ECONOMICALLY OPTIMUM ACCEPTANCE SAMPLING PLANS DEPENDING ON HOW REJECTED LOTS ARE HANDLED

Yiannis Nikolaidis
Department of Technology Management, University of Macedonia,
Logou - Tourpali, GR-59200 Naoussa, Greece
nikolai@uom.gr, Tel.: +30 23320 52465

Theodore H. Kaskalis
Department of Technology Management, University of Macedonia,
Logou - Tourpali, GR-59200 Naoussa, Greece
kaskalis@uom.gr, Tel.: +30 23320 52532

ABSTRACT

It is well known that the purpose of Acceptance Sampling by attributes is to separate inspected batches of raw materials/final products according to the results of sampling. Depending on the outcome of the control every inspected batch may be accepted or rejected. The most frequent choice in practice for handling rejected batches is to apply 100% inspection on them. This potential has been studied thoroughly by academia. However, this is not what always happens to rejected batches in real life situations. Sometimes, the nonconforming units of a rejected batch are cannibalized, recycled or, even worse, landfilled. In some cases, when a batch is rejected its units are sold at a discount or a secondary market at a reduced price.

In this study we develop several formulations which allow the determination of the optimum Acceptance Sampling plans (n^*, c^*) and the optimum average total quality cost corresponding to these plans. Moreover, in the numerical investigation that we conduct, we calculate the percentage cost penalty of using the sampling plan (n, c) set by ISO 2859 standard, instead of the economically optimum (n^*, c^*).

Keywords: Acceptance Sampling; Quality Cost; Discount; Recycling; Cannibalization

INTRODUCTION – LITERATURE REVIEW

It is widely known that Acceptance Sampling (AS) is applied whenever large quantities of similar items or, in other words, large batches of products are being bought or sold. Unlike Statistical Process Control where the objective is to monitor production as it proceeds, AS is applied to batches of goods which have already been produced.

Our main focus in this paper is single sampling by attributes, which is carried out as follows: from a batch of N units, a sample of size n is taken and every item of the sample is characterized as conforming or nonconforming based on an attribute-type quality characteristic. Depending on the outcome of the control and the acceptance-rejection criterion (numbers c and r respectively), every inspected batch may be accepted or rejected. In practice, the most frequent choice for handling rejected batches is to apply 100% inspection (rectifying inspection). However, this is not the only option as far as rejected batches are concerned, in real life situations. Sometimes, the nonconforming units of a rejected batch are cannibalized, recycled or, worse, landfilled.

In order to monitor the quality of purchased or produced batches, companies often use sampling plans - (n, c) in case of single sampling by attributes or (n_1, c_1, r_1, n_2, c_2) in case of double sampling by attributes, etc. - that are recommended by specific standards, which are easily applied in practice. An indicative list of those standards should certainly include ISO 2859-10 (2006), ANSI/ASQ Z1.4 (2008) and the older MIL-STD-105E (1989).

However, the use of standards does not take quality costs directly into account, but only indirectly. More specifically, the aim of standards, for example, to decrease the producer's risk (namely the risk of rejecting an AQL batch) is an indirect endeavor to reduce the appraisal costs, which are associated with measuring, evaluating or auditing products so as to assure their conformance to quality standards. Consequently, it is only fair that one should wonder whether to reduce quality costs indirectly, choosing sampling plans through the use of AS standards, or directly, through the minimization of the expected quality related cost, which leads to the optimum AS policy.

As AS was, but, most importantly, still is in many cases a significant Quality Control (QC) process, detailed information about its principles, rules and procedures can be found in many books such as the editions of Stephens (2001), Schilling and Neubauer (2009) and Montgomery (2012). Specialized information about the economic dimension of AS, can be found many decades before, in the book of Hald (1981).

On the other hand, significant reviews of papers regarding AS and specifically about economically optimum sampling plans are those of Wetherrill and Chiu (1975) and Wall and Elshennawy (1989). Quite detailed review on economic issues of AS and economically optimum sampling plans can be found in Nikolaidis and Nenes (2009). However, some recent contributions are the following: Hsu and Hsu (2012) design an economic model to determine the optimal sampling plan in a two-stage supply chain, which minimizes at the same time the producer's and the consumer's total quality cost, while satisfying both the producer's and the consumer's quality and risk requirements. Chen (2012) proposes the economic selection of quality investment for designing a combined continuous lot by lot AS plan with average outgoing quality limit protection. The optimal set of parameters for single and continuous sampling plan and the quality investment level are jointly determined. Battini et al (2012) design an optimal acceptance policy. More specifically, they introduce a new model that estimates the percentage of items to be checked based on elements such as the inspection unit cost and the unit penalty cost.

ALTERNATIVE FORMULATIONS OF THE QUALITY COST FUNCTION

For designing an AS sampling plan using economic criteria or for determining the economic impact of using a specific sampling plan (n, c), it is necessary first to determine the economic elements of the AS process and then to formulate the average total quality cost function. The form of this function depends on the characteristics of the QC process. The minimization (or the calculation) of that function, using either analytical or numerical methods, ensues.

Focusing on AS by attributes, the average total quality cost per batch that corresponds to the use of a single sampling plan (n, c), for the inspection of a batch with a fraction nonconforming p, is in general:

$$K(n,c|p) = [\mathit{cost\ of\ accepting\ a\ batch}] \cdot P_a(p) + [\mathit{cost\ of\ rejecting\ a\ batch}] \cdot [1 - P_a(p)] \tag{1}$$

where $P_a(p)$ is the probability of accepting a batch with a fraction nonconforming p. Since most of the times the fraction nonconforming p per batch is not deterministic, but is distributed according to a probability density function (pdf) $\varphi(p)$, the average total quality cost function is given by

$$K(n,c) = \int_p K(n,c|p)\varphi(p)dp \tag{2}$$

Usually, in the average total quality cost function the following partial costs can be found: i) the sampling cost, ii) the cost of handling nonconforming units (e.g., repair cost, returns etc.) and iii) the cost of nonconforming items not detected during inspection (e.g., use of nonconforming raw materials in the production process, defamation of a company in case that nonconforming products reach customers etc.). The notation of the respective cost elements is the following:

c_i: inspection cost per item,

c_r: replacement cost, i.e., purchase cost, in case a nonconforming unit is immediately replaced upon detection at a company's expense, or repair cost per nonconforming item,

c_d: cost per nonconforming item that is not detected during inspection.

In what follows, depending on the form of the average total quality cost function, the following notation may be used:

c_p purchase cost per item

c_s the average salvage value per recycled item

c_c the average salvage value per cannibalized unit, where $c_c > c_s$

λ_s customer's profit (as a percentage of the average salvage cost per recycled item c_s) for recycling the nonconforming units of a rejected batch, where $0 \leq \lambda_s \leq 1$

λ_c customer's profit (as a percentage of the average salvage value per cannibalized unit c_c) for cannibalizing the nonconforming units of a rejected batch, where $0 \leq \lambda_c \leq 1$

Case 1 - Classic Case

In the most common case in practice, which have been studied initially by Hald (1981), the following assumptions have been made:

1. The nonconforming units in a sample are repaired.
2. The customer is economically burdened by this repair.
3. The nonconforming units of a rejected batch are repaired too.
4. The customer is once again economically burdened by this repair.
5. Every batch (of size *N*) that is rejected through the examination of a sample is submitted to 100% inspection.

In this case the analytical form of (1) becomes (Case 1.1):

$$K(n,c|p) = [nc_i + npc_r + (N-n)pc_d] \cdot P_a(p) + (Nc_i + Npc_r) \cdot [1 - P_a(p)] \quad (3)$$

Changing assumption 4 into the following one:

6. The *manufacturer* is economically burdened by the repair of the nonconforming units of a rejected batch,

the average total quality cost function is determined by the following equality (Case 1.2):

$$K(n,c|p) = [nc_i + npc_r + (N-n)pc_d] \cdot P_a(p) + Nc_i \cdot [1 - P_a(p)] \quad (4)$$

By also differentiating assumption 2 this way:

7. The *manufacturer* is economically burdened by the repair of the nonconforming units in a sample,

the analytical form of (1) becomes as follows (Case 1.3):

$$K(n,c|p) = [nc_i + (N-n)pc_d] \cdot P_a(p) + Nc_i \cdot [1 - P_a(p)] \quad (5)$$

Case 2

In this case the underlying assumptions are the following:

- 1, 2 and 5, mentioned previously.

