \tag{3.9}$$
$$\mathrm{E}_4 - 4X_1 - 3X_2 - X_3 - 5X_4 - 16X_5 - 7X_6 - 7X_7 - 15X_8 - X_9 - 7X_{10} - 2X_{11} - 9X_{12} = 0 \tag{3.10}$$
$$\mathrm{E}_5 - 2X_1 - X_2 - X_3 - X_4 - 4X_5 - 2X_6 - 3X_7 - 4X_8 - 2X_9 - 2X_{10} - 2X_{11} - 6X_{12} = 0 \tag{3.11}$$

$$X_1 \leq 1{,}600 \tag{3.12}$$
$$X_2 \leq 2{,}400 \tag{3.13}$$
$$X_3 \leq 7{,}500 \tag{3.14}$$
$$X_4 \leq 3{,}000 \tag{3.15}$$
$$X_5 \leq 2{,}000 \tag{3.16}$$
$$X_6 \leq 4{,}000 \tag{3.17}$$
$$X_7 \leq 5{,}000 \tag{3.18}$$
$$X_8 \leq 6{,}500 \tag{3.19}$$
$$X_9 \leq 1{,}000 \tag{3.20}$$
$$X_{10} \leq 2{,}200 \tag{3.21}$$
$$X_{11} \leq 1{,}000 \tag{3.22}$$
$$X_{12} \leq 1{,}800 \tag{3.23}$$

$$\mathrm{E}_1 \leq 60{,}000 \tag{3.24}$$
$$\mathrm{E}_2 \leq 8\mathrm{R}_2 \tag{3.25}$$
$$\mathrm{E}_3 \leq 6\,(X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} + X_{12}) \tag{3.26}$$
$$\mathrm{E}_5 = 50{,}000 + \mathrm{E}_5^+ - \mathrm{E}_5^- \tag{3.27}$$

$$X_j = \mathrm{x}_j\mathrm{X}_j \quad \mathrm{j} = 1,\ldots,12 \tag{3.28}$$
$$\mathrm{x}_j = 0, 1 \quad \mathrm{j} = 1,\ldots,12 \tag{3.29}$$
$$\mathrm{x}_1 + \ldots + \mathrm{x}_{12} = \mathrm{p} \quad \mathrm{p} = \text{integer } (1,\ldots,11) \tag{3.30}$$

$$\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \mathrm{E}_4, \mathrm{E}_5, \mathrm{E}_5^+, \mathrm{E}_5^-, \mathrm{R}_1, \mathrm{R}_2, \mathrm{R}_3, \mathrm{R}_4, \mathrm{R}_5 \geq 0 \tag{3.31}$$
$$\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3, \mathrm{X}_4, \mathrm{X}_5, \mathrm{X}_6, \mathrm{X}_7, \mathrm{X}_8, \mathrm{X}_9, \mathrm{X}_{10}, \mathrm{X}_{11}, \mathrm{X}_{12} \geq 0. \tag{3.32}$$

In (3), the $X_j = \mathrm{x}_j\mathrm{X}_j$ represents the inclusion ($\mathrm{x}_j = 1$) of mix product j in the amount X_j (> 0) within the subsets of p mix products and $\mathrm{x}_j = 0$ indicates otherwise, j=1,…,12. Note the italics. The best subset of p mix variables has superior objective among variants of (3) with $\mathrm{x}_1 + \ldots + \mathrm{x}_{12} = \mathrm{p}$. The p may have a minimum value q (≥ 1) and maximum value r (< 12) with r < q.

