AN EFFECTIVE ALGORITHM FOR SOLVING A SINGLE-SUPPLIER MULTIPLE-HETEROGENEOUS CUSTOMERS NETWORK PROBLEM

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ABSTRACT

This paper focuses on a single-supplier multiple-heterogamous customer network problem with the consideration of dynamic demand. We analyze a two-period special case, and then develop an effective algorithm for solving this problem. The performance of the proposed algorithm is evaluated through simulation of 100 consultative planning cycles with 10 time periods in each cycle.

Keywords: Integrated Production and Distribution, heterogeneous customer, dynamic demand

INTRODUCTION

The integrated production and distribution problem (PDP) is concerned with coordinating the production, inventory and distribution operations in a supply chain process with respect to a given performance measure. Optimal solution for a PDP can be generally obtained by solving a large MIP at the beginning of each planning cycle to develop a Master Production Schedule (MPS) that minimizes the total cost of production, inventory and distribution. This MPS solution assumes that the demand information at each of the distribution centers (DCs) is given deterministically at the beginning of each planning cycle. Often, in practice, demand at DCs is uncertain and can change within a planning cycle, which can make the MPS solution developed at the beginning of the planning cycle a sub-optimal or infeasible solution. A common solution approach to this problem is to revise the MPS solution every time the updated demand information arrives to the central planning system (Lin and Krajewski 1992; Lin et al. 1994; Vairaktarakis 1997; Keung et al. 2001; Robinson et al. 2008; Gelogullari and Logendran 2010). While revising MPS solution leads to lower operating costs, the downside of this approach is that frequent changes to the MPS solution causes instability in the system, often referred to as system nervousness. It is generally believed that the system nervousness is mainly due to frequent changes in the production schedule (Tang and Grubbstrom 2003; Koh and Gunasekaran 2006; Filho and Fernandes 2009; etc.), because the production schedule directly interacts with many other systems, such as material requirements planning (MRP) and enterprise resource planning (ERP), and not so much due to changes in the inventory or distribution schedules. In this study we focus on revising only the inventory and distribution schedule while keeping the production schedule intact; this ensures that revising of schedule leads to lower overall operating cost without affecting system stability. Two major contributions of this paper are: (1) an optimal revised inventory and distribution solution can be obtained as a close-form solution in a special case of the problem and (2) empirical studies suggest that the cost savings by revising the inventory and distribution schedule are significant. Our results suggest that, as the updated demand data arrives, the operations managers could focus only on revising the inventory and distribution schedule for PDP, while keeping the production schedule same as one given by original MPS solution, thus not worrying about system nervousness.

LITERATURE REVIEW AND PROBLEM DEFINITION

Existing literature related to this study can be classified into two categories: integrated PDPs and dynamic scheduling. The work on *integrated PDPs* (Martin, et al. 1993; Chandra and Fisher 1994; Ozdamar and Yazgac 1999; Lei, et al. 2003; Lei and Zhong 2004; Nonino ,and Panizzolo 2007; Bilgen and Günther 2010; Jolai et al. 2011) usually deal with large scale networks with multiple time periods but assumes static demand. Majority of the work on *dynamic scheduling* focuses on machine scheduling and job-shop scheduling problems with dynamic job arrivals. Representative work in this category can be found in Cowling and Johansson (2002), Rangsaritratsamee et. al. (2004), and Wu et al. (1993). Dynamic scheduling work on PDPs that assumes stochastic demand has mostly focused on developing revised production schedule as the updated demand information arrives. Most of this work aims at finding the optimal trade-off between the reduced operating cost and increased instability of the production system due to frequent changes in the production schedule. Representative work in this category can be found in Tang and Grubbstrom (2003), Lin and Krajewski (1992), and Bauso et al. (2010). The work of Cachon and Fisher (2000) comes close to our study. They find the value of updated demand and inventory information from a set of retailers to the manufacturer in the supply chain. Their results are similar to this study, but they focus only on inventory holding and backorder costs. The other works (Lei, et al. 2003, Lei and Zhong 2004; Nonino and Panizzolo 2007; Bilgen and Günther 2010; Jolai et al. 2011) deal with some specific problems and the approaches are diffirent from our proposed method. We study the effect of updated demand and inventory information sharing to the total inventory and distribution costs in the supply chain. To the best of our knowledge, there is no any published work on a dynamic scheduling that addresses the combined effect of information sharing on inventory and distribution aspects of large scale integrated PDPs in stochastic demand setting.