8. The nonconforming units of a rejected batch are *scrapped* (*landfilled*) (with no profit either for the customer or the manufacturer) and not repaired. Consequently, they should be replaced, i.e. repurchased.
9. The customer is economically burdened by this purchase.

10. The units that are bought in order to replace the nonconforming units of a rejected batch (*Np* on average) are not submitted in AS.

A justification of assumption 10 is the following: considering that *p* has a small and rather constant value, the nonconforming units of the extra purchase of *Np* units are few, namely around Np^2. Note that assumption 10 allows the average total quality cost function to keep a simple form (Case 2.1):

$$K(n,c|p) = [nc_i + npc_r + (N-n)pc_d] \cdot P_a(p) + (Nc_i + Npc_p) \cdot [1 - P_a(p)] \tag{6}$$

Replacing assumption 9 by the following one:

11. The *manufacturer* is economically burdened by the purchase conducted in order to replace the nonconforming units of a rejected batch that are scrapped,

the average total quality cost function is determined by (4) - Case 2.2.

Moreover, replacing assumption 2 by 7, the analytical form of the average total quality cost function is determined by (5) - Case 2.3.

Case 3

The underlying assumptions in this case are as follows:

- 1, 2 and 5

12. The nonconforming units of a rejected batch are *recycled* by the customer (for instance, because they can not be repaired or because they are not worth dealing with etc.) and not repaired. Consequently, they should be replaced, or, in other words, repurchased.

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13. The reward of customer for recycling the nonconforming units of a rejected batch is the whole average salvage value c_s per recycled unit.

- 10.

The analytical form of (1) is modified accordingly (Case 3.1):

$$K(n,c|p) = [nc_i + npc_r + (N-n)pc_d] \cdot P_a(p) + [Nc_i + Np(c_p - c_s)] \cdot [1 - P_a(p)] \tag{7}$$

Again, replacing assumptions 9 and 13 by the following ones, respectively:

14. The *manufacturer* is economically burdened by the purchase conducted in order to replace the nonconforming units of a rejected batch that are *recycled*,
15. The reward of customer for recycling the nonconforming units of a rejected batch and saving some of their value for the manufacturer, is the following profit per unit: $\lambda_s \cdot c_s$.

the average total quality cost function is modified as follows (Case 3.2):

$$K(n,c|p)=[nc_i+npc_r+(N-n)pc_d]\cdot P_a(p)+[Nc_i-Np\lambda_s c_s]\cdot[1-P_a(p)] \tag{8}$$

Additionally, replacing assumption 2 by 7, the analytical form of the average total quality cost function is determined as follows (Case 3.3):

$$K(n,c|p)=[nc_i+(N-n)pc_d]\cdot P_a(p)+[Nc_i-Np\lambda_s c_s]\cdot[1-P_a(p)] \tag{9}$$

Case 4

In the last case that we study in this paper, the underlying assumptions are the following:

- 1, 2 and 5

16. The nonconforming units of a rejected batch are *cannibalized* (for instance, because they can not be repaired, but there are parts on them that still have value and can be removed for future use etc.) and not repaired. Consequently, they should be replaced or, in other words, repurchased.

- 9

17. The reward of customer for *cannibalizing* any such unit is the whole average salvage value c_c per *cannibalized* unit.

- 10.

The analytical form of (1) is modified as follows (Case 4.1):

$$K(n,c|p)=[nc_i+npc_r+(N-n)pc_d]\cdot P_a(p)+[Nc_i+Np(c_p-c_c)]\cdot[1-P_a(p)] \tag{10}$$

In this case too, by replacing assumptions 9 and 17 by the following ones, respectively:

18. The *manufacturer* is economically burdened by the purchase conducted in order to replace the nonconforming units of a rejected batch that are *cannibalized*,
19. The reward of customer for *cannibalizing* the nonconforming units of a rejected batch and saving some of their value for the manufacturer, is the following profit per unit: $\lambda_c \cdot c_c$.

the average total quality cost function is the following (Case 4.2):

$$K(n,c|p)=[nc_i+npc_r+(N-n)pc_d]\cdot P_a(p)+(Nc_i-Np\lambda_c c_c)\cdot[1-P_a(p)] \tag{11}$$

Additionally, by replacing assumption 2 by 7, the analytical form of the average total quality cost function is modified as follows (Case 4.3):

$$K(n,c|p)=[nc_i+(N-n)pc_d]\cdot P_a(p)+(Nc_i-Np\lambda_c c_c)\cdot[1-P_a(p)] \tag{12}$$

TABLE 1

Combinations of assumptions for each of the (2) to (11) quality cost functions

	(3)	(4)		(5)		(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. The nonconforming units in a sample are repaired	x	x	x	x	x	x	x	x	x	x	x	x
2. The customer is economically burdened by this repair	x	x	x			x	x	x		x	x	
3. The nonconforming units of a rejected batch are *repaired*	x	x		x								
4. The customer is once again economically burdened by this repair	x											
5. Every batch (of size *N*) that is rejected through the examination of a sample is submitted to 100% inspection	x	x	x	x	x	x	x	x	x	x	x	x
6. The manufacturer is economically burdened by the repair of the nonconforming units of a rejected batch		x		x								
7. The manufacturer is economically burdened by the repair of the nonconforming units in a sample				x	x				x			x
8. The nonconforming units of a rejected batch are *scrapped* (no profit for the customer or the manufacturer) and they should be replaced			x		x	x						
9. The customer is economically burdened by this purchase						x	x			x		
10. The units that are bought to replace the nonconforming units of a rejected batch are not submitted in AS			x		x	x	x	x	x	x	x	x
11. The manufacturer is economically burdened by the purchase conducted in order to replace the nonconforming units of a rejected batch that are *scrapped*			x		x							
12. The nonconforming units of a rejected batch are *recycled* by the customer and they should be replaced							x	x	x			
13. The reward of customer for recycling any such unit is the whole average salvage value c_s							x					
14. The manufacturer is economically burdened by the purchase conducted in order to replace the nonconforming units of a rejected batch that are *recycled*								x	x			
15. The reward of customer for *recycling* the nonconforming units of a rejected batch and saving some value for the manufacturer is $\lambda_s \cdot c_s$								x	x			
16. The nonconforming units of a rejected batch are *cannibalized* and they should be replaced										x	x	x
17. The reward of customer for *cannibalizing* any such unit is the whole average salvage value c_c										x		
18. The manufacturer is economically burdened by the purchase conducted in order to replace the nonconforming units of a rejected batch that are *cannibalized*											x	x
19. The reward of customer for *cannibalizing* the nonconforming units of a rejected batch and saving some value for the manufacturer is $\lambda_c \cdot c_c$											x	x

In Table 1 we present in brief the set of assumptions that have been made for each one of the quality cost functions (3) to (12).

NUMERICAL INVESTIGATION

The economic evaluation of the AS plans recommended by ISO 2859 that we conduct is indicative. It is achieved in a number of combinations of parameters that Nikolaidis and Nenes (2009) examine. More specifically, we focus on a specific combination of cost elements, namely

$c_i = 1$, $c_r = 80$, $c_d = 122.5$, while for the additional quality costs we make the following choices:

- $c_p = c_r = 80$[1]: this choice is justified by the fact that c_r may be the purchase cost, as explained previously, in case a nonconforming unit is immediately replaced upon detection.
- $c_s = 0.1 \cdot c_p = 8$: for several electronic products, salvage cost varies uniformly from 5 to 15% of the purchase cost. Consequently, we consider a 10% value
- $c_c = 0.4 \cdot c_p = 32$: as $c_c > c_s$ due to the fact that cannibalization is a much more advanced recovery option than recycling, we consider - intuitively - a larger value for c_c, i.e. 40% of the purchase cost
- $\lambda_s = 0.3 \cdot c_s = 2.4$: obviously customer's profit is a matter of negotiations between the latter and the manufacturer; however, we consider arbitrarily a 30% value of c_s
- $\lambda_c = 0.3 \cdot c_c = 9.6$ similarly

Moreover, in accordance with Nikolaidis and Nenes (2009), we investigate 40 different values of *N*, three from each one of the 13 first classes of the ISO 2859 standard and one from the 14th class, covering a wide range of *N* values. Finally, we study three different AQL values and as far as the fraction nonconforming *p* is concerned we consider a uniform pdf, namely U(0.0015, 0.04).