For a given p, there are ${}_{12}\mathrm{C}_\mathrm{p}$ possible variants of (3) with $\mathrm{x}_1 + \ldots + \mathrm{x}_{12} = \mathrm{p}$ where ${}_{12}\mathrm{C}_\mathrm{p}$ denotes the number of combinations of the 12 indices (1,2,…,12) taking p at a time. Let S denote the set of indices j indicating the X_j that are free to be included or otherwise among each subset of p indices and S_c be its complement set. The latter may represent the mix products to be included (fixed) in each variant of (3). For example, the analyst may investigate variants of (3) in which X_1, X_2, X_4, X_5 are included and X_3, $\mathrm{X}_6 – \mathrm{X}_{12}$ are included one at a time, in pairs, triples, etc. In this situation, S = {3, 6, 7, 8, 9, 10, 11, 12} and S_c = {1, 2, 4, 5}. Note that S_c may be the null set. For each set of p indices obtained from set S, (3) is edited to include only the X_j for the j so selected and then solved. The sets of p indices may be generated using Step 3 of the interior analysis methodology presented in Section 3. For S = {1,2,3,4,5,6,7,8,9,10,11,12} and p = 3, the objective function value of the best solution for (3) is $3,156,625 with $X_1 = 11{,}625$, $X_3 = 3{,}875$, and $X_5 = 2{,}000$ under accommodation of all emission controls. Of course, for p = 7, the objective function value for the best solution to (3) is $1,781,188, see row 31 of Table 3.

Page limitation and the scope of the paper do not permit discussion of efficient ways to identify the best solution for $1 \leq p \leq 11$ of the Lemathe and Balakrishnan (2005) model and the green product mix problems in general. The point is that interior analysis produces alternatives that may not be otherwise examined.

4.2 Structure of the problems

For model (1), the market constraints (1.12)-(1.23) and the emission restrictions (1.24)-(1.27) bound the solution space. Apart from the usual non-negativity restrictions, the remaining statements are definitional. Consequently, as emission controls are included/excluded in the optimizations, some upper bounds (1.12)-(1.23) of the $X_1,\ldots,X_{12}$ quantities will be under-achieved. Model (1) is then very focused on the impact of greening on market position and identification of the best that can be expected there. If the availabilities of resources 1-5 were specified with right hand side (RHS) constants, the focus would broaden to include resource utilization and operations.

Due to the bounding structure of problem (1), the solution values for the X_j among the thirty-two scenarios of emission accommodation are generally at their lower (=0) or upper (=U_j) bound values, j=1,…,12. The exceptions arise with scenarios that include emissions 1 or 2 or both. The former may be explained in the following manner. Let X_j' denote the complement of X_j such that $X_j + X_j' = U_j$ where U_j is specified in (1.12)-(1.23), j=1,…,12, and let E_m' represent the under-achievement of emission restrictions (1.24)-(1.26) where m is (are) index (indices) of the included emission controls appearing in column 1 of Tables 3-5. For any scenario of (1), $X_1,\ldots,X_{12}$, $R_1,\ldots,R_5$, E_m, E_m' may constitute an initial basis. Iteration(s) taken to secure optimality from the initial basis often consist of an efficient exchange between a basic X_j and its non-basic complement X_j' with U_j as the solution value for the incoming variable. When X_j' is basic at the value U_j, $X_j = 0$ and the contrary is true. Except for emission 3, the emission restrictions in every scenario are binding so once in basis the E_i remain basic, i=1,2,4, as well as $R_1,\ldots,R_5$. Consequently, the operations required to obtain optimality generally but not exclusively consist of $X_j' = U_j$ replacing X_j in the bases encountered in the convergence to optimality. When X_j or X_j' are not basic at the value U_j, its value is determined among the (1.7)-(1.11) or (1.24)-(1.27).

The above is similarly true for model (2) and its solutions.

To obtain the mix solution for each scenario generated by the enumeration scheme given in the Appendix, the procedure began with the manual editing of (1) or (2), was followed with its submission to the Lindo LP application package for solution, and concluded with calculation of the operational impact measures described in Step 4 of the proposed methodology of Section 3.1. A spreadsheet with the results for each scenario in each illustration is available from the corresponding author. Although it is beyond the scope of this paper, an interface can be developed that assists in automatically generating, solving, and computing mix features for all variants of formulations (1) and (2) discussed in this paper.