In this study we consider a hypothetical supply network that consists of a single supplier/manufacturer and *J* heterogeneous distribution centers. We consider a planning horizon that consists of T time periods. The demand at *DC j* in time period t is given by $d_{j,t}$, j = 1,2,…J; t=1,2,…,T. The shipping cost from supplier to a *DC*, inventory holding cost and shortage cost at DC j are given by fj, hj and pj, respectively, *j = 1,2,…J*. We assume that a master production schedule (that can be obtained by solving a large MIP) is given, which specifies the optimal production lot size for each time period in the planning horizon as $< Q_1, Q_2, ..., Q_T >,\ Q_t \geq 0,\ t = 1,2,...,T,$ where Q_t=0 means that there is no production run in time period t. If the production quantity for a time period is positive, i.e., Q_t > 0, then it can be adjusted within a given range $Q_t(1 \pm \theta),\ where\ 0 \leq \theta < 1$. The transportation cost for a *DC* is directly proportional to the number of truck trips required between the manufacturer's plant and the DC site in each time period. All *DCs* are required to send the plant their demand forecast for each time period in a planning horizon at the beginning of the planning horizon, upon which the plant makes its master production schedule that specifies the production, inventory and distribution schedule for the entire planning horizon. The production schedule determined by this MPS is fixed and does not change during the planning horizon. At each time point during the planning horizon, all *DCs* provide the updated information about their demand and current inventory level to the plant. While the plant is not allowed to change the production schedule once the planning horizon has started, it may revise its inventory and distribution schedule in order to lower its total operating cost. Our goal is to find a revised inventory and distribution schedule that minimizes the total operating cost, given the latest demand and inventory information from the *DCs* at each time point of the planning horizon.

THE TWO-PERIOD PROBLEM – A SPECIAL CASE

We consider a special case of the dynamic scheduling problem introduced above, where *T=2*. Let $\{Q_1, Q_2\}$ be the given master production plan, and let $Z_t = 1 \text{ } if \text{ } Q_t > 0,\ t = 1,2.$ A formal definition of this two-period problem (**P₂**) is follows.

$$Min.\ G(I_{0,0}, Z_1, Z_2) = \sum_{j=1,2,\ldots,J} [f_j(Y_{j,1} + Y_{j,2}) + h_j \cdot I_{j,1} + p_j \cdot U_{j,2}] + h_0 \cdot I_{0,1}$$

$st.$

$$\sum_{j=1,2,\ldots,J} X_{j,1} \le I_{0,0} + M \cdot Z_1$$

$$\sum_{j=1,2,\ldots,J} X_{j,2} \le I_{0,0} + M \cdot Z_1 + M \cdot Z_2$$

$$X_{j,1} \le C \cdot Y_{j,1} \qquad j = 1,2,\ldots,J$$

$$X_{j,2} \le C \cdot Y_{j,2} \qquad j = 1,2,\ldots,J$$

$$X_{j,1} - d_{j,1} = I_{j,1} \qquad j = 1,2,\ldots,J$$

$$X_{j,1} + X_{j,2} + U_{j,2} \ge d_{j,2} \qquad j = 1,2,\ldots,J$$

Let $S_{j,1}$ be the *spare truck capacity* for DC_j in *t=1*, and $S_{j,1} = \left\lceil \frac{d_{j,1}}{C} \right\rceil C - d_{j,1}$; $\Delta_{j,2}$ be the *residual load* of DC_j in *t=2*, and $\Delta_{j,2} = d_{j,2} - \left\lfloor \frac{d_{j,2} + I_{j,2}}{C} \right\rfloor \cdot C$; $\delta_{j,2}$ be the *feasible shifting amount* (*decision variables*) to be shipped out of plant to DC_j in *t=1*, that will be inventoried and then used to meet the demand in *t+1*. Since all the DCs' order change must be one period ahead, we assume $I_{0,0} + Q_1 \ge \sum_{j=1,2,\ldots,J} d_{j,1}$ and $d_{j,2},\ j = 1,2,..,J,$ are known at the beginning of *t=1*.