For every *N* and AQL values, the sampling plan (n, c) recommended by ISO 2859 is first determined. Then, for the cost elements presented previously and the selected pdf, the average cost of using the plan (n, c), namely $K(n,c)$, is calculated by (2) and one of the formulations (3) to (12), depending on the examined case. For the same combination of cost elements and pdf, the optimum sampling plan (n^*, c^*), along with the minimum expected quality cost $K(n^*,c^*)$ are found; this has been done by calculating (2) and one of (3) to (12) for all possible *n*s and *c*s and, then, by finding the minimum $K(n,c)$. Finally, using $\Delta K = \frac{K(n,c) - K(n^*,c^*)}{K(n^*,c^*)} \cdot 100\%$, the percentage cost penalty of using the sampling plan (n, c) of ISO 2859 instead of the optimum (n^*, c^*), is calculated. Tables 2 to 10 present the results that correspond to the various cost functions. It should be noted that all calculations were carried out through a standard 32-bit GNU C Compiler environment combined with the necessary GNU Scientific Library functions.

The comparisons that could be made and the conclusions that could be raised are numerous. However, a brief comment describes the general picture: the percentage cost penalty ΔK reaches even 242.2%! This totally confirms the observation of Nikolaidis and Nenes (2009) that the use of the sampling plans recommended by the ISO 2859 standard instead of the economically optimum ones (n^*, c^*) in AS, results in significant increase of the quality cost met by companies.

[1] In this case, Case 2.1 becomes identical with Case 1.1 and will not be further investigated.

TABLE 2

Results for Case 1.1 (as well as Case 2.1)

	Optimum values AQL			0.40%				1.50%				6.50%			
N	*n**	*c**	*K*(*n**,*c**)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)
3	0	0	7.63	3	0	7.98	4.65	3	0	7.98	4.65	2	0	7.86	3.03
5	0	0	12.71	5	0	13.30	4.65	5	0	13.30	4.65	2	0	12.93	1.73
7	0	0	17.79	7	0	18.62	4.65	7	0	18.62	4.65	2	0	18.00	1.17
10	0	0	25.42	10	0	26.60	4.65	8	0	26.33	3.57	2	0	25.61	0.76
12	0	0	30.50	12	0	31.92	4.65	8	0	31.37	2.86	2	0	30.68	0.60
14	0	0	35.59	14	0	37.24	4.65	8	0	36.42	2.34	2	0	35.76	0.48
17	0	0	43.21	17	0	45.22	4.65	8	0	43.99	1.80	8	1	44.12	2.11
20	0	0	50.84	20	0	53.20	4.65	8	0	51.56	1.42	8	1	51.74	1.77
24	0	0	61.01	24	0	63.84	4.65	8	0	61.65	1.07	8	1	61.89	1.45
27	0	0	68.63	27	0	71.82	4.65	8	0	69.23	0.87	8	1	69.51	1.28
38	0	0	96.59	32	0	100.15	3.68	8	0	96.98	0.41	8	1	97.43	0.87
49	2	0	124.53	32	0	127.70	2.55	8	0	124.74	0.17	8	1	125.35	0.65
55	3	0	139.74	32	0	142.73	2.14	8	0	139.88	0.10	13	2	141.28	1.10
70	6	0	177.70	32	0	180.30	1.46	8	0	177.73	0.02	13	2	179.39	0.95
85	8	0	215.58	32	0	217.87	1.06	8	0	215.58	0.00	13	2	217.50	0.89
100	10	0	253.41	32	0	255.45	0.81	32	1	255.60	0.87	20	3	256.50	1.22
120	12	0	303.77	32	0	305.54	0.58	32	1	305.75	0.65	20	3	307.32	1.17
140	14	0	354.08	32	0	355.64	0.44	32	1	355.89	0.51	20	3	358.14	1.15
160	16	0	404.35	32	0	405.74	0.34	32	1	406.03	0.42	32	5	410.46	1.51
215	19	0	542.45	32	0	543.50	0.19	32	1	543.92	0.27	32	5	550.26	1.44
270	52	1	680.13	32	0	681.26	0.17	32	1	681.81	0.25	32	5	690.06	1.46
300	55	1	754.65	32	0	756.41	0.23	50	2	760.28	0.75	50	7	768.46	1.83
390	61	1	977.93	32	0	981.84	0.40	50	2	986.10	0.84	50	7	997.22	1.97
480	65	1	1200.95	32	0	1207.27	0.53	50	2	1211.92	0.91	50	7	1225.98	2.08
550	68	1	1374.30	125	1	1387.72	0.98	80	3	1388.13	1.01	80	10	1407.47	2.41
850	114	2	2112.83	125	1	2132.59	0.94	80	3	2138.34	1.21	80	10	2170.03	2.71
1150	160	3	2848.51	125	1	2877.45	1.02	80	3	2888.55	1.41	80	10	2932.58	2.95
1400	164	3	3460.25	125	1	3498.17	1.10	125	5	3524.19	1.85	125	14	3573.38	3.27
2200	254	5	5410.19	125	1	5484.47	1.37	125	5	5526.82	2.16	125	14	5606.88	3.64
3000	301	6	7354.29	125	1	7470.78	1.58	125	5	7529.45	2.38	125	14	7640.37	3.89
3500	345	7	8567.16	200	2	8687.35	1.40	200	7	8733.36	1.94	200	21	8920.18	4.12
6600	521	11	16067.11	200	2	16348.43	1.75	200	7	16437.67	2.31	200	21	16800.00	4.56
9700	651	14	23547.89	200	2	24009.51	1.96	200	7	24141.98	2.52	200	21	24679.81	4.81
12500	738	16	30295.77	315	3	31044.74	2.47	315	10	30877.06	1.92	200	21	31797.05	4.96
22500	996	22	54359.46	315	3	55834.92	2.71	315	10	55529.63	2.15	200	21	57215.80	5.25
32500	1209	27	78392.97	315	3	80625.11	2.85	315	10	80182.20	2.28	200	21	82634.54	5.41
40000	1378	31	96406.92	500	5	99243.50	2.94	500	14	97561.35	1.20	200	21	101698.59	5.49
92500	2101	48	222378.48	500	5	229381.70	3.15	500	14	225463.78	1.39	200	21	235146.99	5.74
145000	2612	60	348248.45	500	5	359519.90	3.24	500	14	353366.21	1.47	200	21	368595.39	5.84
160000	2739	63	384201.40	800	7	400154.26	4.15	800	21	387015.87	0.73	200	21	406723.50	5.86

TABLE 3

Results for Case 1.2 (as well as Case 2.2)