5. Summary and Remarks

The implementation of environmentally friendlier production challenges the analyst and the decision maker in very profound ways. Whether it is accomplished through restrictions on emissions, their transformations to more green friendly forms, or some combination of both as discussed in this paper, the result will likely have unattractive features of objective, resource use, product/process fit, and others.

In this paper, we presented and illustrated a method for accommodating emission controls and emission transformations one at a time or in combinations within the framework of the green LP product mix problem. We pointed out how the methodology could provide a basis for devising a strategy of partial/stepwise accommodation of emission controls. We referred to the methodology as interior analysis and included a scheme for exhaustively enumerating the possibilities. Measures for comparing product mix solutions were also presented. We discussed the structure of Models (1) and (2) that lends understanding to the solutions it produces.

The value of the proposed methodology lies in the features of the solutions it produces. If a prohibitive reduction in the objective accompanies simultaneous accommodation of emission controls, the decision maker may frame implementation strategies based on stepwise/partial enforcement or even satisficing of emission targets with softer impact on the objective. If the green product mix affects badly the alignment of product and process, the analyst may turn to interior analysis for alternatives with more appealing features of fit. If the transformation of emissions to product can offset some loss in objective due to emission controls, interior analysis can assist in assessing its capacity to do so. Interior analysis may also reveal features of the solution set under stepwise implementation that would not be otherwise evident.

The accommodation of emission controls and transformations has significant consequences. When the product mix model is used for analyzing its impact, the needs of the analyst and the decision maker go beyond just formulation and solution, the treatment most commonly given in the literature. Tools of analysis that assist the analyst and decision maker in the framing of implementation strategy, in assessing impact on process fit, or in the search for accommodations with more appealing aspects are needed. Interior analysis is proposed as a step in this direction.

The model of Letmathe and Balakrishnan (2005) was utilized for illustration purposes only.

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Table 1. Solutions to (1) with and without control of all emissions: the product mix.

Emission Controls	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	**Net Profit ($)**
	(in thousands)												
None	1.6	2.4	0	3.0	2.0	4.0	5.0	6.5	1.0	2.2	1.0	1.8	5,386,000
All	1.6	2.4	3.0[a]	3.0	2.0	0	0	0	9	0	0[b]	1.8	1,781,188

[a] 3.056 [b] 0.107

Table 2. Solutions to (1) with and without control of all emissions: resource usage and emission production.

Emission Controls	**Conventional Resource Usage**					**Emissions**				
	R_1	R_2	R_3	R_4	R_5	E_1	E_2	E_3	E_4	E_5
	(in thousands)									
None	212.2	166.2	1,928	2,090	1,525	291.6	2,929	170.9	255.7	84.8
All	66.304	36.178	552.999	699.044	698.162	60.0	289.424	63.9	80.071	30.671
Percentage Change	-68.78	-78.23	-71.32	-66.55	-54.22	-79.42	-90.12	-62.61	-68.69	-63.83

Table 3. Interior analysis of solutions to (1): product mix values.