Definition 1. The supply to the two-period problem is *sufficient* if

$I_{0,0} + Q_1 + Q_2 \ge \sum_{j=1,2,\ldots,J} d_{j,1} + d_{j,2} + I_{j,2}$ which occurs when any of the following holds

$$I_{0,0} \ge \sum_{j=1,2,\ldots,J} d_{j,1} + d_{j,2} + I_{j,2} \qquad if\ Q_1 = Q_2 = 0$$

$$Q_2 > 0 \qquad if\ Q_1 = 0$$

$$Q_1 > 0 \qquad regardless\ Q_2$$

Lemma 1. If the supply is sufficient, then there exists an optimal shifting amount $\delta^*_{j,2}$ such that $\delta^*_{j,2} = \Delta_{j,2}$ if $\Delta_{j,2} \cdot (h_j - h_0) < f_j$ and $\Delta_{j,2} \le S_{j,1}$, and $\delta^*_{j,2} = 0$ otherwise.

Proposition 1. If the supply is sufficient, then the optimal solution to the two-period dynamic scheduling problem has a close-form solution with

$$Y^*_{j,1} = \left\lceil \frac{d_{j,1}}{C} \right\rceil, \quad Y^*_{j,2} = \left\lfloor \frac{d_{j,2} + I_{j,2} - \delta^*_{j,2}}{C} \right\rfloor; \quad X^*_{j,1} = d_{j,1} + \delta^*_{j,2}, \; X^*_{j,2} = d_{j,2} + I_{j,2} - \delta^*_{j,2}, ;$$

$$I^*_{j,1} = \delta^*_{j,2} \qquad j = 1,2,..., J$$

where $Y^*_{j,\tau}$, $X^*_{j,\tau}$, *and* $I^*_{j,\tau}$ are optimal truck-trips (integers), shipping quantities, and ending DC inventories for time period t=1, 2.

For any given instance of the two-period problem, since $I_{0,0} + Q_1 \ge \sum_{j=1,2,...,J} d_{j,1}$, the insufficient supply case may occur only when $Q_1 = Q_2 = 0$. That is, the insufficient supply case occurs when either of the following conditions holds

$$I_{0,0} < \sum_{j=1,2,...,J} d_{j,1} + d_{j,2} + I_{j,2} \;\; but \;\; I_{0,0} \ge \sum_{j=1,2,...,J} d_{j,1} + d_{j,2} \qquad \text{(1a)}$$

or

$$I_{0,0} < \sum_{j=1,2,...,J} d_{j,1} + d_{j,2} \;\; but \;\; I_{0,0} \ge \sum_{j=1,2,...,J} d_{j,1} \qquad \text{(1b)}$$

Lemma 2. Problem $\mathbf{P_2}$ with insufficient supply is a common knapsack problem.
<u>*Proof*</u>*:* (skipped)

Proposition 2. If the supply is insufficient, then the following heuristic solution to $\mathbf{P_2}$

$$Y^*_{j,1} = \left\lceil \frac{d_{j,1}}{C} \right\rceil, \quad Y^*_{j,2} = \left\lfloor \frac{d_{j,2} + I_{j,2} - \delta^*_{j,2}}{C} \right\rfloor,$$

$$X^*_{j,1} = d_{j,1} + \lambda_j \cdot \delta^*_{j,2}, \; X^*_{j,2} = d_{j,2} + I_{j,2} - \lambda_j \cdot \delta^*_{j,2},$$

$$I^*_{j,1} = \lambda_j \cdot \delta^*_{j,2} \qquad j = 1,2,..., J \text{ and } 0 \le \lambda_j \le 1$$

has a worst case error bound $2 \cdot G^*_\alpha$ where G^*_α is the optimal objective function value of the following linear programming.

P$_{\mathbf{a}}$

$$
\begin{aligned}
& Min.\ G_\alpha = \sum_{j=1,2,\ldots,J} [\, f_j (Y_{j,1} + Y_{j,2}) + h_j \cdot I_{j,1} - g_j \cdot \lambda_j \,] \\
& st. \\
& \qquad \sum_{j=1,2,\ldots,J} \delta^*_{j,2} \cdot \lambda_j \le I_{0,0} - \sum_{j=1,2,\ldots,J} d_{j,1} \\
& \qquad X_{j,1} = d_{j,1} + \delta^*_{j,2} \cdot \lambda_j \qquad j = 1,2,\ldots, J \\
& \qquad X_{j,2} = d_{j,2} + I_{j,2} - \delta^*_{j,2} \cdot \lambda_j \qquad j = 1,2,\ldots, J \\
& \qquad X_{j,1} - d_{j,1} = I_{j,1} \qquad j = 1,2,\ldots, J \\
& \qquad X_{j,1} \le C \cdot Y_{j,1} \qquad j = 1,2,\ldots, J \\
& \qquad X_{j,2} \le C \cdot Y_{j,2} \qquad j = 1,2,\ldots, J \\
& \qquad \lambda_j \in \{0,1\} \qquad j = 1,2,\ldots,J \\
& \qquad \text{All variables are non - negative.}
\end{aligned}
$$

where $\lambda_j = 1$ means that the $\delta_{j,2}$ will be shipped to DC_j in t, that will be used in t+1, 0 otherwise, g_j is a saving factor and defined by $g_j = f_j - \delta_{j,2} \cdot (h_j - h_0)$.