	Optimum values *AQL*			0.40%				1.50%				6.50%			
N	*n**	*c**	*K*(*n**,*c**)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)
3	3	0	7.59	3	0	7.59	0.00	3	0	7.59	0.00	2	0	7.59	0.02
5	5	0	12.26	5	0	12.26	0.00	5	0	12.26	0.00	2	0	12.49	1.93
7	7	0	16.63	7	0	16.63	0.00	7	0	16.63	0.00	2	0	17.39	4.55
10	10	0	22.72	10	0	22.72	0.00	8	0	23.13	1.82	2	0	24.74	8.89
12	12	0	26.49	12	0	26.49	0.00	8	0	27.54	3.97	2	0	29.64	11.90
14	14	0	30.05	14	0	30.05	0.00	8	0	31.95	6.31	2	0	34.53	14.93
17	17	0	35.04	17	0	35.04	0.00	8	0	38.56	10.02	8	1	43.57	24.32
20	20	0	39.68	20	0	39.68	0.00	8	0	45.17	13.84	8	1	51.08	28.75
24	24	0	45.37	24	0	45.37	0.00	8	0	53.98	18.99	8	1	61.11	34.69
27	27	0	49.33	27	0	49.33	0.00	8	0	60.59	22.84	8	1	68.62	39.12
38	38	0	62.16	32	0	64.91	4.42	8	0	84.84	36.48	8	1	96.18	54.73
49	49	0	73.26	32	0	82.26	12.29	8	0	109.08	48.89	8	1	123.74	68.91
55	55	0	78.89	32	0	91.73	16.27	8	0	122.30	55.03	13	2	140.76	78.43
70	70	0	92.28	32	0	115.39	25.04	8	0	155.36	68.35	13	2	178.73	93.68
85	85	0	105.29	32	0	139.05	32.06	8	0	188.41	78.94	13	2	216.70	105.80
100	100	0	118.31	32	0	162.71	37.53	32	1	219.81	85.79	20	3	256.05	116.42
120	120	0	135.94	32	0	194.26	42.91	32	1	262.79	93.32	20	3	306.79	125.69
140	140	0	153.94	32	0	225.81	46.69	32	1	305.78	98.63	20	3	357.52	132.25
160	160	0	172.30	32	0	257.36	49.37	32	1	348.76	102.42	32	5	410.35	138.17
215	215	0	224.11	32	0	344.12	53.55	32	1	466.96	108.36	32	5	550.11	145.46
270	270	0	277.13	32	0	430.88	55.48	32	1	585.17	111.16	32	5	689.87	148.94
300	300	0	306.33	32	0	478.21	56.11	50	2	678.51	121.49	50	7	768.37	150.83
390	287	0	394.29	32	0	620.18	57.29	50	2	879.80	123.14	50	7	997.10	152.89
480	277	0	482.20	32	0	762.15	58.06	50	2	1081.09	124.20	50	7	1225.84	154.22
550	273	0	550.57	125	1	669.04	21.52	80	3	1214.06	120.51	80	10	1407.41	155.63
850	265	0	843.55	125	1	1021.89	21.14	80	3	1869.33	121.60	80	10	2169.94	157.24
1150	262	0	1136.50	125	1	1374.74	20.96	80	3	2524.59	122.14	80	10	2932.46	158.03
1400	380	1	1377.94	125	1	1668.78	21.11	125	5	3155.80	129.02	125	14	3573.33	159.32
2200	497	2	2147.78	125	1	2609.72	21.51	125	5	4947.92	130.37	125	14	5606.79	161.05
3000	495	2	2912.07	125	1	3550.66	21.93	125	5	6740.04	131.45	125	14	7640.25	162.37
3500	617	3	3388.88	200	2	3919.18	15.65	200	7	7410.17	118.66	200	21	8920.17	163.22
6600	736	4	6328.97	200	2	7357.03	16.24	200	7	13942.51	120.30	200	21	16799.97	165.45
9700	981	6	9255.66	200	2	10794.88	16.63	200	7	20474.85	121.21	200	21	24679.77	166.65
12500	1103	7	11893.30	315	3	12853.15	8.07	315	10	24576.53	106.64	200	21	31797.00	167.35
22500	1469	10	21288.62	315	3	23090.06	8.46	315	10	44188.69	107.57	200	21	57215.71	168.76
32500	1714	12	30662.92	315	3	33326.97	8.69	315	10	63800.84	108.07	200	21	82634.41	169.49
40000	1959	14	37685.62	500	5	39689.63	5.32	500	14	69031.82	83.18	200	21	101698.44	169.86
92500	2939	22	86754.46	500	5	91663.36	5.66	500	14	159489.24	83.84	200	21	235146.63	171.05
145000	3673	28	135750.17	500	5	143637.10	5.81	500	14	249946.67	84.12	200	21	368594.83	171.52
160000	3918	30	149741.86	800	7	152811.61	2.05	800	21	251012.41	67.63	200	21	406722.88	171.62

TABLE 4

Results for Case 1.3 (as well as Case 2.3)

	Optimum values AQL			0.40%				1.50%				6.50%			
N	*n**	*c**	*K*(*n**,*c**)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)
3	3	0	3.00	3	0	3.00	0.00	3	0	3.00	0.00	2	0	4.45	48.30
5	5	0	5.00	5	0	5.00	0.00	5	0	5.00	0.00	2	0	9.35	86.95
7	7	0	7.00	7	0	7.00	0.00	7	0	7.00	0.00	2	0	14.25	103.51
10	10	0	10.00	10	0	10.00	0.00	8	0	12.41	24.08	2	0	21.59	115.93
12	12	0	12.00	12	0	12.00	0.00	8	0	16.82	40.13	2	0	26.49	120.76
14	14	0	14.00	14	0	14.00	0.00	8	0	21.22	51.59	2	0	31.39	124.21
17	17	0	17.00	17	0	17.00	0.00	8	0	27.83	63.73	8	1	30.55	79.70
20	20	0	20.00	20	0	20.00	0.00	8	0	34.45	72.23	8	1	38.07	90.33
24	24	0	24.00	24	0	24.00	0.00	8	0	43.26	80.25	8	1	48.09	100.37
27	27	0	27.00	27	0	27.00	0.00	8	0	49.87	84.71	8	1	55.60	105.94
38	38	0	38.00	32	0	41.46	9.12	8	0	74.11	95.03	8	1	83.16	118.85
49	49	0	49.00	32	0	58.82	20.03	8	0	98.35	100.72	8	1	110.72	125.97
55	55	0	55.00	32	0	68.28	24.15	8	0	111.58	102.87	13	2	119.31	116.92
70	70	0	70.00	32	0	91.94	31.35	8	0	144.63	106.62	13	2	157.27	124.68
85	85	0	85.00	32	0	115.61	36.01	8	0	177.69	109.05	13	2	195.24	129.69
100	100	0	100.00	32	0	139.27	39.27	32	1	178.14	78.14	20	3	222.94	122.94
120	120	0	120.00	32	0	170.82	42.35	32	1	221.13	84.27	20	3	273.68	128.06
140	140	0	140.00	32	0	202.37	44.55	32	1	264.11	88.65	20	3	324.41	131.72
160	152	0	159.96	32	0	233.92	46.23	32	1	307.09	91.98	32	5	357.26	123.34
215	173	0	214.41	32	0	320.68	49.56	32	1	425.30	98.36	32	5	497.01	131.81
270	188	0	268.51	32	0	407.44	51.74	32	1	543.50	102.41	32	5	636.77	137.15
300	194	0	297.95	32	0	454.76	52.63	50	2	609.14	104.44	50	7	685.38	130.03
390	207	0	386.12	32	0	596.73	54.55	50	2	810.43	109.89	50	7	914.12	136.75
480	289	1	473.98	32	0	738.71	55.85	50	2	1011.72	113.45	50	7	1142.85	141.12
550	299	1	541.72	125	1	624.87	15.35	80	3	1106.58	104.27	80	10	1274.62	135.29
850	324	1	831.37	125	1	977.73	17.60	80	3	1761.85	111.92	80	10	2037.15	145.03
1150	428	2	1118.94	125	1	1330.58	18.91	80	3	2417.11	116.02	80	10	2799.67	150.21
1400	439	2	1358.10	125	1	1624.62	19.62	125	5	2981.19	119.51	125	14	3365.83	147.83
2200	560	3	2119.68	125	1	2565.56	21.04	125	5	4773.31	125.19	125	14	5399.30	154.72
3000	678	4	2878.00	125	1	3506.50	21.84	125	5	6565.43	128.12	125	14	7432.76	158.26
3500	790	5	3350.98	200	2	3859.65	15.18	200	7	7153.78	113.48	200	21	8588.17	156.29
6600	1042	7	6274.02	200	2	7297.49	16.31	200	7	13686.12	118.14	200	21	16467.97	162.48
9700	1286	9	9187.87	200	2	10735.34	16.84	200	7	20218.47	120.06	200	21	24347.77	165.00
12500	1525	11	11815.56	315	3	12788.68	8.24	315	10	24212.41	104.92	200	21	31465.01	166.30
22500	2018	15	21181.93	315	3	23025.59	8.70	315	10	43824.56	106.90	200	21	56883.71	168.55
32500	2388	18	30533.30	315	3	33262.50	8.94	315	10	63436.71	107.76	200	21	82302.41	169.55
40000	2633	20	37541.18	500	5	39604.05	5.49	500	14	68558.44	82.62	200	21	101366.44	170.01
92500	4101	32	86531.44	500	5	91577.79	5.83	500	14	159015.86	83.77	200	21	234814.64	171.36
145000	5082	40	135469.32	500	5	143551.52	5.97	500	14	249473.28	84.15	200	21	368262.83	171.84
160000	5328	42	149446.45	800	7	152720.33	2.19	800	21	250364.43	67.53	200	21	406390.89	171.93