Row	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	Scenario of included emission controls[a]	**Object. function value ($)**	**In thousands**											
			X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
1	1	2,395,000	1.6	2.4	0	3	2	1.3	0	0	1	2.2	0	0
2	2	1,947,167	1.6	2.4	7.500	3	2	0	1.883	0	0	0	0	1.8
3	3	5,386,000	1.6	2.4	0	3	2	4	5	6.5	1	2.2	1	1.8
4	4	5,130,300	1.6	2.4	0	3	2	4	5	6.5	1	2.2	1	1.8
5	5	5,212,000	1.6	2.4	0	3	2	4	5	6.5	1	2.2	1	1.8
6	1,2	1,785,104	1.6	2.4	3.092	3	2	0.152	0	0	0	0	0	1.8
7	1,3	2,395,000	1.6	2.4	0	3	2	1.3	0	0	1	2.2	0	0
8	1,4	2,308,900	1.6	2.4	0	3	2	1.3	0	0	1	2.2	0	0
9	1,5	2,492,600	1.6	2.4	0	3	2	1.3	0	0	1	2.2	0	0
10	2,3	1,947,167	1.6	2.4	7.500	3	2	0	1.883	0	0	0	0	1.8
11	2,4	1,849,683	1.6	2.4	7.500	3	2	0	1.883	0	0	0	0	1.8
12	2,5	1,984,967	1.6	2.4	7.500	3	2	0	1.883	0	0	0	0	1.8
13	3,4	5,130,300	1.6	2.4	0	3	2	4	5	6.5	1	2.2	1	1.8
14	3,5	5,212,000	1.6	2.4	0	3	2	4	5	6.5	1	2.2	1	1.8
15	4,5	4,956,300	1.6	2.4	0	3	2	4	5	6.5	1	2.2	1	1.8
16	1,2,3	1,785,104	1.6	2.4	3.092	3	2	0.152	0	0	0	0	0	1.8
17	1,2,4	1,704,300	1.6	2.4	3.011	3	2	0	0.138	0	0	0	0	1.8
18	1,2,5	1,861,921	1.6	2.4	3.092	3	2	0.152	0	0	0	0	0	1.8
19	1,3,4	2,308,900	1.6	2.4	0	3	2	1.3	0	0	1	2.2	0	0
20	1,3,5	2,492,600	1.6	2.4	0	3	2	1.3	0	0	1	2.2	0	0
21	1,4,5	2,406,500	1.6	2.4	0	3	2	1.3	0	0	1	2.2	0	0
22	2,3,4	1,849,683	1.6	2.4	7.500	3	2	0	1.883	0	0	0	0	1.8
23	2,3,5	1,984,967	1.6	2.4	7.500	3	2	0	1.883	0	0	0	0	1.8
24	2,4,5	1,887,483	1.6	2.4	7.500	3	2	0	1.883	0	0	0	0	1.8
25	3,4,5	4,956,300	1.6	2.4	0	3	2	4	5	6.5	1	2.2	1	1.8
26	1,2,3,4	1,704,300	1.6	2.4	3.011	3	2	0	0.138	0	0	0	0	1.8
27	1,2,3,5	1,861,921	1.6	2.4	3.092	3	2	0.152	0	0	0	0	0	1.8
28	1,2,4,5	1,781,188	1.6	2.4	3.056	3	2	0	0	0	0	0	0.107	1.8
29	1,3,4,5	2,406,500	1.6	2.4	0	3	2	1.3	0	0	1	2.2	0	0
30	2,3,4,5	1,887,483	1.6	2.4	7.500	3	2	0	1.883	0	0	0	0	1.8
31	1,2,3,4,5	1,781,188	1.6	2.4	3.056	3	2	0	0	0	0	0	0.107	1.8
32	-	5,386,000	1.6	2.4	0	3	2	4	5	6.5	1	2.2	1	1.8

[a] 1 represents inclusion of constraint (1.24) in (1); 2, inclusion of (1.25); 3, inclusion of (1.26); 4, inclusion of $-E_4$ in objective function (1.1); and 5, inclusion of constraint (1.27) and $-5E_5^+ + 4E_5^-$ in objective function (1.1).

Table 4. Interior analysis of solutions to (1): impact on mix, resource use, emissions.