ALGORITHM DEVELOPMENT FOR SOLVING PROBLEM P

According to the analysis of two-period problem, we propose an iterative dynamic scheduling algorithm for solving **P** (see Figure 1). In each iteration, it applies Propositions to solve a two-period problem with periods *t* and *t+1*, t=1, 2,…,T-1, and uses the resulting production quantities, $Q'_t = \sum_{j=1,2,\ldots,J} X_1 \ and\ Q'_{t+1} = \sum_{j=1,2,\ldots,J} X_{j,2}$, as the updated production lot sizes for periods *t* and *t+1*, where $Q'_t > 0\ iff\ Z_t = 1$ and $Q'_{t+1} > 0\ iff\ Z_{t+1} = 1$ in the original master production plan.

The input to this dynamic scheduling algorithm is the master production plan $< Z_1, Z_2, Z_3, \ldots, Z_T >$**,** where if and only if the respective lot size $Q_t > 0$ in the optimal solution to the following mixed integer programming problem.

$$
Minimize: \ \sum_{t\in T} (f_0 \cdot Z_t + h_0 \cdot I_{0,t}) + \sum_{\forall j \forall J} \sum_{\forall t\in T} f_j \cdot Y_{j,t} + \sum_{\forall j\in J} \sum_{\forall t\in T} h_j \cdot I_{j,t}
$$

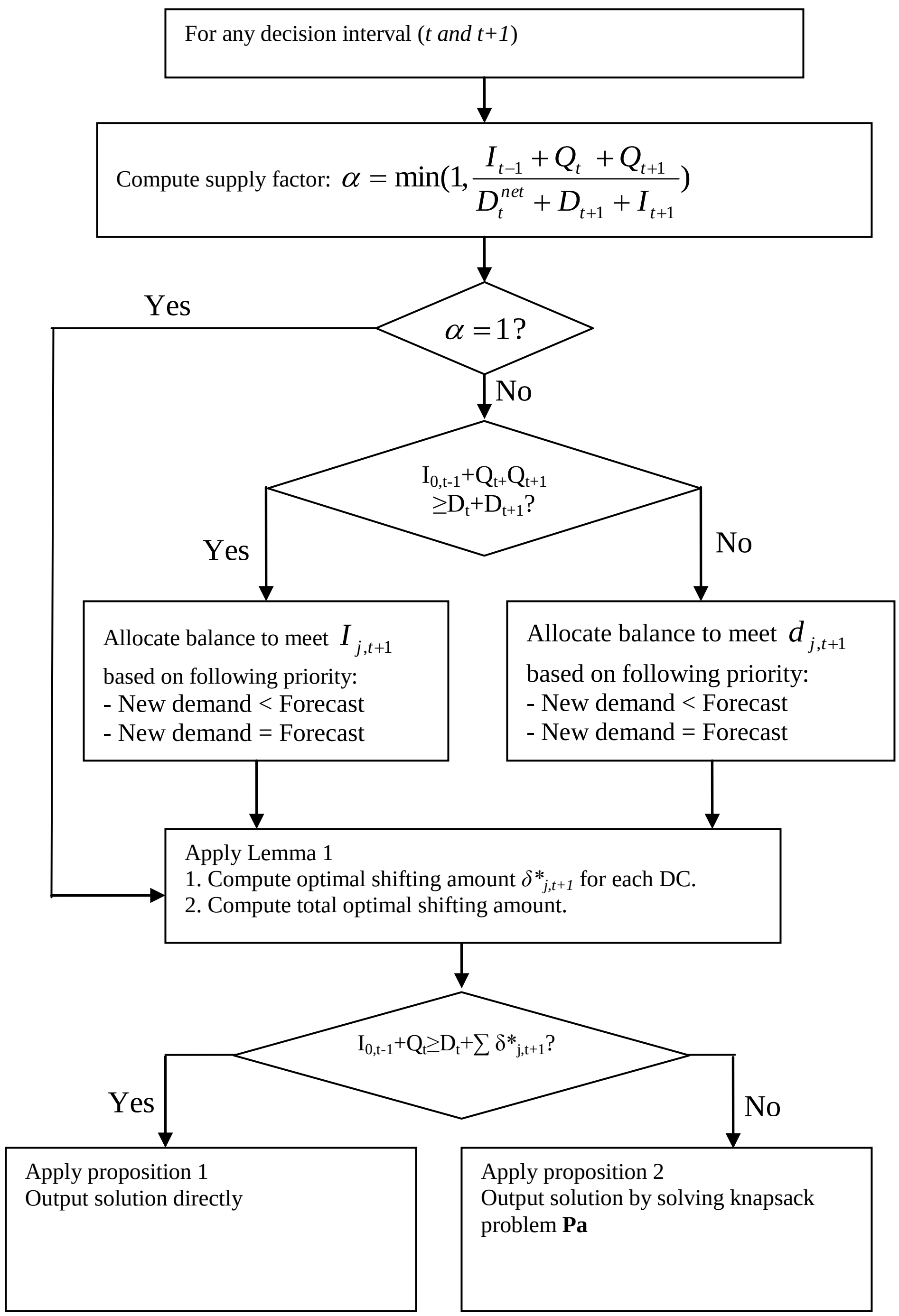
s.t.