TABLE 5

Results for Case 3.1

	Optimum values		AQL	0.40%				1.50%				6.50%			
N	*n**	*c**	*K*(*n**,*c**)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)
3	0	0	7.63	3	0	7.94	4.14	3	0	7.94	4.14	2	0	7.83	2.68
5	0	0	12.71	5	0	13.20	3.83	5	0	13.20	3.83	2	0	12.89	1.39
7	0	0	17.79	7	0	18.42	3.53	7	0	18.42	3.53	2	0	17.94	0.83
10	0	0	25.42	10	0	26.21	3.12	8	0	26.01	2.32	2	0	25.52	0.41
12	0	0	30.50	12	0	31.38	2.87	8	0	30.99	1.60	2	0	30.58	0.25
14	0	0	35.59	14	0	36.52	2.63	8	0	35.97	1.09	2	0	35.63	0.14
17	1	0	43.21	17	0	44.20	2.30	8	0	43.45	0.55	8	1	44.07	1.99
20	3	0	50.80	20	0	51.85	2.06	8	0	50.92	0.24	8	1	51.67	1.72
24	5	0	60.86	24	0	61.99	1.87	8	0	60.89	0.05	8	1	61.81	1.57
27	7	0	68.36	27	0	69.57	1.77	8	0	68.36	0.00	8	1	69.42	1.55
38	13	0	95.64	32	0	96.62	1.03	8	0	95.77	0.14	8	1	97.30	1.74
49	19	0	122.64	32	0	123.16	0.42	8	0	123.17	0.43	8	1	125.19	2.08
55	21	0	137.29	32	0	137.63	0.25	8	0	138.12	0.61	13	2	141.23	2.87
70	27	0	173.73	32	0	173.81	0.05	8	0	175.50	1.02	13	2	179.33	3.22
85	31	0	209.99	32	0	209.99	0.00	8	0	212.87	1.37	13	2	217.42	3.54
100	35	0	246.14	32	0	246.17	0.02	32	1	252.02	2.39	20	3	256.45	4.19
120	39	0	294.21	32	0	294.41	0.07	32	1	301.45	2.46	20	3	307.27	4.44
140	43	0	342.19	32	0	342.66	0.14	32	1	350.88	2.54	20	3	358.08	4.64
160	45	0	390.11	32	0	390.90	0.20	32	1	400.30	2.61	32	5	410.45	5.21
215	51	0	521.71	32	0	523.56	0.35	32	1	536.23	2.78	32	5	550.25	5.47
270	54	0	653.17	32	0	656.23	0.47	32	1	672.15	2.91	32	5	690.04	5.64
300	55	0	724.84	32	0	728.59	0.52	50	2	752.10	3.76	50	7	768.45	6.02
390	104	1	938.22	32	0	945.67	0.79	50	2	975.47	3.97	50	7	997.21	6.29
480	108	1	1150.24	32	0	1162.76	1.09	50	2	1198.84	4.23	50	7	1225.97	6.58
550	111	1	1315.06	125	1	1315.85	0.06	80	3	1370.72	4.23	80	10	1407.47	7.03
850	165	2	2018.55	125	1	2021.52	0.15	80	3	2111.44	4.60	80	10	2170.02	7.50
1150	217	3	2719.12	125	1	2727.18	0.30	80	3	2852.15	4.89	80	10	2932.57	7.85
1400	221	3	3300.19	125	1	3315.23	0.46	125	5	3487.35	5.67	125	14	3573.38	8.28
2200	277	4	5155.26	125	1	5197.00	0.81	125	5	5468.93	6.08	125	14	5606.87	8.76
3000	330	5	7004.11	125	1	7078.77	1.07	125	5	7450.51	6.37	125	14	7640.36	9.08
3500	380	6	8157.19	200	2	8210.53	0.65	200	7	8601.04	5.44	200	21	8920.18	9.35
6600	535	9	15288.98	200	2	15449.29	1.05	200	7	16188.16	5.88	200	21	16799.99	9.88
9700	686	12	22402.54	200	2	22688.05	1.27	200	7	23775.27	6.13	200	21	24679.80	10.17
12500	787	14	28819.32	315	3	29225.58	1.41	315	10	30247.01	4.95	200	21	31797.05	10.33
22500	1041	19	51701.54	315	3	52560.44	1.66	315	10	54395.54	5.21	200	21	57215.79	10.67
32500	1243	23	74554.94	315	3	75895.29	1.80	315	10	78544.07	5.35	200	21	82634.53	10.84
40000	1394	26	91684.11	500	5	93288.11	1.75	500	14	94708.40	3.30	200	21	101698.58	10.92
92500	2102	40	211464.78	500	5	215609.87	1.96	500	14	218866.33	3.50	200	21	235146.96	11.20
145000	2657	51	331145.39	500	5	337931.62	2.05	500	14	343024.25	3.59	200	21	368595.33	11.31
160000	2808	54	365330.11	800	7	375420.00	2.76	800	21	373415.52	2.21	200	21	406723.44	11.33

TABLE 6

Results for Case 3.2

	Optimum values *AQL*			0.40%				1.50%				6.50%			
N	*n**	*c**	*K*(*n**,*c**)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)
3	3	0	7.58	3	0	7.58	0.00	3	0	7.58	0.00	2	0	7.59	0.07
5	5	0	12.22	5	0	12.22	0.00	5	0	12.22	0.00	2	0	12.48	2.08
7	7	0	16.57	7	0	16.57	0.00	7	0	16.57	0.00	2	0	17.37	4.82
10	10	0	22.60	10	0	22.60	0.00	8	0	23.03	1.92	2	0	24.71	9.34
12	12	0	26.32	12	0	26.32	0.00	8	0	27.42	4.18	2	0	29.60	12.47
14	14	0	29.83	14	0	29.83	0.00	8	0	31.81	6.63	2	0	34.50	15.64
17	17	0	34.74	17	0	34.74	0.00	8	0	38.39	10.52	8	1	43.55	25.36
20	20	0	39.27	20	0	39.27	0.00	8	0	44.98	14.53	8	1	51.06	30.03
24	24	0	44.81	24	0	44.81	0.00	8	0	53.75	19.95	8	1	61.08	36.31
27	27	0	48.65	27	0	48.65	0.00	8	0	60.34	24.01	8	1	68.60	40.99
38	38	0	60.99	32	0	63.85	4.69	8	0	84.47	38.49	8	1	96.14	57.63
49	49	0	71.55	32	0	80.90	13.07	8	0	108.61	51.79	8	1	123.69	72.88
55	55	0	76.87	32	0	90.20	17.34	8	0	121.77	58.42	13	2	140.75	83.11
70	70	0	89.46	32	0	113.44	26.80	8	0	154.69	72.91	13	2	178.71	99.76
85	85	0	101.67	32	0	136.69	34.44	8	0	187.60	84.52	13	2	216.67	113.12
100	100	0	113.88	32	0	159.93	40.44	32	1	218.73	92.07	20	3	256.04	124.83
120	120	0	130.44	32	0	190.92	46.37	32	1	261.50	100.48	20	3	306.77	135.19
140	140	0	147.39	32	0	221.92	50.57	32	1	304.27	106.45	20	3	357.51	142.56
160	160	0	164.70	32	0	252.91	53.56	32	1	347.04	110.72	32	5	410.35	149.16
215	215	0	213.68	32	0	338.14	58.25	32	1	464.65	117.46	32	5	550.11	157.45
270	270	0	263.89	32	0	423.37	60.43	32	1	582.27	120.64	32	5	689.86	161.42
300	300	0	291.58	32	0	469.86	61.14	50	2	676.06	131.86	50	7	768.36	163.51
390	299	0	375.13	32	0	609.33	62.43	50	2	876.61	133.68	50	7	997.10	165.80
480	286	0	458.65	32	0	748.80	63.26	50	2	1077.16	134.86	50	7	1225.83	167.27
550	281	0	523.60	125	1	647.48	23.66	80	3	1208.84	130.87	80	10	1407.41	168.80
850	272	0	801.90	125	1	988.57	23.28	80	3	1861.26	132.11	80	10	2169.93	170.60
1150	269	0	1080.18	125	1	1329.66	23.10	80	3	2513.67	132.71	80	10	2932.46	171.48
1400	389	1	1309.89	125	1	1613.90	23.21	125	5	3144.75	140.08	125	14	3573.33	172.80
2200	509	2	2041.30	125	1	2523.48	23.62	125	5	4930.55	141.54	125	14	5606.79	174.67
3000	506	2	2766.91	125	1	3433.06	24.08	125	5	6716.35	142.74	125	14	7640.25	176.13
3500	630	3	3219.93	200	2	3776.13	17.27	200	7	7370.48	128.90	200	21	8920.17	177.03
6600	752	4	6010.94	200	2	7087.28	17.91	200	7	13867.66	130.71	200	21	16799.97	179.49
9700	1001	6	8789.12	200	2	10398.44	18.31	200	7	20364.84	131.70	200	21	24679.77	180.80
12500	1125	7	11292.47	315	3	12307.40	8.99	315	10	24387.52	115.96	200	21	31797.00	181.58
22500	1499	10	20208.48	315	3	22107.71	9.40	315	10	43848.46	116.98	200	21	57215.71	183.13
32500	1748	12	29103.55	315	3	31908.03	9.64	315	10	63309.40	117.53	200	21	82634.41	183.93
40000	1998	14	35767.14	500	5	37903.01	5.97	500	14	68175.94	90.61	200	21	101698.44	184.33
92500	2997	22	82322.00	500	5	87531.81	6.33	500	14	157510.01	91.33	200	21	235146.62	185.64
145000	3746	28	128804.49	500	5	137160.61	6.49	500	14	246844.08	91.64	200	21	368594.81	186.17
160000	3871	29	142078.15	800	7	145391.33	2.33	800	21	246932.31	73.80	200	21	406722.87	186.27