Row	(1)	(2)	(3)	(4)	(5)					(6)				
	Scenario of included emission controls[a]	Object. function value ($)	% change in mix volume	No. of zero values	% reduction in resource utilization with emission controls					% reductions in emissions with controls				
					R1	R2	R3	R4	R5	E1	E2	E3	E4	E5
1	1	2,395,000	44.26	5	69.23	75.69	67.32	68.33	55.74	79.42	84.06	57.40	66.33	69.81
2	2	1,947,167	66.17	5	55.62	71.22	63.49	54.12	33.83	70.36	86.94	54.88	61.88	52.18
3	3	5,386,000	100.00	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	4	5,130,300	100.00	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	5	5,212,000	100.00	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	1,2	1,785,104	46.05	5	68.31	77.49	71.16	66.18	53.95	79.42	89.78	62.61	68.34	63.68
7	1,3	2,395,000	44.26	5	69.23	75.69	67.32	68.33	55.74	79.42	84.06	57.40	66.33	69.81
8	1,4	2,308,900	44.26	5	69.23	75.69	67.32	68.33	55.74	79.42	84.06	57.40	66.33	69.81
9	1,5	2,492,600	44.26	5	69.23	75.69	67.32	68.33	55.74	79.42	84.06	57.40	66.33	69.81
10	2,3	1,947,167	66.17	5	55.62	71.22	63.49	54.12	33.83	70.36	86.94	54.88	61.88	52.18
11	2,4	1,849,683	66.17	5	55.62	71.22	63.49	54.12	33.83	70.36	86.94	54.88	61.88	52.18
12	2,5	1,984,967	66.17	5	55.62	71.22	63.49	54.12	33.83	70.36	86.94	54.88	61.88	52.18
13	3,4	5,130,300	100.00	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14	3,5	5,212,000	100.00	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	4,5	4,956,300	100.00	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	1,2,3	1,785,104	46.05	5	68.31	77.49	71.16	66.18	53.95	79.42	89.78	62.61	68.34	63.68
17	1,2,4	1,704,300	45.73	5	68.55	78.12	71.25	66.41	54.27	79.42	90.07	62.61	68.41	63.65
18	1,2,5	1,861,921	46.05	5	68.31	77.49	71.16	66.18	53.95	79.42	89.78	62.61	68.34	63.68
19	1,3,4	2,308,900	44.26	5	69.23	75.69	67.32	68.33	55.74	79.42	84.06	57.40	66.33	69.81
20	1,3,5	2,492,600	44.26	5	69.23	75.69	67.32	68.33	55.74	79.42	84.06	57.40	66.33	69.81
21	1,4,5	2,406,500	44.26	5	69.23	75.69	67.32	68.33	55.74	79.42	84.06	57.40	66.33	69.81
22	2,3,4	1,849,683	66.17	5	55.62	71.22	63.49	54.12	33.83	70.36	86.94	54.88	61.88	52.18
23	2,3,5	1,984,967	66.17	5	55.62	71.22	63.49	54.12	33.83	70.36	86.94	54.88	61.88	52.18
24	2,4,5	1,887,483	66.17	5	55.62	71.22	63.49	54.12	33.83	70.36	86.94	54.88	61.88	52.18
25	3,4,5	4,956,300	100.00	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
26	1,2,3,4	1,704,300	45.73	5	68.55	78.12	71.25	66.41	54.27	79.42	90.07	62.61	68.41	63.65
27	1,2,3,5	1,861,921	46.05	5	68.31	77.49	71.16	66.18	53.95	79.42	89.78	62.61	68.34	63.68
28	1,2,4,5	1,781,188	45.78	5	68.75	78.23	71.32	66.55	54.22	79.42	90.12	62.61	68.69	63.83
29	1,3,4,5	2,406,500	44.26	5	69.23	75.69	67.32	68.33	55.74	79.42	84.06	57.40	66.33	69.81
30	2,3,4,5	1,887,483	66.17	5	55.62	71.22	63.49	54.12	33.83	70.36	86.94	54.88	61.88	52.18
31	1,2,3,4,5	1,781,188	45.78	5	68.75	78.23	71.32	66.55	54.22	79.42	90.12	62.61	68.69	63.83
32	-	5,386,000	0	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

[a] 1 represents inclusion of constraint (1.24) in (1); 2, inclusion of (1.25); 3, inclusion of (1.26); 4, inclusion of $–E_4$ in objective function (1.1); and 5, inclusion of constraint (1.27) and $-5E_5^+ + 4E_5^-$ in objective function (1.1).

Table 5. Interior analysis of solutions to (1): impact on emission controls.