$$
\begin{aligned}
& Q_t \le M \cdot Z_t \quad \forall\ t = 1,2,\ldots,T \qquad \text{(Relationship between } Q_t \text{ and } Z_t\text{)} \\
& I_{0,t} \ge 0.1 \cdot \sum_{\forall j \forall J} \sum_{\forall t\in T} d_{j,t} \quad \forall\ j \in J, t = 1,2,\ldots,T \ \text{(Safety stock)} \\
& I_{0,t-1} + Q_t - \sum_{j \forall J} X_{j,t} = I_{0,t} \quad \forall\ t = 1,2,\ldots,T \qquad \textit{(The plant's flow balance)} \\
& I_{j,t-1} + X_{j,t} = d_{j,t} + I_{j,t} \quad \forall\ j \in J, t = 1,2,\ldots,T \quad \textit{(DCs's flow balance)} \\
& X_{j,t} \le C \cdot Y_{j,t} \quad \forall\ j \in J, t = 1,2,\ldots,T \qquad \text{(Shipping capacity constraint)} \\
& Z_t \in \{0,1\} \quad \forall\ t = 1,2,\ldots,T
\end{aligned}
$$

$Y_{j,t} \geq 0$ Integer

All variables are non-negative

Figure 1. Algorithm Flow Chart



EMPIRICAL OBSERVATIONS FROM THE INITIAL EVALUATION

The proposed algorithm is applied to each decision interval in a given planning cycle. Its performance is evaluated through computer simulation of certain consecutive planning cycles. Each simulation experiment, defined by respective parameter settings generated from Table 1, consists of 100 consecutive planning cycles, each with 10 time-periods. The total operational cost (inventory, shipping, and penalty due to shortages) is used as a performance measure upon which the performance of this dynamic scheduling algorithm (denoted by Gs) and the optimal value based on the actual demand (G*) are compared. The total operational cost without using the actual ending inventory and updated demand information (G) and the optimal cost based on the actual demand are also compared to show the gap of these costs.

Table 1. Base Values of Parameters

# of plants	I={1}
# of DCs	J={10}
# of periods in each planning cycle	T={10}
Production setup cost at plant	f_0=20,000
Capacity of truck load	C=100
Shipping cost per truck load	f_j=UNIF[100, 400]
Unit holding cost at plant	$h_0=0.02\cdot\min\{f_j\}$
Unit holding cost at DCs	$h_j=2.5\ h_0$
Unit penalty cost at DCs	$p_j=4\ h_0$
Anticipated demand	$d_{j,\tau}$=UNIF[0, 2000]

Our empirical observations are listed in Figures 1A&B and 2A&B. As part of the experiment design, we defined the following dynamic changes to the static DC demand over each planning cycle. Let $d^a_{j,\tau} = d_{j,\tau} + \Delta d_{j,\tau}$ be the updated demand information shared between DC_j and the plant at the beginning of time period, Δn be the ratio of number of changed orders and total number of orders, where $\Delta d_{j,\tau} \geq 0$ and Δn reflect market dynamics such as promotion sales and product recalls. The performance of this dynamic scheduling algorithm is given by performance factor $\eta = Gs / G*$.