TABLE 7

Results for Case 3.3

	Optimum values	AQL		0.40%				1.50%				6.50%			
N	*n**	*c**	*K*(*n**,*c**)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)
3	3	0	2.99	3	0	2.99	0.00	3	0	2.99	0.00	2	0	4.44	48.62
5	5	0	4.97	5	0	4.97	0.00	5	0	4.97	0.00	2	0	9.33	87.86
7	7	0	6.94	7	0	6.94	0.00	7	0	6.94	0.00	2	0	14.23	104.99
10	10	0	9.88	10	0	9.88	0.00	8	0	12.31	24.57	2	0	21.57	118.21
12	12	0	11.84	12	0	11.84	0.00	8	0	16.70	41.08	2	0	26.46	123.53
14	14	0	13.78	14	0	13.78	0.00	8	0	21.09	52.99	2	0	31.35	127.45
17	17	0	16.69	17	0	16.69	0.00	8	0	27.67	65.75	8	1	30.53	82.89
20	20	0	19.59	20	0	19.59	0.00	8	0	34.25	74.81	8	1	38.05	94.17
24	24	0	23.45	24	0	23.45	0.00	8	0	43.03	83.53	8	1	48.06	105.00
27	27	0	26.33	27	0	26.33	0.00	8	0	49.61	88.46	8	1	55.58	111.12
38	38	0	36.83	32	0	40.41	9.71	8	0	73.75	100.23	8	1	83.13	125.69
49	49	0	47.29	32	0	57.45	21.50	8	0	97.88	107.00	8	1	110.68	134.05
55	55	0	52.98	32	0	66.75	26.00	8	0	111.05	109.62	13	2	119.29	125.17
70	70	0	67.18	32	0	90.00	33.96	8	0	143.96	114.29	13	2	157.25	134.07
85	85	0	81.38	32	0	113.24	39.16	8	0	176.88	117.36	13	2	195.22	139.89
100	100	0	95.57	32	0	136.49	42.81	32	1	177.07	85.28	20	3	222.93	133.26
120	120	0	114.50	32	0	167.48	46.27	32	1	219.84	91.99	20	3	273.66	139.00
140	140	0	133.45	32	0	198.47	48.73	32	1	262.61	96.79	20	3	324.39	143.09
160	156	0	152.39	32	0	229.46	50.57	32	1	305.37	100.39	32	5	357.25	134.43
215	178	0	204.12	32	0	314.70	54.17	32	1	422.99	107.23	32	5	497.01	143.49
270	193	0	255.51	32	0	399.93	56.52	32	1	540.60	111.58	32	5	636.77	149.21
300	199	0	283.47	32	0	446.42	57.48	50	2	606.68	114.02	50	7	685.38	141.78
390	213	0	367.21	32	0	585.88	59.55	50	2	807.24	119.83	50	7	914.11	148.93
480	222	0	450.84	32	0	725.35	60.89	50	2	1007.79	123.54	50	7	1142.85	153.49
550	306	1	515.40	125	1	603.31	17.06	80	3	1101.36	113.69	80	10	1274.62	147.31
850	332	1	790.45	125	1	944.41	19.48	80	3	1753.78	121.87	80	10	2037.14	157.72
1150	438	2	1063.87	125	1	1285.50	20.83	80	3	2406.19	126.17	80	10	2799.67	163.16
1400	449	2	1290.92	125	1	1569.74	21.60	125	5	2970.14	130.08	125	14	3365.83	160.73
2200	572	3	2014.22	125	1	2479.32	23.09	125	5	4755.94	136.12	125	14	5399.29	168.06
3000	693	4	2734.34	125	1	3388.90	23.94	125	5	6541.75	139.24	125	14	7432.76	171.83
3500	807	5	3183.59	200	2	3716.60	16.74	200	7	7114.09	123.46	200	21	8588.17	169.76
6600	1064	7	5958.20	200	2	7027.75	17.95	200	7	13611.27	128.45	200	21	16467.97	176.39
9700	1312	9	8723.80	200	2	10338.90	18.51	200	7	20108.45	130.50	200	21	24347.77	179.10
12500	1441	10	11217.52	315	3	12242.93	9.14	315	10	24023.39	114.16	200	21	31465.00	180.50
22500	1941	14	20105.67	315	3	22043.24	9.64	315	10	43484.33	116.28	200	21	56883.71	182.92
32500	2436	18	28978.64	315	3	31843.56	9.89	315	10	62945.27	117.21	200	21	82302.41	184.01
40000	2686	20	35627.71	500	5	37817.43	6.15	500	14	67702.56	90.03	200	21	101366.44	184.52
92500	4061	31	82106.72	500	5	87446.24	6.50	500	14	157036.63	91.26	200	21	234814.62	185.99
145000	5061	39	128533.37	500	5	137075.04	6.65	500	14	246370.70	91.68	200	21	368262.81	186.51
160000	5311	41	141793.00	800	7	145300.05	2.47	800	21	246284.32	73.69	200	21	406390.87	186.61