Row	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Scenario of included emission controls[a]	**Object. function value ($)**	**Is $E_1 \le 60,000$?**	**Is $E_2 \le 8R_2$?**	**Is $E_3 \le 6(X_1 + \ldots + X_{12})$?**	**Cost**[b] **of emission 4 ($)**	**Economic gain**[c] **with trading emission 5 ($)**
1	1	2,395,000	Yes	No	Yes	86,100	97,600
2	2	1,947,167	No	Yes	Yes	97,483	37,800
3	3	5,386,000	No	No	Yes	255,700	-174,000
4	4	5,130,300	No	No	Yes	255,700	-174,000
5	5	5,212,000	No	No	Yes	255,700	-174,000
6	1,2	1,785,104	Yes	Yes	Yes	80,956	76,817
7	1,3	2,395,000	Yes	No	Yes	86,100	97,600
8	1,4	2,308,900	Yes	No	Yes	86,100	97,600
9	1,5	2,492,600	Yes	No	Yes	86,100	97,600
10	2,3	1,947,167	No	Yes	Yes	97,483	37,800
11	2,4	1,849,683	No	Yes	Yes	97,483	37,800
12	2,5	1,984,967	No	Yes	Yes	97,483	37,800
13	3,4	5,130,300	No	No	Yes	255,700	-174,000
14	3,5	5,212,000	No	No	Yes	255,700	-174,000
15	4,5	4,956,300	No	No	Yes	255,700	-174,000
16	1,2,3	1,785,104	Yes	Yes	Yes	80,956	76,817
17	1,2,4	1,704,300	Yes	Yes	Yes	80,775	76,702
18	1,2,5	1,861,921	Yes	Yes	Yes	80,956	76,817
19	1,3,4	2,308,900	Yes	No	Yes	86,100	97,600
20	1,3,5	2,492,600	Yes	No	Yes	86,100	97,600
21	1,4,5	2,406,500	Yes	No	Yes	86,100	97,600
22	2,3,4	1,849,683	No	Yes	Yes	97,483	37,800
23	2,3,5	1,984,967	No	Yes	Yes	97,483	37,800
24	2,4,5	1,887,483	No	Yes	Yes	97,483	37,800
25	3,4,5	4,956,300	No	No	Yes	255,700	-174,000
26	1,2,3,4	1,704,300	Yes	Yes	Yes	80,775	76,702
27	1,2,3,5	1,861,921	Yes	Yes	Yes	80,956	76,817
28	1,2,4,5	1,781,188	Yes	Yes	Yes	80,071	77,318
29	1,3,4,5	2,406,500	Yes	No	Yes	86,100	97,600
30	2,3,4,5	1,887,483	No	Yes	Yes	97,483	37,800
31	1,2,3,4,5	1,781,188	Yes	Yes	Yes	80,071	77,318
32	-	5,386,000	No	No	Yes	255,700	-174,000

[a] 1 represents inclusion of constraint (1.24) in (1); 2, inclusion of (1.25); 3, inclusion of (1.26); 4, inclusion of $-E_4$ in the objective function (1.1); and 5, inclusion of constraint (1.27) and $-5E_5^+ + 4E_5^-$ in objective function (1.1).

[b] $-E_4$ [c] $- 5E_5^+ + 4E_5^-$

Table 6. Volume/quantity transformation of emissions to products.

Emission	Transformed by-product per unit of emission			
	1	2	3	4
1	0.05	0	0	0
2	0	0.025	0	0
3	0	0	0.05	0
4	0	0	0.048	0
5	0	0	0	0.05
Quantity	TP_1	TP_2	TP_3	TP_4

Table 7. Per unit resource requirements of transformed by-products.