Figure 1A and 1B indicate that, when the demand forecast error is 20%, the performance factor of the proposed algorithm increased from 1.001 to 1.004 as the number of changed orders increased from 10% to 80%. Figure 2A and 2B shows that, when the number of changed orders is 50%, the performance factor of the dynamic scheduling algorithm increased from 1.003 to 1.008 as the demand forecast error increased from 10% to 80%.

Figure 1A. The performance vs. number of changed orders (Δd=20%)

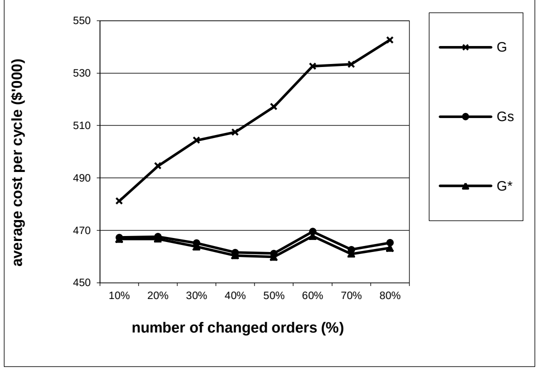


Figure 1B. The performance vs. number of changed orders (Δd=20%)

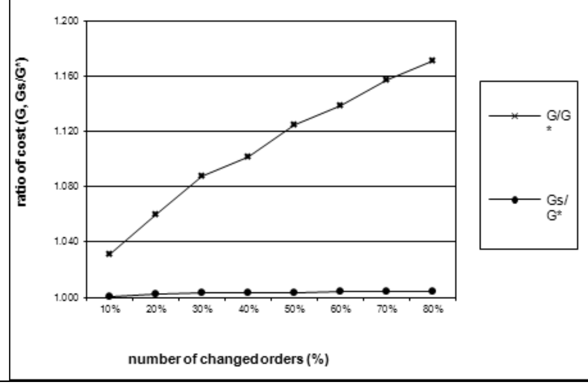


Figure 2A. The performance vs. forecast error (Δn=50%)

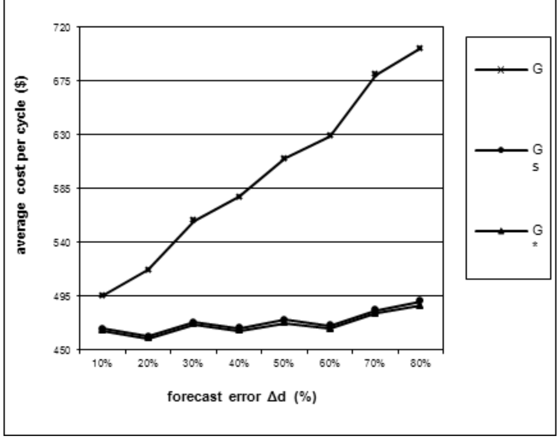
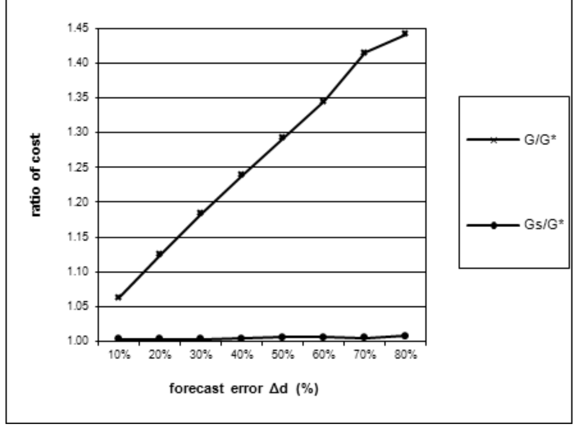


Figure 2B. The performance vs. forecast error (Δn=50%)



COCLUSION

We analyzed the two-period problem (**P_2**) and developed a fast algorithm that reconstructs the distribution schedules between the plant and DCs to minimize the total operation cost for the single-supplier multiple-heterogamous customer network problem with the consideration of dynamic demand. It solves a sequence of two-stage problems and finds a close-form optimal solution to the two-period problem if there is sufficient supply at the plant to meet demand from all DCs. The performance of the proposed algorithm is evaluated through simulation of certain consultative planning cycles with certain time periods in each cycle. This study has two major contributions to the integrated production and distribution field. One is that an optimal revised inventory and distribution solution can be obtained as a close-form solution in a special case of the problem and the other one is that the cost savings by revising the inventory and distribution schedule are significant. The result of this study will also contribute to industries as well. Companies can apply this algorithm to update their production and distribution scheduling under the dynamic demand situations.

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