TABLE 8

Results for Case 4.1

	AQL Optimum values			0.40%				1.50%				6.50%			
N	*n**	*c**	*K(n*,c*)*	*n*	*c*	*K(n,c)*	*ΔK* (%)	*n*	*c*	*K(n,c)*	*ΔK* (%)	*n*	*c*	*K(n,c)*	*ΔK* (%)
3	0	0	7.63	3	0	7.83	2.62	3	0	7.83	2.62	2	0	7.75	1.65
5	0	0	12.71	5	0	12.88	1.36	5	0	12.88	1.36	2	0	12.75	0.36
7	3	0	17.75	7	0	17.83	0.40	7	0	17.83	0.40	2	0	17.76	0.02
10	9	0	25.04	10	0	25.05	0.01	8	0	25.05	0.02	2	0	25.26	0.87
12	12	0	29.75	12	0	29.75	0.00	8	0	29.84	0.31	2	0	30.27	1.74
14	14	0	34.36	14	0	34.36	0.00	8	0	34.63	0.78	2	0	35.27	2.63
17	17	0	41.15	17	0	41.15	0.00	8	0	41.82	1.62	8	1	43.90	6.69
20	20	0	47.79	20	0	47.79	0.00	8	0	49.00	2.54	8	1	51.48	7.71
24	24	0	56.45	24	0	56.45	0.00	8	0	58.59	3.78	8	1	61.58	9.08
27	27	0	62.82	27	0	62.82	0.00	8	0	65.77	4.70	8	1	69.15	10.08
38	38	0	85.51	32	0	86.05	0.63	8	0	92.12	7.73	8	1	96.93	13.35
49	49	0	107.51	32	0	109.53	1.88	8	0	118.48	10.20	8	1	124.71	16.00
55	55	0	119.34	32	0	122.33	2.51	8	0	132.85	11.32	13	2	141.08	18.22
70	70	0	148.63	32	0	154.34	3.84	8	0	168.78	13.56	13	2	179.13	20.52
85	85	0	177.78	32	0	186.35	4.82	8	0	204.72	15.15	13	2	217.18	22.16
100	100	0	206.92	32	0	218.35	5.52	32	1	241.29	16.61	20	3	256.32	23.87
120	110	0	245.84	32	0	261.03	6.18	32	1	288.56	17.38	20	3	307.11	24.92
140	115	0	284.72	32	0	303.71	6.67	32	1	335.84	17.95	20	3	357.90	25.70
160	119	0	323.58	32	0	346.39	7.05	32	1	383.12	18.40	32	5	410.42	26.84
215	125	0	430.38	32	0	463.75	7.75	32	1	513.14	19.23	32	5	550.20	27.84
270	129	0	537.14	32	0	581.11	8.19	32	1	643.15	19.74	32	5	689.98	28.46
300	130	0	595.36	32	0	645.13	8.36	50	2	727.57	22.21	50	7	768.42	29.07
390	133	0	770.01	32	0	837.18	8.72	50	2	943.58	22.54	50	7	997.17	29.50
480	135	0	944.65	32	0	1029.22	8.95	50	2	1159.59	22.75	50	7	1225.93	29.78
550	136	0	1080.48	125	1	1100.25	1.83	80	3	1318.50	22.03	80	10	1407.45	30.26
850	210	1	1656.39	125	1	1688.31	1.93	80	3	2030.73	22.60	80	10	2169.99	31.01
1150	282	2	2230.68	125	1	2276.37	2.05	80	3	2742.97	22.97	80	10	2932.54	31.46
1400	284	2	2706.07	125	1	2766.42	2.23	125	5	3376.83	24.79	125	14	3573.36	32.05
2200	360	3	4224.20	125	1	4334.57	2.61	125	5	5295.26	25.36	125	14	5606.84	32.73
3000	434	4	5737.07	125	1	5902.73	2.89	125	5	7213.68	25.74	125	14	7640.32	33.17
3500	435	4	6680.70	200	2	6780.08	1.49	200	7	8204.09	22.80	200	21	8920.18	33.52
6600	657	7	12514.92	200	2	12751.87	1.89	200	7	15439.61	23.37	200	21	16799.98	34.24
9700	733	8	18332.68	200	2	18723.66	2.13	200	7	22675.13	23.69	200	21	24679.79	34.62
12500	881	10	23580.15	315	3	23768.10	0.80	315	10	28356.85	20.26	200	21	31797.03	34.85
22500	1179	14	42290.47	315	3	42736.98	1.06	315	10	50993.25	20.58	200	21	57215.76	35.29
32500	1402	17	60974.69	315	3	61705.85	1.20	315	10	73629.66	20.75	200	21	82634.49	35.52
40000	1551	19	74978.07	500	5	75421.95	0.59	500	14	86149.54	14.90	200	21	101698.53	35.64
92500	2371	30	172890.15	500	5	174294.36	0.81	500	14	199073.97	15.14	200	21	235146.85	36.01
145000	2967	38	270711.73	500	5	273166.78	0.91	500	14	311998.39	15.25	200	21	368595.16	36.16
160000	3116	40	298651.90	800	7	301217.20	0.86	800	21	332614.49	11.37	200	21	406723.25	36.19

TABLE 9

Results for Case 4.2

	Optimum values		*AQL*	0.40%				1.50%				6.50%			
N	*n**	*c**	*K*(*n**,*c**)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)
3	3	0	7.55	3	0	7.55	0.00	3	0	7.55	0.00	2	0	7.56	0.22
5	5	0	12.13	5	0	12.13	0.00	5	0	12.13	0.00	2	0	12.44	2.55
7	7	0	16.40	7	0	16.40	0.00	7	0	16.40	0.00	2	0	17.32	5.63
10	10	0	22.25	10	0	22.25	0.00	8	0	22.75	2.22	2	0	24.63	10.70
12	12	0	25.83	12	0	25.83	0.00	8	0	27.08	4.81	2	0	29.51	14.23
14	14	0	29.18	14	0	29.18	0.00	8	0	31.41	7.62	2	0	34.39	17.83
17	17	0	33.82	17	0	33.82	0.00	8	0	37.90	12.06	8	1	43.50	28.61
20	20	0	38.05	20	0	38.05	0.00	8	0	44.40	16.68	8	1	51.01	34.04
24	24	0	43.15	24	0	43.15	0.00	8	0	53.06	22.97	8	1	61.01	41.39
27	27	0	46.63	27	0	46.63	0.00	8	0	59.56	27.73	8	1	68.52	46.94
38	38	0	57.49	32	0	60.68	5.55	8	0	83.38	45.03	8	1	96.03	67.04
49	49	0	66.41	32	0	76.81	15.66	8	0	107.20	61.41	8	1	123.55	86.04
55	55	0	70.80	32	0	85.61	20.92	8	0	120.19	69.76	13	2	140.70	98.73
70	70	0	81.01	32	0	107.60	32.82	8	0	152.67	88.46	13	2	178.65	120.53
85	85	0	90.80	32	0	129.59	42.73	8	0	185.15	103.92	13	2	216.60	138.56
100	100	0	100.59	32	0	151.58	50.70	32	1	215.51	114.25	20	3	256.00	154.50
120	120	0	113.94	32	0	180.91	58.77	32	1	257.64	126.11	20	3	306.72	169.19
140	140	0	127.72	32	0	210.23	64.60	32	1	299.76	134.69	20	3	357.45	179.86
160	160	0	141.90	32	0	239.56	68.82	32	1	341.89	140.94	32	5	410.34	189.18
215	215	0	182.38	32	0	320.20	75.57	32	1	457.73	150.98	32	5	550.09	201.63
270	270	0	224.20	32	0	400.84	78.79	32	1	573.57	155.83	32	5	689.85	207.70
300	300	0	247.34	32	0	444.82	79.85	50	2	668.70	170.36	50	7	768.35	210.65
390	347	0	317.51	32	0	576.78	81.66	50	2	867.04	173.08	50	7	997.09	214.04
480	318	0	387.84	32	0	708.74	82.74	50	2	1065.39	174.70	50	7	1225.82	216.06
550	309	0	442.51	125	1	582.79	31.70	80	3	1193.17	169.64	80	10	1407.41	218.05
850	294	0	676.74	125	1	888.60	31.31	80	3	1837.05	171.46	80	10	2169.93	220.65
1150	289	0	910.92	125	1	1194.41	31.12	80	3	2480.92	172.35	80	10	2932.45	221.92
1400	417	1	1105.27	125	1	1449.26	31.12	125	5	3111.59	181.52	125	14	3573.32	223.30
2200	411	1	1720.28	125	1	2264.75	31.65	125	5	4878.45	183.59	125	14	5606.78	225.92
3000	539	2	2330.36	125	1	3080.25	32.18	125	5	6645.31	185.16	125	14	7640.24	227.86
3500	538	2	2711.17	200	2	3347.00	23.45	200	7	7251.39	167.46	200	21	8920.17	229.02
6600	798	4	5054.26	200	2	6278.06	24.21	200	7	13643.09	169.93	200	21	16799.97	232.39
9700	1061	6	7385.58	200	2	9209.12	24.69	200	7	20034.80	171.27	200	21	24679.76	234.16
12500	1193	7	9484.89	315	3	10670.16	12.50	315	10	23820.47	151.14	200	21	31797.00	235.24
22500	1455	9	16958.51	315	3	19160.68	12.99	315	10	42827.77	152.54	200	21	57215.70	237.39
32500	1852	12	24411.75	315	3	27651.20	13.27	315	10	61835.08	153.30	200	21	82634.40	238.50
40000	1983	13	29994.18	500	5	32543.16	8.50	500	14	65608.28	118.74	200	21	101698.42	239.06
92500	3040	21	68984.31	500	5	75137.16	8.92	500	14	151572.30	119.72	200	21	235146.59	240.87
145000	3832	27	107904.06	500	5	117731.16	9.11	500	14	237536.32	120.14	200	21	368594.76	241.59
160000	3964	28	119017.23	800	7	123130.49	3.46	800	21	234692.00	97.19	200	21	406722.81	241.73