Resource type	Item	Per unit resource requirement of transformed by-products				Resource quantity
		1	2	3	4	
Standard	1	1	0	0.1	0.2	RT_1
	2	1	0.5	0.1	0.2	RT_2
	3	0.6	0.1	0.75	0.65	RT_3
	4	1	0	0	1.4	RT_4
	5	0	0.5	0.50	0.5	RT_5
Transformation specific	1	10	7	9	11	ST_1
	2	11	11	7	8	ST_2
	3	6	9	6	6	ST_3
	4	12	12	10	13	ST_4

Table 8. Solutions to (2) with transformed emissions.

Row	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12	(13)	(14)
	Scenario of included transformed emissions[a]	Object. function value ($)	In thousands											
			X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
1	-	1,781,188	1.6	2.4	3.056	3	2	0	0	0	0	0	0.107	1.8
2	1	1,780,024	1.6	2.4	0.938	3	2	0	0	0	0	0	0.460	1.8
3	1,2	2,791,286	1.6	0	0	3	2	3.828	0	0	0	0	0	0.198
4	1,2,3	3,608,369	1.6	2.4	0	3	2	2.453	0	0	0	0	0	0.578
5	1,2,3,4	3,712,643	1.6	2.4	0	3	2	2.553	0	0	0	0	0	0.498
6	1,2,4	2,888,065	1.6	0	0	3	2	3.921	0	0	0	0	0	0.124
7	1,3	2,647,095	1.6	2.4	1.457	3	2	0	0	0	0	0.897	0	1.8
8	1,3,4	2,766,823	1.6	2.4	1.309	3	2	0	0	0	0	0.957	0	1.8
9	1,4	1,893,904	1.6	2.4	0.522	3	2	0	0.636	0	0	0	0	1.8
10	2	2,672,045	1.6	0	0	3	2	2.603	0	0	0	0	0	1.178
11	2,3	3,496,566	1.6	2.4	0	3	2	1.231	0	0	0	0	0	1.555
12	2,3,4	3,615,127	1.6	2.4	0	3	2	1.345	0	0	0	0	0	1.464
13	2,4	2,783,224	1.6	0	0	3	2	2.710	0	0	0	0	0	1.092
14	3	2,629,272	1.6	2.4	2.914	3	2	0	0	0	0	0.314	0	1.8
15	3,4	2,750,137	1.6	2.4	2.764	3	2	0	0	0	0	0.374	0	1.8
16	4	1,899,152	1.6	2.4	2.779	3	2	0	0.184	0	0	0	0	1.8

[a] 1 denotes transformed emission 1; 2 transformed emission 2; 3 transformed emissions 3 and 4; 4 transformed emission 5.

Table 9. Objective function contributions of the transformed emissions.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
			In thousands				
Row	Scenario of included transformed emissions[a]	Object. function value ($)	TP_1	TP_2	TP_3	TP_4	% of object. f. value
1	-	1,781,188	-	-	-	-	-
2	1	1,780,024	3.000	-	-	-	[b]
3	1,2	2,791,286	3.000	13.792	-	-	26.62
4	1,2,3	3,608,369	3.000	11.935	7.116	-	39.69
5	1,2,3,4	3,712,643	3.000	12.106	7.111	1.235	41.09
6	1,2,4	2,888,065	3.000	13.950	-	1.139	28.72
7	1,3	2,647,095	3.000	-	7.409	-	26.20
8	1,3,4	2,766,823	3.000	-	7.433	1.531	28.87
9	1,4	1,893,904	3.000	-	-	1.491	[b]
10	2	2,672,045	-	11.710	-	-	28.35
11	2,3	3,496,566	-	9.857	7.117	-	41.59
12	2,3,4	3,615,127	-	10.052	7.171	1.404	43.16
13	2,4	2,783,224	-	11.892	-	1.309	30.80
14	3	2,629,272	-	-	7.182	-	31.07
15	3,4	2,750,137	-	-	7.205	1.546	33.57
16	4	1,899,152	-	-	-	1.537	5.42

[a] 1 denotes transformed emission 1; 2 transformed emission 2; 3 transformed emissions 3 and 4; 4 transformed emission 5.

[b] negative contribution to the objective.