TABLE 10

Results for Case 4.3

	Optimum values AQL			0.40%				1.50%				6.50%			
N	*n**	*c**	*K*(*n**,*c**)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)	*n*	*c*	*K*(*n*,*c*)	*ΔK* (%)
3	3	0	2.95	3	0	2.95	0.00	3	0	2.95	0.00	2	0	4.42	49.57
5	5	0	4.87	5	0	4.87	0.00	5	0	4.87	0.00	2	0	9.29	90.68
7	7	0	6.76	7	0	6.76	0.00	7	0	6.76	0.00	2	0	14.17	109.60
10	10	0	9.53	10	0	9.53	0.00	8	0	12.02	26.12	2	0	21.49	125.38
12	12	0	11.35	12	0	11.35	0.00	8	0	16.35	44.12	2	0	26.37	132.34
14	14	0	13.14	14	0	13.14	0.00	8	0	20.69	57.46	2	0	31.24	137.82
17	17	0	15.78	17	0	15.78	0.00	8	0	27.18	72.27	8	1	30.48	93.18
20	20	0	18.38	20	0	18.38	0.00	8	0	33.68	83.26	8	1	37.99	106.71
24	24	0	21.78	24	0	21.78	0.00	8	0	42.34	94.37	8	1	47.99	120.32
27	27	0	24.30	27	0	24.30	0.00	8	0	48.84	100.96	8	1	55.50	128.38
38	38	0	33.33	32	0	37.24	11.72	8	0	72.66	117.99	8	1	83.01	149.07
49	49	0	42.15	32	0	53.36	26.60	8	0	96.47	128.88	8	1	110.53	162.23
55	55	0	46.91	32	0	62.16	32.51	8	0	109.47	133.35	13	2	119.24	154.19
70	70	0	58.73	32	0	84.15	43.29	8	0	141.95	141.70	13	2	157.19	167.66
85	85	0	70.50	32	0	106.15	50.56	8	0	174.43	147.41	13	2	195.14	176.79
100	100	0	82.28	32	0	128.14	55.74	32	1	173.85	111.29	20	3	222.89	170.90
120	120	0	98.01	32	0	157.46	60.66	32	1	215.97	120.36	20	3	273.61	179.17
140	140	0	113.78	32	0	186.79	64.16	32	1	258.10	126.83	20	3	324.34	185.05
160	160	0	129.60	32	0	216.11	66.75	32	1	300.22	131.65	32	5	357.24	175.64
215	192	0	173.11	32	0	296.75	71.42	32	1	416.06	140.34	32	5	497.00	187.09
270	208	0	216.36	32	0	377.39	74.43	32	1	531.90	145.84	32	5	636.75	194.30
300	215	0	239.89	32	0	421.38	75.65	50	2	599.33	149.83	50	7	685.37	185.70
390	230	0	310.36	32	0	553.33	78.29	50	2	797.67	157.02	50	7	914.10	194.53
480	239	0	380.72	32	0	685.29	80.00	50	2	996.02	161.61	50	7	1142.84	200.18
550	244	0	435.42	125	1	538.63	23.70	80	3	1085.69	149.34	80	10	1274.61	192.73
850	355	1	667.31	125	1	844.44	26.54	80	3	1729.56	159.18	80	10	2037.14	205.28
1150	468	2	898.08	125	1	1150.25	28.08	80	3	2373.44	164.28	80	10	2799.66	211.74
1400	479	2	1088.72	125	1	1405.10	29.06	125	5	2936.98	169.76	125	14	3365.83	209.15
2200	609	3	1696.83	125	1	2220.59	30.87	125	5	4703.84	177.21	125	14	5399.29	218.20
3000	737	4	2301.93	125	1	3036.09	31.89	125	5	6470.70	181.10	125	14	7432.74	222.89
3500	745	4	2679.03	200	2	3287.47	22.71	200	7	6995.00	161.10	200	21	8588.17	220.57
6600	1129	7	5007.65	200	2	6218.53	24.18	200	7	13386.70	167.33	200	21	16467.97	228.86
9700	1270	8	7326.74	200	2	9149.59	24.88	200	7	19778.41	169.95	200	21	24347.76	232.31
12500	1528	10	9417.30	315	3	10605.69	12.62	315	10	23456.34	149.08	200	21	31465.00	234.12
22500	2057	14	16865.75	315	3	19096.21	13.22	315	10	42463.65	151.77	200	21	56883.70	237.27
32500	2454	17	24299.10	315	3	27586.72	13.53	315	10	61470.95	152.98	200	21	82302.40	238.71
40000	2718	19	29868.48	500	5	32457.58	8.67	500	14	65134.90	118.07	200	21	101366.42	239.38
92500	4042	29	68789.81	500	5	75051.59	9.10	500	14	151098.92	119.65	200	21	234814.59	241.35
145000	5098	37	107658.87	500	5	117645.59	9.28	500	14	237062.94	120.20	200	21	368262.76	242.06
160000	5362	39	118759.22	800	7	123039.21	3.60	800	21	234044.01	97.07	200	21	406390.81	242.20

SUMMARY AND FUTURE RESEARCH

In this study we extend the research of Nikolaidis and Nenes (2009) by formulating and examining numerically several quality cost functions, depending on how rejected lots are handled. We also consider rectifying inspection, but focus on scrapping, recycling and cannibalizing the nonconforming units of rejected batches, while we create variations according to who is responsible for handling the nonconforming units.

Naturally, in practice these are not the only options as far as rejected batches are concerned. Sometimes, when a batch is rejected (through the examination of a sample), then it is **not** submitted to rectifying inspection, but it is scrapped, recycled or cannibalized. In other cases, when a batch is rejected through single sampling, its units are sold at a discount or a secondary market at a reduced price. These are excellent cases for future research. The latter could also head towards a much more extensive numerical investigation and data mining, and/or real life applications and conclusions.

References

ANSI/ASQ Z1.4 (2008). *Sampling Procedures and Tables for Inspection by Attributes*.

Battini, D., Faccio, M., Persona, A., and Sgarbossa, F. (2012). Design of an Integrated Quality Assurance Strategy in Production Systems, *International Journal of Production Research*, 50(6), 1682-1701.

Chen, C.-H. (2012). Economic Design of a Combined Continuous Lot by Lot Acceptance Sampling Plan under the Quality Investment, *Applied Mechanics and Materials*, 145, 410-414.

Hald, A. (1981). *Statistical Theory of Sampling Inspection by Attributes*, Academic Press.

Hsu, L-F, and Hsu, J-T (2012). Economic Design of Acceptance Sampling Plans in a Two-Stage Supply Chain, *Advances in Decision Sciences*.

ISO 2859-10 (2006). *Sampling Procedures for Inspection by Attributes - Part 10: Introduction to the ISO 2859 Series of Standards for Sampling for Inspection by Attributes*.

MIL-STD-105E (1989). *Sampling Procedures and Tables for Inspection by Attributes*.

Montgomery, D.C. (2012). *Introduction to Statistical Quality Control*, 7th Edition, John Wiley.

Nikolaidis, Y., and Nenes, G. (2009). Economic Evaluation of ISO 2859 Acceptance Sampling Plans used with Rectifying Inspection of Rejected Lots, *Quality Engineering*, 21(1), 10-23.

Schilling, E.G., and Neubauer, D.V. (2009). *Acceptance Sampling in Quality Control*, 2nd Edition, CNC Press.

Stephens, K.S. (2001). *The Handbook of Applied Acceptance Sampling: Plans, Procedures and Principles*, ASQ Quality Press.